

BILEVEL GAME-THEORETIC FRAMEWORK FOR THE OPTIMAL PLANNING OF BIOFUEL REFUELING STATIONS UNDER MARKET EQUILIBRIUM

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ABSTRACT

As a new entrant of fuel market, biofuel refuelling stations are faced with the keen competition from existing petrol stations. The optimal planning of biofuel refuelling stations strongly affects the profit of biofuel company. However, there are few studies to deal with this issue. This paper develops a bilevel game-theoretic framework that consists of an upper-level model for the petroleum company and a lower-level model for the biofuel company. The mutual restraint relations among biofuel company, petroleum company and the market are all formulated into the models. Finally, the proposed method is validated by a real-world case in China.

Keywords: bilevel game-theoretic framework, biofuel refuelling station, petrol station, optimal planning

NONMENCLATURE

Abbreviations	
PC/BC	Petroleum/Biofuel company
RS	Refuelling station
GRS/BRS	Petrol/Biofuel refueling station
Sets and indices	
$a \in A$	Set of BRS scale
$i \in I$	Set of demand sites
$s \in S$	Set of all station sites
$s \in S^B$	Set of alternative sites for BRSs
$s \in S^P$	Set of existing PSs
Parameters	
c_a^{BE}	The construction cost of a a -scale station, CNY

d_i	The energy demand in site i , MJ/d
$l_{s,i}$	The distance between sites s and i , km
p^{BP}, p^{PP}	The production cost of 1 m ³ biofuel/gasoline, CNY
p^{FB}	The cost of 1 m ³ biofuel paid by PC, CNY
p_s^{BT}, p_s^{PT}	The cost of transporting 1 m ³ biofuel/gasoline to site s , CNY
v_a^{max}	The capacity of a a -scale BRS, m ³
α	Turnover coefficient of RS
β^B, β^P	Calorific value of fuels, MJ/m ³
β^M	The blending ratio of gasoline
μ^U, γ^U	Coefficients of the price elasticity
Variables	
$B_{s,a}^{BC}$	Binary variable indicating a a -scale BRS is built at site s
$B_{s,i}^Q$	Binary variable indicating site i is supplied by site s
G^B, G^P	The profit of BC/PC, CNY
G^{BC}	The construction cost of BC, CNY
G^{BT}, G^{PT}	The transport cost of BC/PC, CNY
p^U	The unit price of energy, CNY/MJ
Q_s^B, Q_s^P	Sale volume of biofuel/gasoline at site s , m ³
V^B, V^P	The yield of biofuel/gasoline, m ³
V^{PB}	The biofuel purchased by PC, m ³

1. INTRODUCTION

Along with the global energy crisis and environmental pollution brought by fossil fuel, renewable energy is concerned by countries all over the world. Biofuel, derived from plants, agricultural,

commercial and industrial wastes, is the most competitive type of alternative energy for fossil fuel. There are a great variety of biofuels, including biogas, liquid biofuel and solid biomass fuels[1]. For example, bioethanol is a type of liquid fuel that can be used as a substitute and additive for conventional transportation fuel. Blending the bioethanol with gasoline helps to save nonrenewable oil resources, reduce air pollution from automobile exhaust, as well as promote agricultural development. If business operators can effectively integrate biofuel resources and optimize supply chain structure, the sale of biofuel will be greatly increased.

Presently, a large number of studies have been carried out in strategic (long-term), tactical (medium term) and operational (short-term) decisions of fuel supply chains. The common optimization issues include selection, selection of production technologies, refinery location, capacity planning, inventory planning and detailed scheduling[2-5], while the evaluating indicators for BCSs generally involve economy, environment, society, risk and reliability. With the development of modelling technology, recent work considered more practical and complex factors such as multi-objective optimization, uncertainties, and game theory between the supply market (the farmers) and the demand market (the refineries). However, most of the previous work focused on the optimization of upstream industries rather than biofuel refuelling stations (BRSs) in the downstream market. As a new entrant of the fuel market, the layout of BRSs strongly depends on the layout of existing petrol stations (PRSs) and in turn affects the profit of petroleum company (PC). Therefore, this paper regards the decision-making of PC and BC as Stackelberg game, in which PC acts as the leader and BC acts as the follower. And then we propose a bilevel game-theoretic framework for the optimal planning of BRSs under market equilibrium.

2. PROBLEM DESCRIPTION

As mentioned above, there are two sides in the fuel market, one is the BC as the follower, and the other is the PC as the leader (see Fig.1). Generally, the PC will first decide the total supply amount of gasoline to the market and the amount of biofuel purchased from BC. And then, after knowing the PC's decision, the BC will determine the locations and scales of new BRSs, the total supply amount of biofuel to the BRSs for sale and to the PC for blending. The decisions of both sides are restricted by fuel market and meantime they interact with each other. To simplify this issue, some necessary assumptions are presented as follows:

- (1) The sale price of gasoline and biofuel is measured by the price for unit energy and their calorific value.
- (2) According to the price elasticity of supply, the price of unit energy goes down as total supply increases, while goes up as total supply decreases.
- (3) All drivers select the nearest RS to refuel cars.
- (4) All fuels are incompressible.
- (5) The calorific value of blending fuel is estimated by the proportion as well as the calorific value of each pure component.

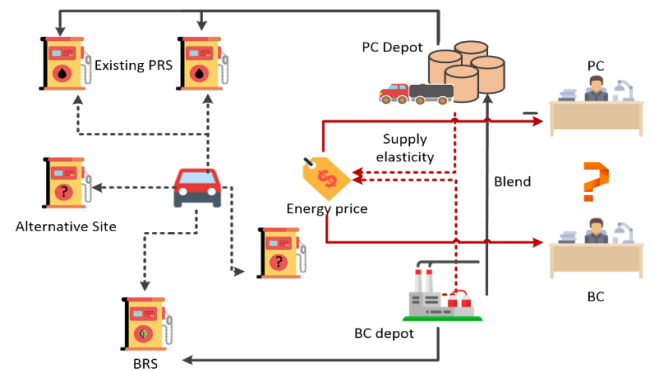


Fig 1 The studied system

3. OPTIMIZATION FRAMEWORK

The proposed game-theoretic framework consists of an upper-level model for PC and a lower-level model for BC. Note that the variables with superscript (*) in the upper-level model are the decision variables obtained by the lower-level model, so they are actually known parameters for the upper-level model. On the contrary, the variables with superscript (*) in the lower-level model are from the upper-level model. By subsequently solving these two models, the optimal results of both sides tend to stable. The detailed solution strategy for the proposed optimization framework is presented in Fig.2

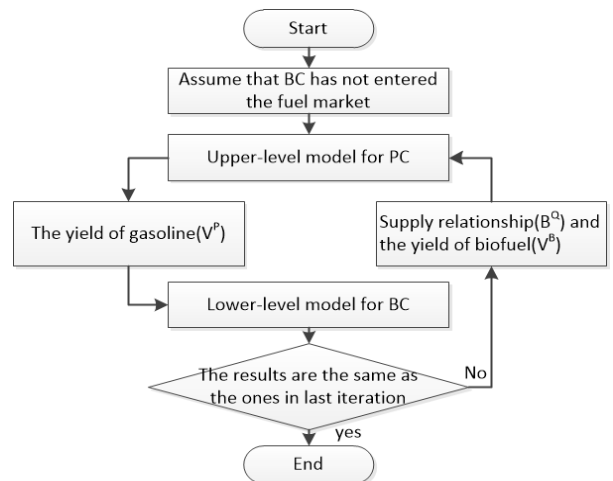


Fig 2 The flowchart of solution strategy

3.1 Upper-level model for a petroleum company

The upper-level model for PC is formulated as eqs.(1)-(7) based on the decision-making of BC. Its objective function eq.(1) is to maximize the gross profit G^P , which is equal to the sale profit of gasoline G^{PS} minus the transport cost G^{PT} . Note that the sale profit G^{PS} is computed as the total income minus the summation of purchase expenses on biofuel and product cost, as stated in eq.(2). Considering different calorific value of gasoline and biofuel, the income is computed by the unit of energy rather than volume. Based on the first assumption, there is a negative relevant relation between unit energy price and supply amount (eq.(3)). Eq.(4) obtains the transport cost which is related to the locations of PRSs and the transport amount. Eq.(5) uses the maximum allowable blending ratio to impose the upper bound on the biofuel sold to PC.

$$\begin{aligned} \max G^P &= G^{PS} - G^{PT} & (1) \\ G^{PS} &= P^Q \sum_{s \in S^P} \sum_{i \in I} d_i \cdot B_{s,i}^{Q*} - p^{PB} V^{PB} - p^{PP} V^P & (2) \\ P^Q &= -\mu^U [\beta^B (V^{PB} + V^{B*}) + \beta^P V^P] + \gamma^U & (3) \\ G^{PT} &= \sum_{s \in S^P} p_s^{PT} Q_s^P & (4) \\ V^{PB} &\leq \beta^M \cdot V^P & (5) \end{aligned}$$

According to actual conditions, drivers are willing to go to the nearest RS. Let the binary variable $B_{s,i}^Q$ indicate if site s is the nearest RS of demand site i . As for upper-the level model, binary variable $B_{s,i}^{Q*}$ is a fixed value after knowing the locations of all BRSs. In this way, we can easily obtain the total sale energy at each PRS, by the sum over all demand sites $i \in I$ of term $(d_i \cdot B_{s,i}^{Q*})$. And then the sale energy can be converted to corresponding volume through the calorific value after bleeding (eq.(6)).

$$Q_s^P = \left(\frac{V^P + V^{PB}}{\beta^P V^P + \beta^B V^{PB}} \right) \cdot \sum_{i \in I} d_i \cdot B_{s,i}^{Q*} \quad s \in S^P, i \in I \quad (6)$$

Assuming that the total volume remains constant after blending, the sale volume at all PRSs cannot exceed the yield of gasoline plus the purchase volume of biofuel (eq.(7)).

$$\sum_{s \in S^P} Q_s^P \leq V^P + V^{PB} \quad (7)$$

3.2 Low-level model for biofuel company

The gross profit G^B of BC is given in eq.(8), where G^{BS} , G^{BC} and G^{BT} stand for sale profit of biofuel, construction cost of BRSs and transport cost. These individual costs are obtained by eqs.(9), (11) and (12), respectively. Eq. (13) functions the same as eq. (5). Eq.(14) computes the sale biofuel at all alternative sites,

while eq. (15) enforces the biofuel yield to be greater than the biofuel sold to drivers and the PC.

$$\begin{aligned} \max G^B &= G^{BS} - (G^{BC} + G^{BT}) & (8) \\ G^{BS} &= P^Q \sum_{s \in S^B} \sum_{i \in I} d_i \cdot B_{s,i}^Q + p^{FB} V^{FB} - p^{BP} V^B & (9) \\ P^Q &= \mu^U [\beta^B (V^{PB} + V^B) + \beta^P V^{P*}] + \gamma^U & (10) \\ G^{BC} &= \sum_{s \in S^B} \sum_{a \in A} c_a^{BE} B_{s,a}^{BS} & (11) \\ G^{BT} &= \sum_{s \in S^B} p_s^{BT} Q_{s,i}^B & (12) \\ V^{PB} &\leq \beta^M \cdot V^{P*} & (13) \\ Q_s^B &= \frac{1}{\beta^B} \cdot \sum_{i \in I} d_i \cdot B_{s,i}^Q \quad s \in S^B, i \in I & (14) \\ \sum_{s \in S^B} Q_s^B + V^{PB} &\leq V^B & (15) \end{aligned}$$

Now let binary variable $B_{s,a}^{BC}$ identify if a a -scale BRS is built at site s . If so, the total sale biofuel at station s cannot exceed its capacity v_a^{max} .

$$\alpha \cdot Q_s^B \leq \sum_a B_{s,a}^{BC} \cdot v_a^{max} \quad s \in S^B, i \in I \quad (16)$$

Eqs.(17)-(19) ensure the drivers in demand site i to refuel their cars at the nearest RS s (in case of $B_{s,i}^Q = 1$). Note that if there is no BRS built in alternative site $s \in S^B$, corresponding binary variable $B_{s,i}^Q$ must be zero (i.e. (20)).

$$l_{s,i} \leq l_{s',i} + l_{s,i} \cdot (1 - B_{s,i}^Q) \quad s \in S, s' \in S^P, s \neq s', i \in I \quad (17)$$

$$l_{s,i} \cdot \sum_{a \in A} B_{s',a}^{BC} \leq l_{s',i} + l_{s,i} \cdot (1 - B_{s,i}^Q) \quad s \in S, s' \in S^B, s \neq s', i \in I \quad (18)$$

$$\begin{aligned} \sum_{s \in S} B_{s,i}^Q &= 1 & i \in I \quad (19) \\ \sum_{i \in I} B_{s,i}^Q &\leq |I| \cdot \sum_{a \in A} B_{s,a}^{BC} & s \in S^B \quad (20) \end{aligned}$$

4. CASE STUDY

4.1 Basic data

Fig.3 illustrates a real-world district with existing 12 PRS, among which 8 are first-class (red points), and the others are second-class (yellow points). This district is divided into several blocks by 1 km² of the square. Through the urban GPS and the population heat map, the energy demand in each block can be estimated and represented by colour depth. The yellow blocks are high-demand area, and the dark blue ones are the low-demand area. The average calorific value of gasoline is 5.7×10⁴ MJ/ m³, while that of biofuel is 3.4×10⁴ MJ/ m³. In the present system without any BRS, the average daily sale volume of each PRS could be obtained, as noted in the brackets of Fig.3. The coefficients μ^U and γ^U for price elasticity are set as 3×10⁻⁶ and 0.1228, respectively.

Considering the factors of safety, environment protection and urban planning, 18 alternative sites (blue points) are selected to construct BRSs. There are three scales for each BRS, as shown in Table 1.

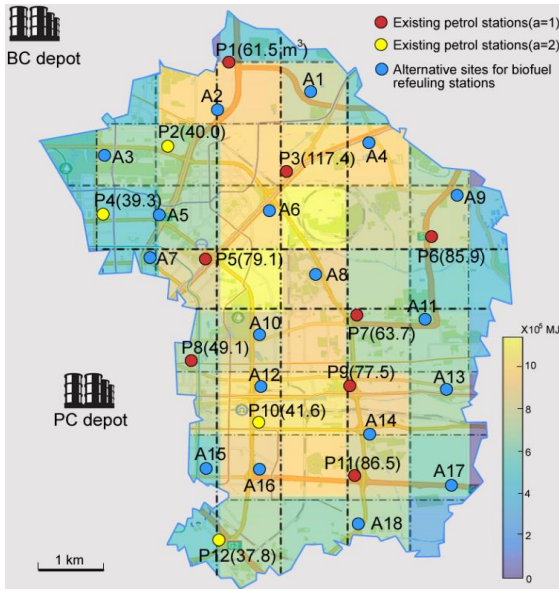


Fig 3 The studied district

Table 1 Information for BRSs of different scales

Scale	I	II	III
Capacity (m ³)	150	60	15
Construction cost (10 ⁴ CNY)	11	7	4

4.2 Computational result

The proposed model is implemented in GAMS on an Intel i7-7700HQ with 16GB. BARON[6] is used to solve the upper-level NLP model, and GloMIQO 2.3 is used to solve the lower-level MUQCP model. Fig.3 displays the optimal results of both companies during each iteration. One can see that the gross profit of PC decreases as the gross profit of BC goes up, and finally, the solutions of both upper-level and lower-level models tend to stable after 6 iterations, which is illustrated in Table.2.

Table 2 Computational results during each iteration

Iteration	1	2	3	4	5	6
GP(10 ⁶ CNY)	1.60	1.20	1.11	1.06	1.05	1.05
GB(10 ⁵ CNY)	1.86	1.99	2.03	2.06	2.06	2.06

The optimal planning of BRSs is displayed in Table.3. Two first-class BRSs are built in high-demand sites A4 and A6 which are relatively nearer with BC depot, and four second-class BRSs are constructed in further sites A11, A14, A15 and A18. Overall, the total sale volume of gasoline is 510.7 m³/d, and that of biofuel is 465.2 m³/d.

Table 3 The optimal planning of BRSs

Site	A4	A6	A11	A14	A15	A18
Scale	I	I	II	II	II	II
Sale(m ³)	108.1	78.3	59.3	57.7	52.1	58.5

Then, we explore the impact of supply elasticity on final decisions. Fig.4 displays the optimal results under different coefficients μ^U that range from 0 to 6.5×10^{-6} .

Most noticeably, the gross profit of GB keeps decreasing, with μ^U increased. This is not surprising since biofuel is new energy and needs high investment. Its profit is greatly influenced by market environmental fluctuations. The risk brought by high supply elasticity discourages its investment willingness and thus leads to lower profit of BC. While for PC that features the perfect system, its profit goes down first, goes up afterwards and finally goes down again. The first stage reduction is caused by the lower energy price as well as the market competition of BC. However, when the energy price drops to a certain value, BC is less willing to construct BRSs, so PC regains the lost customers. Finally, PC is virtually monopolizing the fuel market and its gross profit just decreases with reduction of energy price.

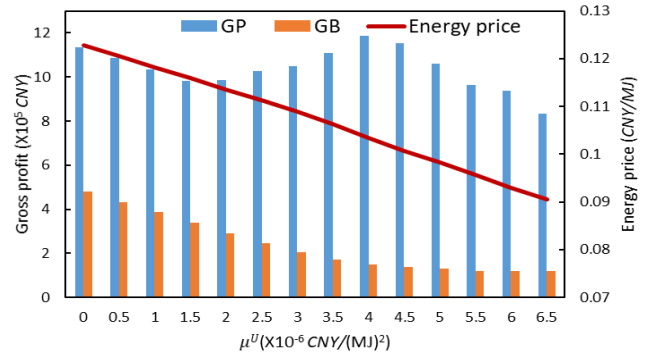


Fig 4 The optimal results with varying supply elasticity

5. CONCLUSION

Aiming at the optimal planning of BRSs under market equilibrium, this paper presents a bilevel game-theoretic framework that involves an upper-level model for PC (leader) and a lower-level for BC (follower). Both models take their respective profit maximization as the objective functions, providing the optimal planning after knowing the decisions of the other side. This decision-making process can be regarded as the Stackelberg game between PC and BC, and is achieved by subsequently solving these two models until the optimal solutions of both sides tend to stable. Finally, a real-word district in China is given as an example to demonstrate the practical value of the proposed optimization framework.

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