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MODELING AND STABILITY ANALYSIS OF DISTRIBUTED CYBER-PHYSICAL POWER SYSTEMS WITH TIME DELAY

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ABSTRACT

As an essential part of the energy system, the modern power system is a high-dimensional, hybrid nonlinear differential dynamic system, which is considered as a typical cyber-physical system(CPS). The time delay of the advanced information system is a significant concern in the stability analysis of cyber-physical power system (CPPS). However, the time-delayed stability of power systems under distributed control has not been studied from a CPS perspective yet. In this regard, we introduce a dynamic cyber-physical power system model based on the time-delayed differential algebraic equation (TDAE). To analyze the time-delayed stability of CPPS, a critical eigenvalue tracking method for solving the time-delay margin of CPPS based on Rekasius substitution is proposed. Case studies illustrate the validity of the proposed method.

Keywords: CPS, smart grids, distributed control, time-delayed stability, TDAE

1. INTRODUCTION

As an essential part of energy system, power system is a high-dimensional, hybrid nonlinear differential dynamic system that includes intermittent components such as synchronous generators and electric motors, and various intermittent operating components such as switches. The application of advanced information systems such as wide area measurement system (WAMS) and energy management system (EMS) in a smart grid make the modern power system become a cyber-physical system (CPS) with a strong coupling of its cyber side and physical side [1]. At the same time, the vulnerability of cyber-physical power system (CPPS) is superimposed because of the wide-area measurement and control interaction with the physical system and the consequences of cyber contingencies are often more catastrophic [2].

One of the most typical and common cyber contingencies is time delay. With the ever-expanding network scale of CPPS and the highly informative process of the power system, the time delay problem of wide-area measurement and control signals is increasingly valued. It has been proved that in certain scenarios, a slight time-delay of information systems can also lead to instability of the power system [3].

The existing researches on the stability analysis of power systems with time delay can be mainly divided into time-domain methods and frequency-domain methods.

The time-domain methods mainly based on the Lyapunov–Krasovskii's stability theorem and the Razumikhin's theorem [4]. However, these methods are conservative and a well-defined Lyapunov function is difficult to find. The frequency-domain methods mainly analyze the eigenvalues and root locus of power systems with time-delay to calculate the stability margin. An explicit infinitesimal generator discretization (EIGD) approach is presented in [5], which fully utilizes sparsity eigenvalue techniques. The Rekasius substitution is introduced in [6] for stability analysis of one-dimensional time-delayed linear time-invariant (LTI) systems to obtain an obvious simplified polynomial expression of characteristic equations instead of solving the transcendental equations.

With the development of renewable energy and multi-agent interaction, traditional centralized energy management cannot meet the requirements of modern power systems, and the future energy management system will gradually tend to be a "distributed autonomous and centralized coordination" architecture [7]. The time-delayed stability of the distributed CPPS highly depends on the cyber network and distributed control algorithm.

However, the existing studies mainly focus on the methods to solve the time-delayed differential algebraic equations (TDAEs) and calculate the time-delayed stability margin of power systems. These studies only consider the time delay of controllers in the component level or the time delay of physical variables, which lacks the detailed modeling of the cyber network and information flow of CPPS [8]. Moreover, the time-delayed stability of power systems under distributed control has not been studied yet.

Therefore, it is critical to combine the distributed control architecture with the physical grid so as to analyze the time-delayed stability of the distributed CPPS more accurately. To analyze the time-delayed stability of the distributed CPPS, the differential algebraic equations (DAE) model of distributed information flow of the CPPS should be established so as to match the dynamic model of the physical power system. Then based on the combined CPPS model, the time-delayed stability margin can be obtained by analyzing the stability of the TDAE model.

The rest of this paper is organized as follows. Section 2 motivates the problem set up and details the modeling of distributed CPPS. Section 3 analyzes the time-delayed stability of the proposed distributed CPPS. The simulation results are shown in Section 4 and followed by the conclusion in Section 5.

2. MODELING OF DISTRIBUTED CPPS

2.1 *Modeling of Distributed Information Flow*

A distributed communication network topology is expressed as $G=(\boldsymbol{v},\boldsymbol{e},\boldsymbol{A})$, where $\boldsymbol{v}=\{v_1,\ldots,v_N\}$ is a set of N nodes of the communication network, $\boldsymbol{e}\subseteq\boldsymbol{v}\times\boldsymbol{v}$ is a set of edges, $\boldsymbol{A}=[A_{ij}]_{n\times n}$ is a weighted adjacency matrix of the communication network.

All neighbor nodes directly connected to node v_i through the communication link can be represented by the set N_i, which can be represented as follows:

$$N_i=\left\{v_j\in\boldsymbol{v}:(v_i,v_j)\in\boldsymbol{e}\right\} \tag{1}$$

where $j\in N_i$ refers to a set of nodes adjacent to node v_i.

A distributed control framework for distributed CPPS is shown in Figure 1. The iteration process involves the specific implementation of the distributed algorithm. The general form is in the discrete time (DT) model [9], which can be expressed as the following iteration formula:

$$\boldsymbol{x}_i^c(k+1)=\boldsymbol{x}_i^c(k)+\varepsilon\boldsymbol{h}_i\left(\boldsymbol{x}_i^c(k)\right) \tag{2}$$

where step size $\varepsilon>0$ and $\boldsymbol{h}_i(k)$ refers to correction function of the distributed algorithm in the k_{th} iteration.

In the dynamic system, continuous-time (CT) agent evolves according to the following formula:

$$\dot{\boldsymbol{x}}_i^c(t)=\boldsymbol{h}_i\left(\boldsymbol{x}_i^c(t)\right) \tag{3}$$

Since the iteration of agent i need to collect the state of its neighboring nodes. Combining with the above measurement process, the dynamic model of the iteration process of agent i is expressed as follows:

$$\dot{\boldsymbol{x}}_i^c(t)=\boldsymbol{h}\left(\boldsymbol{x}_i^c(t),\boldsymbol{x}_{j\in N_i}^c(t-\tau_{ij})\right) \tag{4}$$

where τ_{ij} is time delay of link $(i,j)\in\boldsymbol{e}$.

2.2 *Modeling of Dynamic Power System*

The differential equation of dynamic swing equation model of generator i is expressed as follows [10]:

$$\begin{cases}\dot{\delta}_i=\omega_i-\omega_N\\ \dot{\omega}_i=\dfrac{1}{M_i}\left(-D_i(\omega_i-\omega_N)+P_{m,i}-P_{e,i}\right)\end{cases} \tag{5}$$

where δ_i and ω_i are phase angle and angular frequency of generator i, respectively, $\dot{\delta}_i$ and $\dot{\omega}_i$ represent the derivatives of δ_i and ω_i, respectively. ω_N refers to nominal angular frequency of the power system, $P_{m,i}$ is the mechanical power input at bus i and $P_{e,i}$ is electromagnetic power output at bus i.

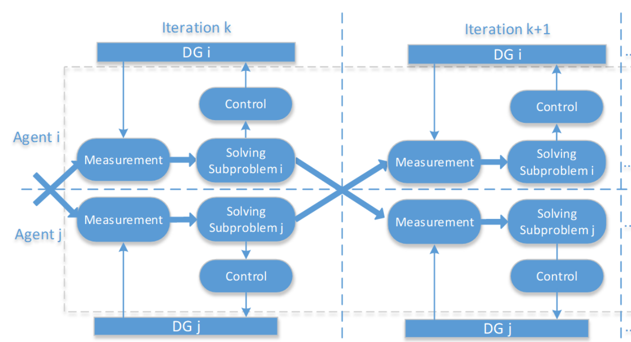

Fig 1 Discrete-Time Distributed Control Framework for CPPS

In this paper, we use DC power flow to calculate $P_{e,i}$, which is expressed as the following equations:

$$P_{e,i} = \sum_{j=1}^{N} B_{ij} \sin(\delta_i - \delta_j) + P_{d,i} \quad (6)$$

where $B_{ij} = -1 / x_{ij}$, x_{ij} is reactance of branch (i, j) of power systems and $P_{d,i}$ denotes the load demands at bus i.

Therefore, the differential algebraic equations of the physical power systems can be described in the form:

$$\begin{aligned} \dot{\boldsymbol{x}}^p &= \boldsymbol{f}(\boldsymbol{x}^p, \boldsymbol{y}^p, \boldsymbol{u}^p) \\ 0 &= \boldsymbol{g}(\boldsymbol{x}^p, \boldsymbol{y}^p, \boldsymbol{u}^p) \end{aligned} \quad (7)$$

where $\boldsymbol{x}^p$ denotes physical state variables, $\boldsymbol{y}^p$ is physical algebraic variables, $\boldsymbol{u}^p$ indicates control system parameters obtained from iterations of the distributed algorithm.

2.3 Modeling of Distributed CPPS

Considering the time-delay of communication with neighboring agents, combined with the distributed information flow model on the cyber side and dynamic power system model on the physical side and the distributed CPPS model is expressed as follows:

$$\begin{cases} \dot{\boldsymbol{x}}^p = \boldsymbol{f}(\boldsymbol{x}^p, \boldsymbol{y}^p, \boldsymbol{u}^p) \\ \dot{\boldsymbol{x}}^c = \boldsymbol{h}(\dot{\boldsymbol{x}}^c, \boldsymbol{x}^c_{N_{\boldsymbol{J}}, \tau_{ij}}) \\ 0 = \boldsymbol{g}(\boldsymbol{x}^p, \boldsymbol{y}^p, \boldsymbol{u}^p) \end{cases} \quad (8)$$

where $N_{\boldsymbol{J}}$ represents a subset of nodes $\boldsymbol{J} \subseteq v$ in the communications network.

Considering the time-delay of communication with neighboring agents, the distributed CPPS model is expressed as follows:

To make it more explicit, the above model is rewritten the following abstract form:

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}_\tau, \boldsymbol{y}_\tau) \\ 0 = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}) \\ 0 = \boldsymbol{g}_\tau(\boldsymbol{x}_\tau, \boldsymbol{y}_\tau) \end{cases} \quad (9)$$

where $\boldsymbol{x}$ contains $\boldsymbol{x}^p$ and $\boldsymbol{x}^c$, $\boldsymbol{y}$ contains $\boldsymbol{y}^p$, $\boldsymbol{u}^p$ and $\boldsymbol{z}^c$, $\boldsymbol{x}_\tau$ and $\boldsymbol{y}_\tau$ represent $\boldsymbol{x}(t-\tau)$ and $\boldsymbol{y}(t-\tau)$

3. TIME-DELAYED STABILITY ANALYSIS

The above distributed CPPS model with time-delay is a form of time-delayed differential algebraic equations (TDAEs). Compared with the traditional time-delayed power system model, the cyber side model is constructed more elaborately, especially the distributed information flow model for distributed control. The distributed CPPS model lays the foundation for the time-delayed stability analysis of distributed CPPS.

Analysis of the small disturbance stability of the distributed CPPS at the equilibrium point can give the necessary condition of time-delayed stability of the distributed CPPS. The equilibrium point is obtained from the following equations:

$$\begin{cases} 0 = \boldsymbol{f}(\boldsymbol{x}_e, \boldsymbol{y}_e, \boldsymbol{x}_{e\tau}, \boldsymbol{y}_{e\tau}) \\ 0 = \boldsymbol{g}(\boldsymbol{x}_e, \boldsymbol{y}_e) \\ 0 = \boldsymbol{g}_\tau(\boldsymbol{x}_{e\tau}, \boldsymbol{y}_{e\tau}) \end{cases} \quad (10)$$

Then, linearize the distributed CPPS model at the equilibrium point:

$$\begin{cases} \Delta\dot{\boldsymbol{x}} = \dfrac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\Delta\boldsymbol{x} + \dfrac{\partial \boldsymbol{f}}{\partial \boldsymbol{y}}\Delta\boldsymbol{y} + \dfrac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}_\tau}\Delta\boldsymbol{x}_\tau + \dfrac{\partial \boldsymbol{f}}{\partial \boldsymbol{y}_\tau}\Delta\boldsymbol{y}_\tau \\ 0 = \dfrac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}}\Delta\boldsymbol{x} + \dfrac{\partial \boldsymbol{g}}{\partial \boldsymbol{y}}\Delta\boldsymbol{y} \\ 0 = \dfrac{\partial \boldsymbol{g}_\tau}{\partial \boldsymbol{x}_\tau}\Delta\boldsymbol{x}_\tau + \dfrac{\partial \boldsymbol{g}_\tau}{\partial \boldsymbol{y}_\tau}\Delta\boldsymbol{y}_\tau \end{cases} \quad (11)$$

TABLE I

REKASIUS-SUBSTITUTION-BASED CRITICAL EIGENVALUE TRACKING METHOD

Step 1: Initialization. **Step 1.1:** Replacing the exponential term of the characteristic equation with Rekasius substitution: $e^{-\tau s}=(1-Ts)/(1+Ts)$ **Step 1.2:** Set the iteration step $\varepsilon \geq 0$ and $k = 1$. **Step 1.3:** Set a small enough T_0; Set a big enough T_N to ensure that there is at least one eigenvalue of the characteristic equation with positive real part.
Step 2: Iteration. $T_k = T_0 + k \cdot \varepsilon$.
Step 3: Solving the characteristic equation Solve the characteristic equation using Rekasius substitution with T_k: $\det(s \cdot \boldsymbol{I} - \tilde{\boldsymbol{A}} - \tilde{\boldsymbol{A}}_\tau \cdot (1-T_k s)/(1+T_k s))$, obtain the rightmost eigenvalue of the characteristic equation $\lambda_{\max}$.
Step 4: Solving the time-delayed stability margin. If $\mathrm{Re}(\lambda_{\max}) \geq 0$, calculate the time-delayed stability margin: $\tau_{mar} = 2/\omega\left[\tan^{-1}(\omega T_{mar}) \pm l\pi\right] \quad l = 0,1,2...$; Else $k = k + 1$ and return to Step 2.

Thus, the state space form of the delay differential algebraic equations is constructed as follows:

$$\begin{aligned} \Delta\dot{\boldsymbol{x}} &= \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\Delta\boldsymbol{x} - \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{y}} \cdot \left(\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{y}}\right)^{-1} \cdot \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}}\Delta\boldsymbol{x} \\ &\quad + \left(\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}_\tau} - \left(\frac{\partial \boldsymbol{g}_\tau}{\partial \boldsymbol{y}_\tau}\right)^{-1}\frac{\partial \boldsymbol{g}_\tau}{\partial \boldsymbol{x}_\tau}\right)\Delta\boldsymbol{x}_\tau \\ &= \tilde{\boldsymbol{A}} \cdot \Delta\boldsymbol{x} + \tilde{\boldsymbol{A}}_\tau \cdot \Delta\boldsymbol{x}_\tau \end{aligned} \quad (12)$$

Then, the characteristic equation of the distributed CPPS with time delay is expressed as follows:

$$\det(s \cdot \boldsymbol{I} - \tilde{\boldsymbol{A}} - \tilde{\boldsymbol{A}}_\tau \cdot e^{-s\tau}) \quad (13)$$

To analyze the time-delayed stability of the distributed CPPS, a critical eigenvalue tracking method based on Rekasius substitution is proposed, as illustrated in Table I.

4. NUMERICAL STUDIES AND DISCUSSIONS

In this section, we first use a single machine infinite bus system to verify the validity of the proposed Rekasius-substitution-based critical eigenvalue tracking method. Then, a three-bus system based on a fully distributed power dispatch method for frequency recovery [11] is also tested.

To illustrate the effectiveness of the proposed model and method in the distributed CPPS, we use a three-bus system with a subgradient-based fully distributed frequency control algorithm. The three-bus system is shown in Fig 2 and the test data can also be obtained from [12]. The bus-3 is considered as infinite bus with nominal frequency. And the phase angle of bus-3 δ_3 is set as $0°$. The black solid line is the transmission line on the physical side, and the grey dashed line denotes the communication links between agents.

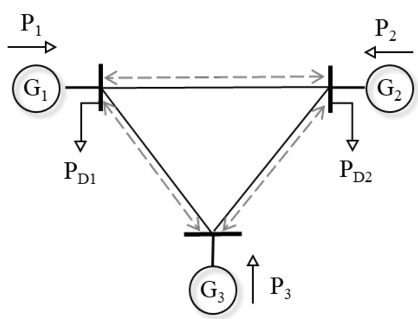

Fig 2 The Tested Three Bus System

The countinous model (CT) of subgradient-based fully distributed frequency control algorithm is expressed as follows:

$$\dot{\lambda}_i(t) = (1 - A_{ii}) \cdot \lambda_i(t) + \sum_{j \in N_i} A_{ij} \cdot \lambda_j(t) - \alpha(\omega_i(t) - \omega_N) \quad (14)$$

where λ_i denotes the cost increment rate of generator i , $\lambda_i = 2a_i P_{m,i} + b_i$, $A_{ij} \in \boldsymbol{A}$ is communication coefficient of subgradient algorithm, α is step size for frequency recovery. In this three bus case, all the elements of $\boldsymbol{A}$ is defined as 1/3.

The above algorithm can rapidly recover frequency with minimal regulating cost, which is very suitable for frequency recovery of low-inertia microgrids, especially a microgrid with multi-agents.

According to the above model, the differential variables $\boldsymbol{x}$ in this system is expressed as follows:

$$\boldsymbol{x} = [\dot{\delta}_1 \quad \dot{\omega}_1 \quad \dot{\lambda}_1 \quad \dot{\delta}_2 \quad \dot{\omega}_2 \quad \dot{\lambda}_2] \quad (15)$$

The equilibrium point of these DDAEs model is obtained according to formula (11):

$$\boldsymbol{x}_e = [-0.2150 \quad 1.0 \quad 6.5 \quad -0.0547 \quad 1.0 \quad 6.5] \quad (16)$$

In this paper, we assume that there is the same time-delay $\tau = \tau_{ij}, (i,j) \in \boldsymbol{e}$ of communication with neighboring agents in the iteration of the distributed algorithm. Thus the differential equations on the cyber is transferred to:

$$\begin{aligned} \dot{\lambda}_1 &= (1 - A_{11}) \cdot \lambda_1 + \sum_{j \in N_1} A_{1j} \cdot \lambda_j(t - \tau) - \alpha(\omega_1 - \omega_N) \\ \dot{\lambda}_2 &= (1 - A_{22}) \cdot \lambda_2 + \sum_{j \in N_2} A_{2j} \cdot \lambda_j(t - \tau) - \alpha(\omega_2 - \omega_N) \end{aligned} \quad (17)$$

Based on the proposed Rekasius-substitution-based critical eigenvalue tracking method, we can obtain the get the eigenvalues of the characteristic equation after Rekasius substitution in the critical stability state, as presented in Fig 3, where the critical eigenvalue refers to the rightmost real part eigenvalue.

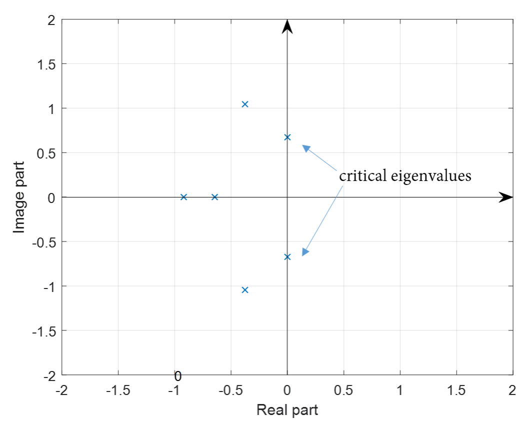

Fig 3 Eigenvalues of the Three-Bus System Considering Time Delay in the Critical Stability State

According to the proposed method, the time-delayed stability margin can be calculated when $T_{mar} = 0.1698$. Therefore, the time-delayed stability margin for this distributed CPPS is calculated by the proposed method as follows:

$$\tau_{mar} = 2 / \omega \left[\tan^{-1}(\omega \cdot T_{mar}) \right] = 0.3381\text{s} \quad (18)$$

The obtained time-delayed stability margin means that when the distributed control system has a time delay longer than 338.1ms, the stability of this distributed system cannot be guaranteed. Since the time-delayed stability margin is a necessary condition for system stability, even small disturbances at the equilibrium point can make the system unstable.

Fig 4 presents the time delay margin of the three bus system under different values of load demands at bus 1 and bus 2. It can be observed that the higher the load level of bus 1 and the lower the load level of bus 2, the larger the time delay allowed by the system.

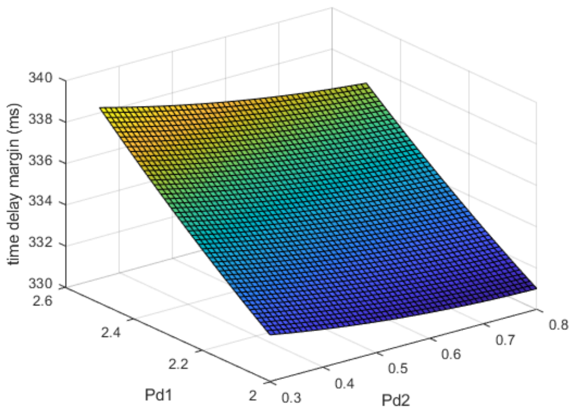

Fig 4 Time Delay Margin of the Tested System Under Different Values of Pd1 and Pd2

It is noteworthy that when the Rekasius-substitution is used to transfer the characteristic equation and the rightmost eigenvalue is on the imaginary axis, the calculation of time-delayed stability margin is accurate.

5. CONCLUSIONS

In this paper, considering the cyber-physical interdependence, we establish a distributed CPPS model with time-delayed based on time-delayed differential algebraic equations (TDAEs) of distributed information flow model on the cyber side and the dynamic power system model on the physical side. Then, a Rekasius-substitution-based critical eigenvalue tracking method is proposed for solving the time-delayed stability margin of the distributed CPPS. Case studies verify the validity of the proposed model and theoretical time-delayed stability analysis, which can give the necessary condition of the time-delayed stability of a distributed CPPS.

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