# TWO-STAGE ROBUST ALLOCATION MODEL OF SOLAR ENERGY EQUIPMENTS IN DISTRICT INTEGRATED ENERGY SYSTEMS

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## ABSTRACT

Large scale utilization of solar energy has become an inevitable trend of an energy-efficient and environment-friendly society. A two-stage robust allocation model of solar energy equipments in district integrated energy systems is proposed in this paper with the uncertainty of solar irradiance and operating constraints of energy networks. To improve the solvability, the above non-convex non-linear model is converted to a 0-1 mixed integer second-order cone problem. The validity of the model is verified by typical cases.

**Keywords:** solar irradiance uncertainty, two-stage robust optimization, allocation of solar energy equipment

## NOMENCLATURE

Abbreviations	
DIES PV SC CHP EB GB	District Integrated Energy System Photovoltaic Solar Collector Combined Heat and Power Electrical Boiler Gas Boiler
Symbols	
$Cap_{PV}, Cap_{SC}$ $P_{CHP}^{t}, P_{PV}^{t}$ $H_{GB}^{t}, H_{EB}^{t},$ $H_{SC}^{t}$	Capacity of PVs/SCs Power Output of CHP Units/PV Thermal Output of GB/EB/SC

$P_{grid,p}^{r}$ , $P_{grid,s}^{r}$	Power Purchased/Sold from/to Grid				
$u^t$ , $i^t$	Square of Voltage/Current Amplitude				
$P_{ij}^{t}$ , $Q_{ij}^{t}$	Active/Reactive Power of branch ij				
$P_i^t$ , $Q_i^t$	Active/Reactive Power of node <i>i</i>				
$H_i^t$	Thermal Power injected into node i				
${H}_{ij}^{\prime\prime}$ , $\Delta {H}_{ij}^{\prime\prime}$	Thermal Power/Loss of Pipeline ij				
$e_{grid}^{t}$	State of Transaction (Binary Variable)				
$P_{\mathit{load},\mathit{i}}^{\mathit{t}}$ , $Q_{\mathit{load},\mathit{i}}^{\mathit{t}}$	Active Load and Reactive Load				
$H^{t}_{load,i}$	Thermal Load				
$Q^{t}_{grid,i}$	Reactive Power from Grid				
$A_{_{PV,i}}$ , $A_{_{SC,i}}$	Installation Area of PV/SC				
$c_{ins,PV}$ , $c_{ins,SC}$	Investment Price of PV/SC				
C <sub>ins,PV</sub> , C <sub>ins,SC</sub> C <sub>fuel</sub>	Investment Price of PV/SC Purchase Price of Fuel				
$\mathcal{C}_{ins,PV}$ , $\mathcal{C}_{ins,SC}$ $\mathcal{C}_{fuel}$ $lpha^t$ , $eta^t$	Investment Price of PV/SC Purchase Price of Fuel Electricity Purchase/Sale Price				
$\mathcal{C}_{ins,PV}$ , $\mathcal{C}_{ins,SC}$ $\mathcal{C}_{fuel}$ $lpha^t$ , $eta^t$ $\mathcal{Q}_{fuel}$	Investment Price of PV/SC Purchase Price of Fuel Electricity Purchase/Sale Price Heating Value of Fuel				
$C_{ins,PV}$ , $C_{ins,SC}$ $C_{fuel}$ $\alpha^{t}$ , $\beta^{t}$ $q_{fuel}$ $r_{CR,i}$	Investment Price of PV/SC Purchase Price of Fuel Electricity Purchase/Sale Price Heating Value of Fuel Capital Recovery Factor				
$C_{ins,PV}$ , $C_{ins,SC}$ $C_{fuel}$ $\alpha^{t}$ , $\beta^{t}$ $q_{fuel}$ $\tilde{r}_{CR,i}$ $\tilde{I}_{i}$ , $I_{s}$	Investment Price of PV/SC Purchase Price of Fuel Electricity Purchase/Sale Price Heating Value of Fuel Capital Recovery Factor Actual/Standard Irradiance Intensity				

## 1. INTRODUCTION

Reasonable allocation of photovoltaics (PVs) and solar collectors (SCs) in the district integrated energy systems (DIESs) is beneficial to improving comprehensive accommodation of renewable energy and reducing energy consumption cost. While the fluctuation of the solar irradiance and energy networks have influence on the allocation of solar energy equipments. Therefore, it is of significance to allocate solar energy equipments appropriately under the uncertain solar irradiance in DIESs with energy networks.

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The allocation of solar energy equipments has been studied all the time. Reference [1] proposed a multiobjective optimal allocation model for PV and solar water heaters, but didn't consider the uncertainty of solar energy. Reference [2] has established an optimal co-allocation model of solar power plants with thermal energy storage to improve the economic viability and the uncertainties of solar irradiation is characterized. While energy network was not considered in reference [1-2]. A method for optimal allocation of PV under the randomness of solar energy is proposed in reference [3], but the model is limited in electrical distribution networks without thermal energy and SCs. Both the thermal network and SCs are rarely mentioned in the DIESs planning studies.

According to the problems above, a two-stage robust economic allocation model of solar energy equipments like PVs and SCs in DIESs considering the uncertainty of solar irradiance and the operating constraints of energy networks is proposed in this paper. The model is converted into a 0-1 mixed integer second-order cone program, and the validity is verified by a typical case study containing district electrical and thermal networks. The results shows that the installation of SCs can decrease the total cost by reducing the operating cost, and when there are no allowance of electricity sale to grid, appropriate curtailment could reduce the total cost of DIES with SCs.

#### 2. DISTRICT INTEGRATED ENERGY SYSTEM MODEL

In this context, the electrical and thermal load of the DIES are supplied by the grid, solar energy, and natural gas through district electrical network and district heating network. The energy conversion equipments contains PVs, SCs, CHP units, GBs and EBs. The models of the constituent parts in DIESs are shown below.

#### 2.1 Equipment model

The general models of PVs and SCs are as follows:

$$P_{PV,i}^{t} \leq A_{PV,i} I_{i} \eta_{PV} = \frac{I_{i}}{I_{s}} Cap_{PV,i}$$
(1)

$$H_{SC,i}^{t} \leq A_{SC,i} I_{i} \eta_{SC} = \frac{I_{i}}{I_{s}} Cap_{SC,i}$$
<sup>(2)</sup>

Furthermore, installation area constraint of PVs and SCs must be considered if installed at the same place:

$$0 \le A_{PV,i} + A_{SC,i} \le A_i^{\max}$$
(3)

The other energy conversion equipments, CHP units, GBs and EBs, are constrained by linear models used in reference [4].

## 2.2 Transaction between the DIES and the grid

Supposing that the DIES operates in the gridconnected mode and has the right to purchase electricity from the grid or sell electricity to the grid at time t. The transaction can be constrained by the following ways:

$$0 \le P_{grid,p}^t \le e_{grid}^t P_{grid}^{\max}$$
(4)

$$0 \le P_{grid,s}^{t} \le (1 - e_{grid}^{t})P_{grid}^{\max}$$
<sup>(5)</sup>

## 2.3 Energy network model

## 2.3.1 District electrical network model

The power flow of district electrical network is generally modeled with the non-convex non-linear Distflow model. A Distflow second-order cone model proposed in reference [5] is used to avoid the NP-hard problem during the solving of model with min-max form:

$$\sum_{i \in \delta(j)} (P_{ij}^{t} - r_{ij} i_{ij}^{t}) + P_{j}^{t} = \sum_{k \in \xi(j)} P_{jk}^{t}$$
(6)

$$\sum_{i \in \delta(j)} (Q_{ij}^{t} - x_{ij} i_{ij}^{t}) + Q_{j}^{t} = \sum_{k \in \xi(j)} Q_{jk}^{t}$$
(7)

$$u_{j}^{t} = u_{i}^{t} - 2(r_{ij}P_{ij}^{t} + x_{ij}Q_{ij}^{t}) + (r_{ij}^{2} + x_{ij}^{2})i_{ij}^{t}$$
(8)

$$\|2P_{ij}, 2Q_{ij}, i_{ij}^{t} - u_{i}^{t}\|_{2} \le i_{ij}^{t} + u_{i}^{t}$$
(9)

$$\underline{U}^2 \le u_i^* \le U^2 \tag{10}$$

$$0 \le i_{ij}^{\tau} \le I^{-2} \tag{11}$$

Furthermore, the electrical power balance should be considered in DIESs at node i, time t:

$$P_{CHP,i}^{t} + P_{PV,i}^{t} + P_{grid,p,i}^{t} - P_{grid,s,i}^{t} - P_{EB,i}^{t} - P_{load,i}^{t} = P_{i}^{t}$$
(12)

$$Q_{grid,i}^{t} - Q_{load,i}^{t} = Q_{i}^{t}$$
(13)

# 2.3.2 District heating network model

The linear district heating network energy flow model proposed in reference [6] is used in this paper. The thermal power balance in DIESs is as follows:

$$H_{i}^{t} + \sum_{i \in I} H_{ij}^{\prime t} = 0$$
 (14)

$$H''_{ij} = -(H''_{ji} - \Delta H'_{ji})$$
(15)

$$H_{ij}^{\prime \min} \le H_{ij}^{\prime\prime} \le H_{ij}^{\prime \max}$$
 if  $H_{ij}^{\prime\prime} > 0$  (16)

$$H_{i}^{t} = H_{SC,i}^{t} + H_{CHP,i}^{t} + H_{GB,i}^{t} + H_{EB,i}^{t} - H_{load,i}^{t}$$
(17)

#### 2.4 The Uncertainty Model of Solar Energy

The output of PVs and SCs is uncertain because of the irradiance intensity with randomness and intermittence. The uncertainty can be described by a box uncertainty set  $\,U$ . The optimal results is generally obtained at the extreme of  $\,U$ , described as

$$\mathbf{U} := \begin{cases} u = u + \left(\theta^{+} - \theta^{-}\right) \Delta u = \left(\tilde{I}\right)^{\mathrm{T}} \\ \theta^{+} + \theta^{-} \leq 1 \\ \sum \left(\theta^{+} + \theta^{-}\right) = \Gamma \end{cases}$$
(18)

where  $\hat{u}$ ,  $\Delta u$  represent predicted value and the fluctuation range of irradiance intensity, respectively;  $\theta^+$ ,  $\theta^-$  are binary auxiliary variables;  $\Gamma$  is a integer, representing the total number of the uncertain variables.

# 3. TWO-STAGE ROBUST PLANNING MODEL FOR SOLAR ENERGY EQUIPMENTS IN DIES

The two-stage robust planning model is aimed to find the optimal allocation of solar energy equipments to minimize the total cost under the worst affection of the irradiance intensity. The objective function is shown as follows:

$$\min_{x} \{C_{ins} + \max_{u \in U} \min_{y \in \Omega(x,u)} [C_{fuel} + C_{grid}]\}$$
(19)

$$C_{ins} = \sum_{i=1}^{n_{PV}} r_{CR,i} c_{ins,PV} Cap_{PV,i} + \sum_{i=1}^{n_{SC}} r_{CR,i} c_{ins,SC} Cap_{SC,i}$$
(20)

$$C_{fuel} = \sum_{D=1}^{365} \sum_{t=1}^{24} c_{fuel} \left[ \sum_{i=1}^{n_{CHP}} \frac{P_{CHP,i}^{t}}{\eta_{CHP,i} q_{fuel}} + \sum_{i=1}^{n_{GB}} \frac{H_{GB,i}^{t}}{\eta_{GB,i} q_{fuel}} \right] \Delta t \quad (21)$$

$$C_{grid} = \sum_{D=1}^{365} \sum_{t=1}^{24} [\alpha^{t} P_{grid, p}^{t} - \beta^{t} P_{grid, s}^{t}] \Delta t$$
 (22)

where  $C_{ins}$  represents the annual cost of the installation of PVs and SCs;  $C_{fuel}$ ,  $C_{grid}$  represent the annual cost of the fuel consumed and the transaction between the DIES and the grid, respectively.

In Eq. (19), x and y are the design variables of the first-stage problem and the second-stage problem, respectively:

$$\begin{cases} x = [Cap_{PV}, Cap_{SC}, e_{grid}^{t}]^{T} \\ y = [P_{CHP}^{t}, H_{GB}^{t}, H_{EB}^{t}, P_{grid,p}^{t}, P_{grid,s}^{t}, P_{PV}^{t}, H_{SC}^{t}, \\ u^{t}, i^{t}, P_{ij}^{t}, Q_{ij}^{t}, P_{i}^{t}, Q_{i}^{t}, H_{ij}^{t}, H_{ij}^{t}, H_{ji}^{t}]^{T} \end{cases}$$
(23)

The constraints of the two-stage robust planning model for solar energy equipments in DIESs are Eqs. (1) - (18), and linear constraints of CHP units, GBs and EBs.

The two-stage robust planning model can be decomposed into a master problem and a sub-problem. The Big-M method is used and the binary variables are introduced to convert the model into a 0-1 mixed integer program, which can be solved by C&CG method with existing solvers, such as YALMIP and CPLEX [7].

## 4. CASE STUDIES

#### 4.1 Introduction

The case in this paper is modified from reference [8], including a district electrical network and a district heating network. The energy stations are all equipped with equipments mentioned in 2.1. The parameters of the case can be found in reference [8-10].

To verify the validity of the model, 6 scenarios are set as Tab. 1 to study the impact of solar curtailment, transaction from DIES to grid, and the installation of SCs on the optimal allocation of solar energy equipments in the DIESs.

Tab 1 Scenarios settings								
Scenario	Solar energy	Electricity	Solar energy					
	curtailment	sale to grid	equipments					
1	Allowed	Not allowed	PV+SC					
2	Not allowed	Not allowed	PV+SC					
3	Allowed	Allowed	PV+SC					
4	Allowed	Not allowed	PV					
5	Not allowed	Not allowed	PV					
6	Allowed	Allowed	PV					

The fluctuation of the irradiance intensity is set to 10%. Electricity purchase price is 0.5RMB/kWh.  $A_i^{max}$  in each station is 9000 m<sup>2</sup>.



a typical day

#### 4.2 Results

The results of 6 scenarios are shown in Tab. 2. The total electrical and thermal output of energy stations are compared in Fig. 1. The output of all energy stations in a typical day in 6 scenarios is shown in Fig. 2-6, respectively.

Tab 2 Part of the results								
Scenario	1	2	3	4	5	6		
Capacity of PVs (MW)	3.52	3.01	4.29	4.73	4.73	4.73		
Capacity of SCs (MW)	3.87	2.91	1.40	-	-	-		
Total cost (10 <sup>4</sup> RMB)	1.62	1.67	1.55	1.76	1.76	1.58		
Investment cost (10 <sup>3</sup> RMB)	4.00	3.23	3.25	2.82	2.82	2.82		
Operating cost (10 <sup>4</sup> RMB)	1.22	1.35	1.23	1.47	1.47	1.29		

## 5.2.1 Scenario 1

According to the Fig. 2, when there is no solar irradiance at night, the electrical and thermal load are mainly supplied by CHP units. During daytime, PVs and SCs play the major role of supplement. The power and thermal supply are assisted by grid and GBs respectively. EBs produce heat only when the output of PVs reaches the maximum and the electricity can't be accommodated. The reason is that annual investment cost is less than operating cost and the dispatch priority of equipments is determined by their operating cost and efficiency.

Furthermore, even though the curtailment of solar energy exists, the current allocation is economically optimal in scenario 1. Here comes the conclusion that appropriate curtailment could reduce the total cost of DIES.



Fig 2 Output and Curtailment of scenario 1 in a typical day

# 5.2.2 Scenario 2

When the curtailment is not allowed, the capacity of PVs and SCs reduce, and the total cost increases. Meanwhile, the capacity of SCs is affected by the output of PVs indirectly.

From the discussion above, reasonable curtailment could improve solar energy accommodation capability and reduce total cost of DIES. Furthermore, the total cost is less if installation area becomes larger.



Fig 3 Output and Curtailment of scenario 2 in a typical day

# 5.2.3 Scenario 3

The allowance of selling electricity to upstream power system eventuates a huge improvement of the capacity of PVs and a significant reduction of SCs. The allocation of solar energy equipments are totally different from scenario 1. There is no use of boilers and there is no solar energy curtailment. CHP units do the main work to supply, cooperating with PVs and SCs. The total cost reduces by 4.55% compared with scenario 1.



Fig 4 Output and Curtailment of scenario 3 in a typical day

# 5.2.4 Scenario 4-6

From Fig. 1 and Tab. 2 we can see that the existence of SCs can decrease the operating cost, especially the cost of supplying thermal load. Compared with scenarios 1-3, the output of CHPs changes little, but there is a significant decrease on output of GBs and EBs.



Fig 5 Output and Curtailment of scenario 4 and scenario 5 in a typical day

In scenarios 4-6, economical characteristic makes the full installation of PVs. In scenario 4, the redundant electricity from PVs will be supplied to EBs to generate thermal energy. So, there is no solar curtailment in scenarios 4. The allocation results of scenario 4 and 5 are same. In scenario 6, the redundant electricity from PVs is sold to grid instead of supplying EBs. So, there is no solar curtailment in scenarios 6 as well.



Fig 6 Output and Curtailment of scenario 6 in a typical day

## 5. CONCLUSIONS

A two-stage robust allocation model of solar energy equipments in DIES is proposed, taking the uncertainty of irradiance intensity and the operation constraints of energy networks into consideration. The impact of curtailment, electricity selling has been analyzed in a case study by 6 different scenarios. Reasonable curtailment can improve solar energy accommodation capability and reduce the total cost of DIES. Allowance of electricity sale to grid with upstream power system makes the allocation of solar energy equipments totally different. Most importantly, the SCs can make the system more economical, which should be encouraged to install for DIES. The energy storage systems will be considered in the future work and the solar energy accommodation capability of DIES will be studied as well.

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