A QP-based Torque Distribution Strategy for All-Wheel-Independent-Drive Electric Vehicles

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ABSTRACT

This paper presented a torque distribution strategy based on linear time-varying quadratic programming (LTV-QP) for yaw stability control of all-wheelindependent-drive electric vehicles. A two-degree-offreedom vehicle dynamic model was established to figure out the desired vehicle states including sideslip angle of the centroid of vehicle and the yaw moment, which was used as the reference signal of the LTV-QP controller. However, the influence of the time-varying steering angle was generally not taken into account. A QP-based torque distribution strategy is put forward to reduce the yaw rate error caused by them. The proposed strategy is evaluated in Matlab/Simulink to track the reference yaw moment and optimize the torque distribution. The results indicate that the LTV-QP controller can effectively distribute the torques of four in-wheel motors and significantly improve the vehicle yaw stability.

Keywords: All-wheel-independent-Drive Electric vehicles, yaw stability control, linear time-varying quadratic programming (LTV-QP), torque distribution strategy.

1. INTRODUCTION

With the fast development of various control strategies, all-wheel-independent-drive electric vehicles have ignited wide-spread interest thanks to the ability to control the wheel torque continuously and accurately [1]. Since the torque distribution should take lots of situations into account, such as road condition, nonlinear tire characteristic, unknown driver's command and so on, it poses a challenge for torque distribution on

distributed drive electric vehicles with four independent in-wheel motors

Recently, numbers of methods were applied in this zone, such as active front-wheel steering (AFS), direct yaw-moment control (DYC) and the combination of the two through differential driving or braking. Wu, in order to calculate the brake pressures and the range of active steering angles, proposed a controller based on the AFS and DYC [2]. Shuai figured out the steady-state response combinng the AFS and DYC [3]. However, they were all acceptable only for the simple conditions. Zhai implemented three strategies for torgue distribution and the yaw rate error root mean square decreased by 75 percent [4]. Lin built a hybrid model predictive control (hMPC)-based yaw stability controller to solve the nonlinear problem and proposed a multi-objective optimal Torque Distribution Strategy for improving the vehicle yaw stability [5]. As the variables to be controlled increase, the torque distribution control becomes complex. A lot of research aim at taking more variables into account, such as the time-varying steering angle. Alexander et al. proposed a linear time-varying modelbased predictive controller (LTV-MPC) for controlling the yaw ability appropriately through steering [6-7]. Quadratic programming (QP) is a process of solving a special mathematical optimization problem, which is to optimize (minimize or maximize) the quadratic function of multiple variables subjected to the linear constraints [8]. LTV-QP can handle the time-varying inputs based on the standard QP.

In this paper, a LTV-RBF controller is developed to improve the performance of yaw moment. In Section 2, a two-degree-of-freedom dynamic model of distributed electric drive vehicle is established. In Section 3, a LTV-QP controller is designed to optimize the optimal torque

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distribution of all wheels acceptablely. Section 4 presents the performance of proposed strategy based on Matlab/Simulink.

2. VEHICLE DYNAMIC MODEL

In this section, a two-degree-of-freedom dynamic model of distributed electric drive vehicle is established, as shown in Fig. 1. The dynamic equations that only consists of lateral and longitudinal movements can be expressed as follows.

$$(\dot{\beta} + \gamma) m v_x = F_{yf} \cos \delta_f + F_{yr} I_z \dot{\gamma} = a F_{yf} \cos \delta_f - b F_{yr}$$
(1)

where *m* is the vehicle mass, β is the sideslip angle of CG, γ is the vehicle yaw rate, *a* and *b* are longitudinal distances between the CG and the front-axle or rear-axle, while δ_f is the steering angle of front wheels, I_z is the vehicle yaw moment of inertia.

Tire sideslip angles are described as follows:

$$\alpha_{f} = \beta - \delta_{f} + \frac{a \cdot \gamma}{v_{x}}$$

$$\alpha_{r} = \beta - \frac{b \cdot \gamma}{v_{x}}$$
(2)

In (1)-(2), the dynamic equations can be expressed as follows:

$$(\dot{\beta}+\gamma)mv_{x}=K_{f}\delta_{f}-(K_{f}+K_{r})\beta-\frac{1}{v_{x}}(aK_{f}-bK_{r})\gamma$$

$$I_{z}\dot{\gamma}=aK_{f}\delta_{f}-(aK_{f}-bK_{r})\beta-\frac{(a^{2}K_{f}+b^{2}K_{r})}{v_{x}}\gamma$$
(3)

The yaw rate and the sideslip angel can be simplified as follows [9]:

$$\begin{aligned} \gamma &= G_{\gamma} \cdot \delta_f \\ \beta &= G_{\beta} \cdot \delta_f \end{aligned}$$
 (4)

where $G_{\gamma} = (1/(1 + Av_x^2)) \cdot (v_x / l), G_{\beta} = (1 - (m/l)/(a/bK_{\gamma}) v_x^2))(a/bK_{\gamma})v_x^2)/(1 + Av_x^2) \cdot (b/l)$ $A = (m/l^2) \cdot ((aK_f - bK_r)/K_rK_f)$



Figure 1 Two-degree-of-freedom dynamic model

3. TORQUE DISTRIBUTION STRATEGY

In this section, the yaw moment is generated from the driver input based on the MPC, the steering angle and the required torque. A LTV-RBF NNs is designed to calculate the acceptable optimal torque of each wheel. Fig. 2 shows the torque distribution strategy control system.



Figure 2 Torque distribution strategy control system

3.1 Control problem conversion

The control of yaw rate and the sideslip angle of CG is the key to improve the vehicle yaw stability. The slip ratio of all tires should be controlled under 0.2 due to the friction coefficient constraint of the wheel. The algorithm we adopted based on the combination of AFS and DYC, is to track the yaw rate efficiently by controlling the vehicle yaw moment and the front wheel active angle. Since the trajectory of angle variation has been determined by the upper controller based on MPC [5], the torque distribution comes down to a linear time-varying problem.

3.2 Torque Distribution Strategy

The vehicle yaw moment consists of two part, the longitudinal yaw moment and the lateral yaw moment . Only the $M_{\rm x}$ can be controlled by the longitudinal forces generated by four in-wheel motors. The lack of $M_{\rm y}$ may cause the oversteer or understeer. However, by adjusting the driving and braking torque, the vehicle lateral stability can be achieved [4]. The influence of the longitudinal force on the yaw moment is shown in Fig. 3. The total vehicle yaw moment can be simplified as follows:

$$M_{z} = I_{z}\dot{\gamma} = F_{x1} \times (a\sin\delta_{f} - l_{s}\cos\delta_{f}) + F_{x2} \times (a\sin\delta_{f} + l_{s}\cos\delta_{f}) + F_{x3} \times (-l_{s}) + F_{x4} \times l_{s}$$

$$T_{reg} = T_{1} + T_{2} + T_{3} + T_{4}$$
(5)



Figure 3 Lateral dynamics of the all-wheel-independent-drive electric vehicles

3.2.1 no-steering situation

When there is no steering angle input, the tires is only restricted by longitudinal forces which are determined by the vehicle vertical load. The vertical load of the four wheel can be seen as a constant as the vehicle runs at a constant speed,. The torque distributed by vertical load of each wheel is:

$$T_{1} = T_{2} = \frac{mgb}{2(a+b)} \cdot r$$

$$T_{3} = T_{4} = \frac{mga}{2(a+b)} \cdot r$$
(7)

3.2.2 time-varying steering situation

When a time-varying steering angle is received, the in-wheel motors from the left and the right sides of the same axle have different torque to generate the yaw moment. The longitudinal yaw moment M_x is distributed to the four motors to maintain the desired yaw moment from the driver's commands generated by MPC to figure out the yaw stability. The torque distribution of the distributed drive electric vehicles is a linear time-varying over-actuated system, the solution is infinite. However by setting a reasonable optimization objective function and minimizing it, the optimal control sequence satisfying the constraints can be obtained. And it can be converted into a standard LTV-QP question. The given objective function based on (5)-(6) is: $\omega=B\xi$

 $J(\omega(\mathbf{t}), \Delta \xi(\mathbf{t})) = \sum_{i=1}^{H_{\mathcal{P}}} ||\omega_r(\mathbf{t}+\mathbf{i} | \mathbf{t}) - \omega_{ref}(\mathbf{t}+\mathbf{i} | \mathbf{t})||_{\mathcal{Q}}^2 + \sum_{i=1}^{H_{\mathcal{C}}} ||\Delta \xi(\mathbf{t}+\mathbf{i} | \mathbf{t})||_{\mathcal{R}}^2$ where $\omega = [M_Z]^T; \xi = [\mathbf{T}_1, \mathbf{T}_2, \mathbf{T}_3, \mathbf{T}_4]^T;$ $B = [\frac{a \sin \delta_f - l_s \cos \delta_f}{\mathbf{r}}, \frac{a \sin \delta_f + l_s \cos \delta_f}{r}, -\frac{l_s}{r}, \frac{l_s}{r}];$ Subj.to. $\omega_{k+1} = B_k \xi_k$ $\xi_{k+1} = \xi_k + \Delta \xi_k$ $\sum_{i=1}^{4} T_i = (fmg + \frac{C_D A v^2}{21.15})$ $-30 \le T_i \le 30$

The first one is used to measure the deviation between the output and the reference output in the predictive time-domain, which reflects the fast tracking ability of the system to the reference trajectory. The second one is used to measure the control increment of the system in the control time-domain, which reflects the requirement of the system for the smooth change of the control quantity. Since the system is time-varying, it is not guaranteed during the control period that the optimization objective function can obtain the optimal solution at each moment. Therefore, it is necessary to add a relaxation factor to the optimization objective function. Then the system can replace the optimal solution with the obtained suboptimal solution to prevent the occurrence of no feasible solution.

The controller will figure out the optimal solution in each control period, and obtain the optimal control increment sequence. Then apply the first one of the optimal control increment series to the system as the actual control increment. The yaw moment tracking control will be received by repeating the above process in next control period.

4. RESULTS AND DISCUSSION

In this section, the performance of the proposed strategy is tested based on dSPACE simulator. The parameters of the distributed drive electric vehicle are shown in Table 1.

Parameters	Values		
Vehicle Mass	1523 kg		
Wheelbase	1.539 m		
Vehicle Moment of Inertia	1558 Nm		
Wheel radius	0.354 m		
Distance from front axle to CG	1.016 m		
Distance from front axle to CG	1.592 m		
Dimensionless coefficient	0.3		
Front area	1.95m ²		
Transmission ratio	7.1		
Friction coefficient	0.01		

Table 1 vehicle parameters

This paper presents a constant longitudinal velocity 15 m/s with a generally sine steering input. The control time period is 0.05s. Since the tracking is the most important, the control weight matrix Q and R is

	1					0.2			
Q =		1	1	;R =	0.2				
					;K =			0.2	
				1_					0.2

Fig. 4 compares the the desired and actual yaw moment. The dot line is desired yaw moment and the line is actual yaw moment output. As the vehicle ran at a constant speed, there is no yaw moment. A time-varying steering angles were applied in the vehicle from the driver's command at the time of 2.5s, the yaw moment changed with the input steering angles changes. Due to the control relaxation Q, the line is basically coincide with the dot line and has a small root mean square. And the output yaw moment has some deviation from the desired yaw moment only at the area near the peak point.

Fig. 5, in which T1, T2, T3 and T4 mean the torque of left-front, right-left, left-rear and right-rear in-wheel motors respectively, presented the torque distribution of four in-wheel motors. When there were no yaw moment the torques in the same axle were the same while the front and the rear one were submitted by the vertical load distribution. As the steering angels inputted, the two inside-motors had less torque than the outside-ones to ensure yaw stability. Basically, the inside ones had braking torques and the outside ones had driving torques. Due to the control relaxation R, the torque of each motor changed smoothly. Therefore, the proposed strategy can achieve an acceptable performance on yaw stability control while taking the time-varying steering angles input into account.





5. CONCLUSION

Prior work has documented the accuracy of vehicle velocity prediction using a LTV-MPC [6]. In this study, we employed LTV-QP controller to achieve the torque distribution of the all-wheel-independent-drive electric vehicles for stabilizing the yaw movement. It can generally maintain the desired yaw moment with time-varying steering angels input. Results showed that the LTV-QP controller we proposed can reach an acceptable performance on stabilizing the vehicle yaw moment via controlling the four wheels torque generated by four in-wheel motors.

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