

# AFFINE ANALYSIS FOR UNCERTAINTY OF GAS LOAD IN INTEGRATED ELECTRICAL AND NATURAL-GAS SYSTEM

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## ABSTRACT

This paper is aimed at analyzing the influence of fluctuation by gas load in the integrated electrical and natural-gas system (IENGs). To address this issue, an affine method is proposed to calculate energy flow in IENGs considering uncertainties. In the distribution network, the power flow is solved by affine arithmetic based on forward-backward sweep method after obtaining the solutions of coupling units in the gas system. A numerical test, in which an IENGs is made up by a 13-bus distribution network and a 7-node gas system, shows the correctness and effectiveness of the proposed method. The results draw the conclusion that the fluctuation of gas load leads to the fluctuation of the whole IENGs.

**Keywords:** integrated electrical and natural-gas system, uncertainty, affine mathematics, energy flow

## NONMENCLATURE

$f_{mn}$	Gas flow from node $m$ to node $n$
$v_m$	Gas pressure at node $m$
$K_{mn}$	Flow factor of pipeline $m-n$
$i_{ij}$	Branch current between bus $i$ and bus $j$
$i_{lj}$	Injected load current of bus $j$
$U_i$	Voltage of bus $i$
$Z_{ij}$	Branch impedance between bus $i-j$
$\dot{S}_j^*, \dot{U}_j^*$	Conjugate values of injected power and voltage of bus $j$
$GHV$	Gross heating value
$\alpha_g, \beta_g, \gamma_g$	Heat rate coefficients of the gas-fired generator

$F_{gas}^m$	Gas flow of the gas-fired generator of node $m$
$P_{G,i}$	Power output of gas-fired generator at bus $i$
$\hat{x}$	Variable $x$ in the affine form
$[x]$	Variable $x$ in the interval form

## 1. INTRODUCTION

The Integrated energy system (IES), which is regarded as a popularized form of energy utilization, is composed of energy supply network and terminal energy-consuming system [1]. IES can not only supply multi-energy and high reliability of energy utilization for users, but also improve energy efficiency.

There are numerous forms of IES, among which integrated electrical and natural-gas system (IENGs) is a common form of energy supply and utilization. It is valuable to analyze the mutual impacts by both of the two subsystems. And energy flow analysis is one of the most important analytical tools for IENGs. However, the study of energy flow for IENGs is more complicated compared by a single energy system such as power system or natural-gas system, as there are multiple energy coupling links between the two subsystems of IENGs.

Several works have investigated energy flow calculation [2-5]. However, since there exists the coupling between the power system and gas system in IENGs, uncertain factors such as fluctuation of power generation, electrical load and gas load will make the operation of the whole system more sophisticated. Hence, it is an attention-worth problem that how the IENGs with uncertainties is analyzed. So as the IENGs is developed, it is urgent to study how to solve its uncertain energy flow.

This paper is aimed at analyzing the influence of fluctuation by gas load in the IENGs. After the gas flow is calculated in the low-pressure gas system considering the fluctuation of gas load and the results of coupling units are delivered to the distribution network, the power flow is calculated by the affine arithmetic based on forward-backward sweep method. In the numerous case, the proposed method proves to be correct with an IENGs consisting of 13-bus distribution network and 7-node natural-gas system. And the impact of the fluctuation by gas load on IENGs is studied.

The remainder of this paper is organized as follows. Section 2 introduces the model of IENGs. The affine method is proposed in Section 3 to analyze energy flow for IENGs, and Section 4 substantiates the proposed method by a case study and analyses the influence of uncertainty in the power subsystem. The work of this paper is summarized in Section 5.

## 2. IENGs MODEL

### 2.1 Natural-gas system model

In a gas system, a great number of gas pipelines make up a tight gas network. [2] gives a detailed model to calculate the gas flow of pipelines. This paper neglects the effects of factors such as temperature and topography in pipelines, and gas equation of low-pressure pipeline can be formulated by:

$$K_{mn}f_{mn}^2 = v_m - v_n \quad (1)$$

where  $f_{mn}$  is the gas flow passing through nodes from  $m$  to  $n$ .  $v_m$  and  $v_n$  are the gas pressure at nodes  $m$  and  $n$  respectively.  $K_{mn}$  is a parameter which is related to gas pipelines and influenced by various factors, such as the length and inner diameter of pipelines, the friction coefficient of natural gas flowing in the pipeline, and the natural gas temperature.

### 2.2 Power system model

The forward-backward sweep method can be used to analyze the power flow for radial distribution network. The forward-backward sweep model can be denoted as:

$$I_{ij} = I_{Lj} + \sum_{k \in d} I_k = \frac{\dot{S}_j^*}{\dot{U}_j^*} + \sum_{k \in d} I_k \quad (2)$$

$$\dot{U}_j = \dot{U}_i - I_{ij}Z_{ij} \quad (3)$$

where  $I_{ij}$  is the branch current between bus  $i$  and bus  $j$ ,  $I_{Lj}$  is the injected load current of bus  $j$ ,  $I_k$  is the

branch current that takes bus  $j$  as the head bus,  $\dot{S}_j^*$  and  $\dot{U}_j^*$  are the conjugate values of injected power and voltage of bus  $j$ ,  $\dot{U}_i$  is the voltage of bus  $i$ , and  $Z_{ij}$  is the branch impedance between bus  $i$  and bus  $j$ .

### 2.3 Coupling unit model

Gas-fired generators, combined heating and power units and P2Gs (power to gas) are relatively common as coupling units which combine the power system and gas system. In this paper, the gas-fired generators are selected as coupling units, whose relation between input gas and output electricity can be expressed as:

$$F_{gas}^m = \frac{1}{GHV}(\alpha_g + \beta_g P_{G,i} + \gamma_g P_{G,i}^2) \quad (4)$$

where  $P_{G,i}$  is the power output of gas-fired generator at bus  $i$  in the power system, and  $F_{gas}^m$  is the gas flow of the gas-fired generator of node  $m$  in the gas system, and  $GHV$  is gross heating value, and  $\alpha_g$ ,  $\beta_g$  and  $\gamma_g$  are heat rate coefficients of the gas-fired generator.

## 3. AFFINE METHOD FOR ENERGY FLOW IN IENGs

### 3.1 Affine arithmetic

There exist uncertainties like fluctuation of loads in IENGs when the system is operating and it is difficult to obtain the probability distribution function of loads. Affine arithmetic provides a proper tool to calculate uncertain quantities. The second-order formulation of an affine quantity  $\hat{x}$  can be denoted as:

$$\hat{x} = x_0 + x_1 \varepsilon_1 \quad (5)$$

where  $x_0$  and  $x_1$  are the center value and partial deviation of  $\hat{x}$ , and  $\varepsilon_1$  is the noisy symbol that lies in the interval  $[-1,1]$ .

An affine quantity can be converted to an interval quantity. That is to say, for an affine quantity  $\hat{x}$  and an interval quantity  $[x]$  whose upper bound and lower bound are  $\bar{x}$  and  $\underline{x}$ , the converting formula can be denoted as:

$$[x] = [\underline{x}, \bar{x}] = [x_0 - x_1, x_0 + x_1] \quad (6)$$

Arithmetic operations for affine arithmetic can be found in [6].

### 3.2 Affine method for energy flow in IENGs

The affine solution process of energy flow in IENGs considering uncertainty of gas load can be summarized in four steps.

(1) parameters initialization

In this step, the injected flow in the gas system and the apparent power in the distribution are initialized in affine forms.

#### (2) gas flow calculation

Generally, the deterministic gas flow in the low-pressure can be solved by Newton nodal method. This method cannot be applied to affine gas flow directly since the Jacobi matrix in the affine cannot be calculated, either. To use the method to get the results, the technique [8] to calculate the inverse matrix of Jacobi in the affine form is utilized. Then the power output of gas-fired generators at the coupling bus are delivered to the distribution network.

#### (3) affine power flow calculation based on forward-backward sweep method

The injected current of bus  $j$  in affine form  $\hat{I}_j$  can be calculated via (7):

$$\hat{I}_j = (\hat{S}_j / \hat{U}_j)^* \quad (7)$$

Then from the branch at the end of the distribution network, calculate the initial affine current at each branch by the injected current  $\hat{I}_j$  and Kirchhoff's Law, which can be expressed as:

$$\hat{I}_{ij} = \hat{I}_j + \sum_{k \in d} \hat{I}_k = \frac{\hat{S}_j^*}{\hat{U}_j^*} + \sum_{k \in d} \hat{I}_k \quad (8)$$

The following process is that from the starting point to the terminal point, update affine voltage  $\hat{U}_j$  at the endpoint of each branch by voltage at the slack bus which has been set. This procedure can be formulated via (9):

$$\hat{U}_j = \hat{U}_i - \hat{I}_{ij} Z_{ij} \quad (9)$$

#### (4) convergence criterion

Convert affine quantities  $\hat{U}_i^k$  (the real part  $\text{Re}(\hat{U}_i^k)$  and the imaginary part  $\text{Im}(\hat{U}_i^k)$ ) and  $\hat{V}_i^k$  of the  $k$ th iteration to interval quantities  $[U_i^k]$ ,  $[V_i^k]$ . If the upper bound and lower bound of voltage and pressure are less than the allowable value  $\varepsilon_{err}$  compared to ones of the last iteration via (10), output the results. Or jump back to (2) to continue the iteration.

$$\begin{cases} \max(|\text{Re}(\bar{U}_i^k) - \text{Re}(\bar{U}_i^{k-1})|, |\text{Re}(\underline{U}_i^k) - \text{Re}(\underline{U}_i^{k-1})|) < \varepsilon_{err} \\ \max(|\text{Im}(\bar{U}_i^k) - \text{Im}(\bar{U}_i^{k-1})|, |\text{Im}(\underline{U}_i^k) - \text{Im}(\underline{U}_i^{k-1})|) < \varepsilon_{err} \\ \max(|\bar{V}_i^k - \bar{V}_i^{k-1}|, |\underline{V}_i^k - \underline{V}_i^{k-1}|) < \varepsilon_{err} \end{cases} \quad (10)$$

## 4. CASE STUDY

An IENGs consisting of a 13-bus distribution network and a 7-node gas system [7] as is shown in Fig 1. Bus

11 and bus 12 in the power system are connected to node 6 and node 5 in the gas system with two gas-fired

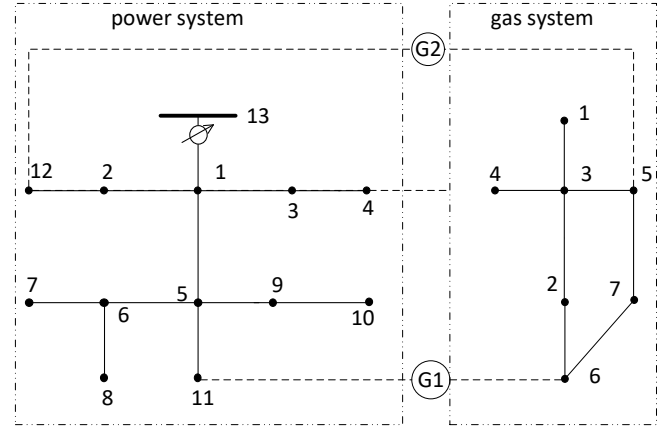


Fig 1 IENGs composed by 13-bus electricity system and 7-node gas system

generators, which are named as G1 and G2.

### 4.1 Effectiveness of the proposed method

Assume  $\pm 10\%$  fluctuation of gas load and electrical load. The proposed method is verified by comparison with Monte Carlo (MC) stochastic simulation which randomly samples fluctuation of electrical load and gas load for  $10^4$  trials to get maximum/minimum results. It is assumed that solutions of MC simulation are regarded as the real results if sufficient number of samples are applied. Given space limitations, this paper only shows the power output of couplings (G1 and G2) in Fig.2 and the real part of voltage in Fig.3, whose results are obtained by the proposed method and MC simulation.

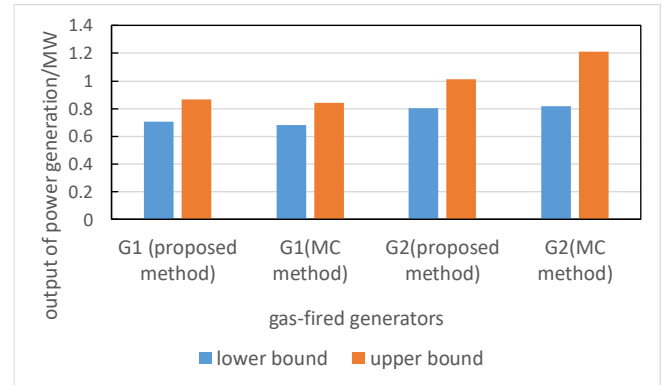


Fig 2 Output of power of gas-fired generators

It can be seen that the results obtained by MC simulation are just included by the ones obtained by the proposed method. That is to say, the range of the proposed method is a little wider than the range of the MC simulation, which illustrates the correctness and completeness of the proposed method. Although the

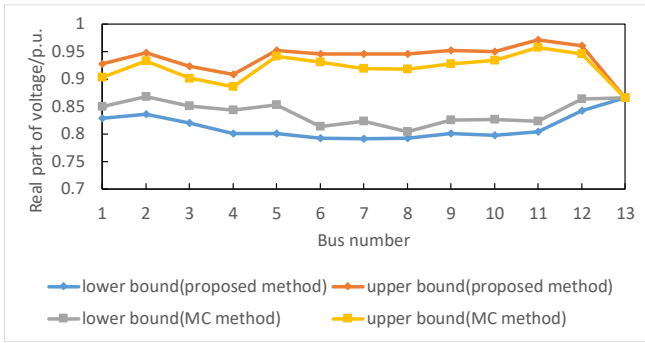


Fig 3 Real part of voltage

uncertain energy flow can be calculated by the proposed method and MC simulation, the solutions can be computed just one time by the proposed method, while the MC simulation takes many times to compute the results.

#### 4.2 Impact of gas load change on IENGs

To analyze the impacts on IENGs by the fluctuation of gas load, assume  $\pm 10\%$  and  $\pm 20\%$  fluctuation of gas load, while the  $\pm 10\%$  fluctuation of electrical load remains unchanged. The real part of voltage are shown in Fig.4.

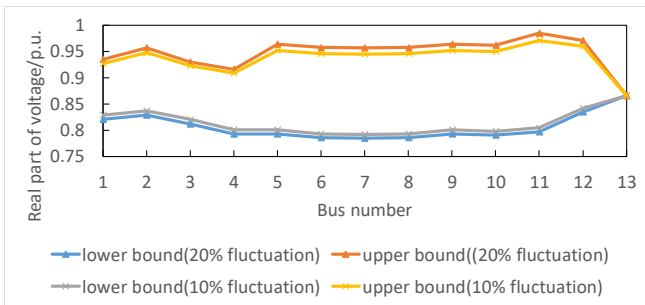


Fig 4 Real part of voltage

First of all, it can be seen that when the fluctuation of gas load changes, voltage will change. This means the uncertainties produced by the fluctuation of gas load in gas system are delivered to gas system, making voltage of buses fluctuate. Since the distribution network is tightly connected to the gas system by the coupling units, i.e. gas-fired generators. The uncertainties will be delivered through coupling units. Besides, when the fluctuation level of gas increases, the fluctuation of the whole system is getting larger. So the uncertainty brought by gas load makes the IENGs operate with uncertainty. In other words, if we want to mitigate the uncertainty in the power system, we should not only reduce the fluctuation of electrical load, but also take the uncertainty delivered by the gas system into consideration.

## 5. CONCLUSION

This paper proposes an affine arithmetic-based method to analyze the influence by fluctuation of gas load for IENGs which is composed of distribution network and low-pressure gas system. An IENGs which contains a 13-bus distribution network coupled with a 7-node low-pressure gas system shows that the proposed method can be effectively applied to analyze the uncertainty brought by fluctuation of gas load.

## REFERENCE

- [1] OMalley M, Kroposki B. Energy comes together: the integration of all systems. IEEE Power and Energy Magazine 2013;11:18-23.
- [2] A. Martinez-Mares, C. R. Fuerte-Esquivel. A unified gas and power flow analysis in natural gas and electricity coupled networks. IEEE Transactions on Power Systems 2012;27: 2156-2166.
- [3] S. Chen, Z. Wei, G. Sun, Y. Sun, H. Zang, Y. Zhu. Optimal power and gas flow with a limited number of control actions. IEEE Transactions on Smart Grid 2018; 9:5371-5380.
- [4] Qing Zeng, Jiakun Fang, Jinghua Li, Zhe Chen. Steady-state analysis of the integrated natural gas and electric power system with bi-directional energy conversion. Applied Energy 2016;184:1483-1492.
- [5] Burcin Cakir Erdener, Kwabena A. Pambour, Ricardo Bolado Lavin, Berna Dengiz. An integrated simulation model for analysing electricity and gas systems. International Journal of Electrical Power & Energy Systems 2014;61:410-420.
- [6] A. Vaccaro, C. A. Canizares, D. Villacci. An affine arithmetic-based methodology for reliable power flow analysis in the presence of data uncertainty. IEEE Transactions on Power Systems 2010;25: 624-632.
- [7] Wang Yingrui, Zeng Bo, Guo Jing, Shi Jiaqi, Zhang Jianhua. Multi-energy flow calculation method for integrated energy system containing electricity, heat and gas. Power System Technology 2016;40: 2942-2951.
- [8] D. Degrauwe, G. Lombaert, G. De Roeck. Improving interval analysis in finite element calculations by means of affine arithmetic. Computers & Structures 2010;88:247-254.