# **COORDINATED OPERATION OF GAS AND ELECTRICITY NETWORKS**

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### ABSTRACT

This paper focuses on the coordinated operation (CO) of a gas network and electricity network and proposed a novel framework for coordinated operation between them. In this paper, credit ranking (CR) indicator for coupling units was established, and gas consumption constraints information of natural gas-fired units (NGFUs) is generated, natural gas network operator (GNO) will deliver this information to electricity network operator (ENO). The greatest advantage of this operation framework is that no frequent information interaction between GNO and ENO is needed. The entire framework contains two participants and three optimization problems, which are GNO optimization sub-problem-A, GNO optimization sub-problem-B and ENO optimization sub-problem. Decision sequence changed from traditional ENO-GNO-ENO to GNO-ENO-GNO. For ENO optimization sub-problem, a Second-order Cone (SOC) relaxation was utilized and reformulated the original problem Mixed-Integer Second-Order Cone as Programming (MISOCP) problem. For GNO optimization sub-problem, an improve Sequential Cone Programming (SCP) method is applied based on SOC relaxation and converts the original sub-problem to MISOCP problem. Simulations demonstrate the effectiveness of the proposed framework.

**Keywords:** Coordinated operation, convex relaxation, gas network, mixed-integer second-order cone programming.

#### NOMENCLATURE

Symbols	
t	Index of time periods.
i	Index of thermal units.
b	Index of buses.
r	Index of renewable energy.
br	Index of power transmission lines.
j,l	Index of buses in electricity network.
m,n	Index of gas nodes.
S	Index of gas wells.
mn	Index of gas pipelines.
gi	Index of resident natural gas load.
GU	Set of NGFUs.
NGU	Set of non-NGFUs.
i	Index of thermal units.
b	Index of buses.
r	Index of renewable energy.
br	Index of power transmission lines.
j,l	Index of buses in electricity network.
m,n	Index of gas nodes.
$N_T$	Number of time periods.
$N_B$	Number of thermal units.
N <sub>R</sub>	Number of types of renewables.
$N_s$	Number of gas wells.
$N_L$	Number of resident gas load.
$P_i^{\min}, P_i^{\max}$	Minimum and maximum output of unit $i$ .
$UR_i, DR_i$	Ramp up/down limits of unit $i$ .
$T_i^{on}, T_i^{o\!f\!f}$	Minimum on/off time of unit $i$ .
$lpha^{c},eta^{c},\gamma^{c}$	Cost coefficient of non-NGFUs.
$lpha^{g},eta^{g},\gamma^{g}$	Fuel coefficient of NGFUs.

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$SU_i$	Startup cost coefficient of unit $i$ .
$\eta$	Natural gas contract price of NGFUs.
$P_{r,t}^W$	Forecast of renewable energy $r$ at hour $t$ .
and max	Maximum power flow of power transmission line
$PJ_{br}$	br .
$x_{jk}$	Reactance between bus $j$ and $k$ .
$D^{ini}$	Initial electrical load.
$p^{ini}$	Initial electricity price.
Ε	Price-elastic matrix of electrical load.
$CS^{\min}$	Minimum customer's satisfaction.
$C_{_{mn}}$	Weymouth constant of pipelines.
$W_s^{\min}$ , $W_s^{\max}$	Min and max production of gas well <i>s</i> .
$ au_{gl}$	Gas price of resident natural gas load.
L	Resident natural gas load.
$K_P$	Bus-thermal unit incidence matrix.
$K_{\scriptscriptstyle W}$	Bus-renewable unit incidence matrix.
K <sub>D</sub>	Bus-electrical load incidence matrix.
$K_L$	Bus-branch incidence matrix.
$T^{w}$	Node-gas well incidence matrix.
$T^{g}$	Node-NGFUs incidence matrix.
$T^{l}$	Node-resident natural gas load incidence matrix.
$T^{f}$	Node-gas pipe incidence matrix.
$G_{i,t}^c$	Cost of non-NGFU $i$ at hour $t$ .
$G^{g}_{i,t}$	Fuel consumption of NGFU $i$ at hour $t$ in electricity network.
$P_{i,t}$	Generation dispatch of unit $i$ at hour $t$ .
$P_{r,t}$	Generation dispatch of renewable energy $r$ at hour $t$ .
$SU_{i,t}$	Startup cost of unit $i$ at hour $t$ .
$u_{i,t}$	Status indicator of unit $i$ at hour $t$ .
$Y_{i,t}, Z_{i,t}$	Startup/ shutdown of unit $i$ at hour $t$ indicator.
$X^{\mathit{on}}_{i,t}, X^{\mathit{off}}_{i,t}$	On/Off time of unit $i$ at hour $t$ .
$\theta$	Bus voltage angle.
$pf_{br}$	Power flow on transmission line br.
D	Adjusted electrical load.
$\Delta D_t$	Variety in electrical load at hour $t$ .
$\Delta p_t$	Variety in electricity price at hour $t$ .
$D^{dev}$	Deviation matrix of electrical load.
$p^{dev}$	Deviation matrix of electricity price.
CS	Electricity customer's satisfaction.
$W_{s,t}$	Production of gas well $s$ at hour $t$ .
$\mathcal{O}_{m,t}$	Gas pressure of gas node $m$ at hour $t$ .
$F_{mn,t}$	Gas flow of pipeline $mn$ at hour $t$ .
$\lambda^0,\lambda^1$	Initial and updated credit value factor.
$CR^0, CR^1$	Initial and updated credit rank.

#### 1. INTRODUCTION

As compared with other primary sources of energy, natural gas has some advantages, especially in traditional thermal power generation. Natural-Gas Fired Units (NGFUs) gradually replace coal-fired units due to the low capital costs, high efficiency, operation flexibility and relatively low prices in many countries [1]. In China, NGFUs occupying 4% of total generating capacity, while in the USA, it reached 39% [2]. Therefore, considering natural gas systems with power systems could have a practical issue.

Most prior researches have focused on the coordination of electricity and natural gas networks at synergistic scheduling level. Electricity network flow rules are summarized in [3] to solve the gas flow problem. A methodology for considering the combination of the natural gas network was proposed in [4]. Coordination the natural gas and electricity infrastructures with stochastic scheduling were reported in [5]. The impact of natural gas prices on the overall system was studied in [6]. A discussion of the operation of electricity and natural gas networks in distribution level by ADMM was given in [2]. From these studies, we can easily find that as the coupling continues to deepen, the impact of natural gas networks on the power system will become significant Error! Reference source not found.

Many research works have focused on the solver of steady-state natural gas flow also. The difficulties mainly come from non-linear, non-convex Weymouth equation; natural gas flow is a nonlinear programming (NLP) problem. References [3][4] used nonlinear solution or some of the more mature commercial solvers to solve gas flow, such as Newton-Raphson iteration or interior point method. Approximate piecewise linearization methods were adopted in [7][10], by converting to a mixed-integer linear programming (MILP) problem. References [2][11] considered the special form of the Weymouth equation, by changing the NLP problem MISOCP, through convex relaxation to eliminate non-convexity.

Electricity network and natural gas network are owned by different operator and stakeholder; however, electricity network and natural gas network are often seen as an integrated system to achieve coordinated operation or establish a third-party agency or department with a lot of information interaction. The main contributions of this paper are summarized as follows:

Novel idea and framework are proposed for CO and change the traditional decision sequence. In this idea and

framework, it avoids frequent information interaction and without a third coordination department.

This paper constructed a reasonable model to generate NGFUs' gas consumption constraints. In order to generate reasonable gas consumption constraints information and deliver from GNO to ENO, a CR indicator and update method were introduced.

## 2. OPERATION PRINCIPLE

## 2.1 Traditional decentralized operation

The structure of traditional decentralized operation (DO) is composed of two steps:

Step 1.1: ENO carries out the optimal local scheduling of electricity network which described as ENO sub-problem, and the corresponding consumption of natural gas information will be delivered to GNO.

Step 1.2: GNO carries out the optimal scheduling of natural gas network which described as GNO subproblem. If GNO is unable to supply enough expected gas volume in Step 1.1 under the premise of resident gas demand, it has a higher priority, ENO needs to be redispatched in the electricity network.

The main defect with this traditional DO is that ENO as an advance decision-maker, cannot guarantee that the local scheduling will result feasible solution in GNO scheduling feasible domain as shown in Fig. 1. The decision sequence is ENO-GNO-ENO.



Fig. 1. The diagram of traditional DO.

## 2.2 A novel framework for coordinated operation

In this paper, a novel framework was established for CO of electricity and natural gas networks. The difference from the traditional DO is the decision sequence, namely, GNO-ENO-GNO, GNO as an advance decision-maker provides NGFUs gas consumption constraints to ENO. When there are multiple coupling units while gas supply capacity is limited, how to generate reasonable constraints for multiple coupling units is extremely important. To solve this problem, this paper introduces CR, and the entire framework for CO is shown in Fig. 2 and composed of three steps:

Step 2.1: Receive NGFUs' CR from the previous period, GNO calculation maximize profit with uncertain gas load of NGFUs, named GNO sub-optimization-A problem. Set the gas supply to NGFUs of optimization result as information delivered to ENO. In fact, this information means that natural gas network's maximum gas supply capacity is given to NGFUs.

Step 2.2: ENO set maximum gas supply of NGFUs as constraints in optimization sub-problem to determine the hourly dispatch, including the dispatching generation and actual gas consumption of NGFUs.

Step 2.3: After receiving the actual gas load of NGFUs in natural gas network, GNO optimizes gas network scheduling in minimum operation cost and updates the CR of NGFUs for the next scheduling period.

Remark:

1) The GNO and ENO do not have dispatching power and networks information of another party, but the information in coupling units is available for both GNO and ENO.

2) In natural gas network, resident natural gas load priority is higher than NGFUs gas load, GNO will cut the gas supply of NGFUs first if there is a lack of natural gas. Thus, in GNO optimization sub-problem, resident natural gas load is the considered parameter and the profit of this part is fixed.

3) In GNO optimization sub-problem-A, we ignore the cost of gas well. It assumes that the gas purchase price of NGFUs is greater than the production cost of gas well, GNO always benefits from it.

4) CR is a gas price-related indicator, we assume that forecast gas contract price of all NGFUs is the same. A higher CR means GNO will provide as much natural gas as possible to this NGFU in Step 2.1, so the gas constraint value delivered to ENO will be larger.



Fig. 2. The structure of the proposed CO framework.

#### 3. MATHEMATICAL MODEL

#### 3.1 GNO optimization sub-problem-A

As shown in (1), the objective of the GNO suboptimization-A problem is to maximize profit. In this subproblem, the resident natural gas load is fixed, and NGFUs gas load is variable. The first term of the objective function represents incomes from resident natural gas load, the second term is the expected profits from NGFUs natural gas load, the third term is the cost of gas wells.

Natural gas network models (2)-(6) are applied by steady-state natural gas flow. Constraint (2) represents the relationship between pressure of gas node and gas flow in pipelines which is Weymouth function. The pressure of gas node upper and lower limits is shown by (3). Nodal natural gas balance for network and the boundary for natural gas well production are given in (4), and (5) respectively. The gas load of NGFUs is limited by units' maximum gas consumption (6). Constraint (7) represents relationship between CR, gas contract price of NGFUs and credit value factor, which are equal to the product of CR and the forecast gas contract price.

$$\operatorname{Max} \quad obj_{-}Ga = \sum_{t=1}^{N_{t}} \left( \sum_{gl=1}^{N_{t}} \tau_{gl} L + \sum_{i \in GU} \lambda_{i}^{o} G_{i,t}^{g} \right)$$
(1)

$$F_{mn,t} = \operatorname{sgn}\left(\omega_{m,t}, \omega_{n,t}\right) \cdot C_{mn} \sqrt{\omega_{m,t}^2 - \omega_{n,t}^2}$$
(2)

$$\omega_{m,t}^{\min} \le \omega_{m,t} \le \omega_m^{\max} \tag{3}$$

$$T^{w}W - T^{g}G^{g} - T^{l}L = T^{f}F$$
(4)

$$W_s^{\min} \le W_{s,t} \le W_s^{\max} \tag{5}$$

$$G_{i,t}^{g} \le G_{i}^{g,\max} \tag{6}$$

$$\lambda_i^0 = \eta_i C R_i^0 \tag{7}$$

After completing the above optimization solution, a series of constraints of NGFUs will be generated, the gas supply to NGFUs of this optimization result is given by (8). This value will deliver to the next step.

$$G_{i,t}^{g,0} = G_{i,t}^g = \arg\max\left(obj\_Ga\right), \forall i \in GU$$
(8)

#### 3.2 ENO optimization sub-problem

#### **Objective function**

As shown in (9), the objective of ENO suboptimization problem is to determine the hourly dispatch to minimize the operating cost over the entire scheduling horizon with security-constrained. The first term of the objective function represents the generation costs and startup cost of NGFUs, the second term is the generation costs and startup cost of non-NGFUs. The shutdown cost of units has been converted into the startup cost.

$$\operatorname{Min} \quad obj_{-}E = \sum_{t=1}^{N_{T}} \left\{ \sum_{i \in GU}^{\sum i \in GU} \left( G_{i,t}^{g} \eta_{i} + SU_{i,t} \right) + \sum_{i \in NGU} \left( G_{i,t}^{c} + SU_{i,t} \right) \right\}$$
(9)

#### Units and network constraints

The equations (10)-(21) are physical constraints of generating unit. Generation scheduling of each thermal unit is limited by minimum and a maximum output of the unit as shown in (10). The dispatched renewable energy at each hour is limited by the forecast renewable energy (11). Thermal units operating ramping up/down constraints are given in (12)-(13). Minimum on/off limits are listed by (14)-(15). The relationships between startup/shutdown indicators and unit states are given in (16). The logic between startup indicators and shutdown indicators are given in (17). Constraint (18) represents the generation costs of non-NGFUs. Constraint (19) represents the fuel consumption NGFUs. Constraint (20) shows the startup costs of thermal units.

$$P_i^{\min} u_{i,t} \le P_{i,t} \le P_i^{\max} u_{i,t} \tag{10}$$

$$P_{r,t} \le P_{r,t}^{\mathsf{W}} \tag{11}$$

$$P_{i,t} - P_{i,t-1} \le UR_i (1 - y_{i,t}) + P_i^{\min} y_{i,t}$$
(12)

$$P_{i,t-1} - P_{i,t} \le DR_i \left( 1 - z_{i,t} \right) + P_i^{\min} z_{i,t}$$
(13)

$$\left[ X_{i,t-1}^{on} - T_i^{on} \right] \left[ u_{i,t-1} - u_{i,t} \right] \ge 0$$
 (14)

$$\left[X_{i,t-1}^{\text{off}} - T_i^{\text{off}}\right] \left[u_{i,t} - u_{i,t-1}\right] \ge 0$$
(15)

$$y_{i,t} - z_{i,t} = u_{i,t} - u_{i,t-1}$$
(16)

$$y_{i,t} + z_{i,t} \le 1$$
 (17)

$$G_{i,t}^{c} = \alpha_{i}^{c} u_{i,t} + \beta_{i}^{c} P_{i,t} + \gamma_{i}^{c} P_{i,t}^{2}, \forall i \in NGU$$
(18)

$$G_{i,t}^{g} = \alpha_{i}^{g} u_{i,t} + \beta_{i}^{g} P_{i,t} + \gamma_{i}^{g} P_{i,t}^{2}, \forall i \in GU$$

$$\tag{19}$$

$$SU_{i,t} = su_i y_{i,t} \tag{20}$$

Constraint (21) limits the gas consumption of NGFUs, the information of this constraint is from GNO optimization sub-problem-A.

$$G_{i,t}^{g} \le G_{i,t}^{g,0}, \forall i \in GU$$

$$(21)$$

For simplicity, the electric power transmission is modeled by (22)-(26) with DC power flow. Constraint (22) shows the system power balance at each hour. Constraint (23) represents the power balance at each bus. Constraint (24) represents the DC power flow for branch *br* in which first and last buses are j and k respectively. Constraint (25) gives line capacity limit,  $pf^{max}$  is power flow limits. Constraint (26) is the phase angle of reference bus.

$$\sum_{i \in GU} P_{i,t} + \sum_{i \in NGU} P_{i,t} + \sum_{r=1}^{N_R} P_{r,t} = \sum_{b=1}^{N_B} D_{b,t}$$
(22)

$$K_P \cdot P_i + K_W \cdot P_r - K_D \cdot D = K_L \cdot pf$$
(23)

$$pf_{br} = \frac{\theta_j - \theta_l}{p_{br}}, (j, l \in br)$$
(24)

$$|pf_{br}| \le pf_{br}^{\max} \tag{25}$$

$$\theta_{ref} = 0 \tag{26}$$

#### 3.3 GNO optimization sub-problem-B

The objective of GNO sub-optimization-B problem is to minimize the cost of gas wells. In this problem, the resident natural gas load and NGFUs gas load are fixed. The natural gas network constraints are similar to GNO sub-optimization-A, comprised of (2)-(5).

$$\operatorname{Min} obj_{-}Gb = \sum_{t=1}^{N_{T}} \sum_{s=1}^{N_{S}} \tau_{s} W_{s,t}$$
(27)

s.t Constraints (2)-(5)

Meanwhile, GNO updates the CR as function (28) as shown below:

$$CR_{i}^{1} = 0.5 \begin{pmatrix} \sum_{t=1}^{N_{T}} G_{i,t}^{g,1} \\ CR_{i}^{0} + \frac{\sum_{t=1}^{N_{T}} G_{i,t}^{g,0}}{\sum_{t=1}^{N_{T}} G_{i,t}^{g,0}} \end{pmatrix}$$
(28)

An improve SCP method based SOC with reasonable initial expansion value was adopted for natural gas network sub-problem. The solution is derived from the special form of the Weymouth function which was replaced by McCormick envelope to make the whole model into a MISOCP problem.

#### 4. CASE STUDIES AND DISCUSSIONS

Simulation results for coordinated operation are tested on a 6-bus electricity system and 6-node natural gas system to demonstrate the proposed CO framework of the electricity network model and natural gas network.

Fig. 3 shows the integrated topology consisted of the 6-bus electricity network and 6-node natural gas network. The test networks have three NGFUs, one non-NGFU, one renewable energy source, seven transmission lines, two natural gas wells and five pipelines. In this case, the fluctuating natural gas loads and electricity load are considered, the test data of networks is provided in http://motor.ece.iit.edu/data/.

The same fuel price for all units is considered. Load shedding is added to avoid imbalance between load and generation. We assume that all NGFUs have an initial credit ranking of 0.5, the entire scheduling time is four days and hour is taken as a time scale. Units information and daily load for of electricity and natural gas networks remain unchanged.

Decentralized and coordinated operations without natural gas pipelines congestion are presented to illustrate the advantage of the proposed framework applied to daily scheduling for a few days.



Fig. 3. The topology of electricity and gas networks.

The gas supply capacity of the natural gas network to NGFUs is redundant. Because the gas consumption of NGFUs in ENO optimal scheduling is feasible for GNO, so the cost of ENO in DO is minimum and there is no load shedding.

Hourly cost, the total cost of ENO, and hourly generation of G1 are shown in Fig. 4. DO hourly cost of electricity network is less than CO in day 1, the main

reason is non-max generation dispatched with relatively cheaper unit G1 as Fig. 5 shown. It can be seen that the first day of CO does not allow electricity network to have optimal scheduling. But after the CR updated once, DO total cost of electricity network is the same with CO, the scheduling results of the two coordinated operation methods are consistent.



Fig. 4. The hourly and total cost of ENO, hourly generation of G1.

From Fig. 5, the CR's variation trend of NGFUs is gradually converging. CR of G1 is significantly higher than other units, G1 has an advantage in GNO optimization sub-problem-A and gas consumption constraint will be slack. So, CO reaches an optimal equilibrium after the second day.



Fig. 5. Variation trend of CR.

#### 5. CONCLUSIONS AND FUTURE WORK

This paper proposed a novel framework for coordinated operation of electricity and natural gas networks. In the long-term, this framework will not affect the local optimal scheduling for both electricity network and natural gas network. An improve SCP method based SOC with reasonable initial expansion value is adopted for GNO sub-problem, this method has fast convergence characteristics. Future work on decentralized and coordinated operation with natural gas pipelines congestion and decentralized and coordinated operation with demand response will be investigated in due course.

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