STATE-OF-CHARGE ESTIMATION OF HYBRID ENERGY STORAGE SYSTEM USING FRACTIONAL ORDER MODELS

Yujie WANG^{*}, Chang Liu, Rui Pan, Zonghai Chen Department of Automation, University of Science and Technology of China, Hefei, Anhui 230027, P.R. China

ABSTRACT

Accurate and robust real-time state estimation is essential to the reliable and safe operation of the hybrid energy storage system. This paper handles a closed-loop method for state-of-charge estimation of lithium-ion battery and ultracapacitor hybrid system. In this work, a fractional-order model is developed to approximate the dynamic behavior of the lithium-ion battery and ultracapacitor. Then, a closed-loop method is proposed for model parameter and state-of-charge estimation. Experiments under dynamic load profiles are used to verify the proposed method. The experimental results indicate that the proposed method can obtain robust estimation results for the hybrid energy storage system, and is appropriate for real-time systems.

Keywords: Lithium-ion battery, ultracapacitor, fractional order model, state estimation.

1. INTRODUCTION

With the rapid development of the electric vehicle (EV) and smart grid (SG), the demands for advanced energy storage systems (ESS) are on the rise. The lithiumion battery which is featured by high energy density and long lifespan has been widely used in EV and SG applications. In EV applications, the batteries usually suffer from frequent charge and discharge which directly affects their lifespan and shortens their service time. To overcome the drawbacks, the ultracapacitors are used to group with the batteries in order to improve the power density of the ESS [1]. By combining with the ultracapacitors, the load current on the batteries can be reduced, and the power capability of the hybrid ESS can be improved which is considered as a better utilization than the individual operation of the battery.

Accurate and robust modeling and state estimation are essential to a reliable and safe operation for both the lithium-ion batteries and ultracapacitors. Diverse model and state estimation algorithms have been reported in the literature. For system modeling, the electrochemical pseudo-two-dimensional model (P2D model) presented by Doyle et al. [2] has been widely used to simulate the electrochemical behavior of the batteries. Tang et al. [3] proposed an equivalent-circuit modeling algorithm for the lithium-ion battery by using a model migration method. Wang et al. [4] investigated the equivalentcircuit modeling of lithium-ion batteries and ultracapacitors. Two of the time domain identification methods were introduced. Zhang et al. [5] compared different equivalent-circuit models for ultracapacitors.

An important character of the lithium-ion batteries is that multiple dynamic processes occur simultaneously during operation and give different timescales in nature. The fastest dynamic process is the movement of charge carriers through electrolytes and collector. With the increase of the timescale, there are electrochemical double layer effects and charge transfer reactions. The second is the solid phase ion diffusion and the stray of the porous electrode inductance. All lithium-ion batteries experience permanent and irreversible aging during static storage or cyclic operation, which is the slowest dynamic process. These dynamic processes are usually coupled. For example, electrochemical reactions accelerate the aging of the system, which in turn affects the charge and discharge performance of the battery. An ideal capacitor is incapable to mimic these dynamic behaviors of the double layer accurately because of capacitance dispersion phenomenon, so by introducing Warburg-like element and constant phase element (CPE) on the basis of the equivalent circuit, this paper establishes corresponding fractional-order models for both the lithium-ion batteries and ultracapacitors. Moreover, a framework for the state-of-charge (SOC) estimation is presented.

2. FOUNDATION OF FRACTIONAL ORDER CALCULUS

The fractional order calculus (FOC) was first proposed by Leibniz in 1965 to deal with arbitrary derivatives and integrals. Some physical systems have been found to have better FOC characteristics, such as the relaxation of organic dielectric materials and permanent magnet motors [6]. The fractional differential equation (FODE) is a natural way to describe distributed parameter systems. For energy storage devices, the FOC is a good representation of the dominant electrochemical behavior. In this work, the Grünald-Letnikov (GL) definition [7] is employed.

The detailed GL derivative formulation is given as follow:

$$\mathfrak{D}^{\mu}x(t) = \lim_{h \to 0} \frac{1}{h^{\mu}} \sum_{j=0}^{t/h} (-1)^{j} \binom{\mu}{j} x(t-jh)$$
(1)

$$\binom{\mu}{j} = \frac{\mu!}{j!(\mu-j)!} = \frac{\Gamma(\mu+1)}{\Gamma(j+1)\Gamma(\mu-j+1)}$$
(2)

where $\mathfrak{D}^{\mu}x(t)$ represents the Integral-differential operator, μ is the integral-differential order, [t/h] represents the integer part of t/h.

The gamma function is defined by:

$$\Gamma(\mu) = \int_{0}^{\infty} t^{\mu-1} e^{-t} dt \qquad (3)$$

3. MODELING OF HYBRID POWER SOURCE SYSTEM

3.1 Model for lithium-ion batteries

The fractional order model for the lithium-ion battery is shown in Fig.1.



Fig 1 Fractional order model of the lithium-ion battery

The impedances of CPE1 and CPE2 can be expressed by:

$$Z_{CPE_1} = \frac{1}{C_1 S^{\alpha_1}} \tag{4}$$

$$Z_{CPE_2} = \frac{1}{C_2 S^{\alpha_2}} \tag{5}$$

where C_1 , C_2 are constants accounting for the main capacitance effect of CPE, *s* is the complex variable, and α is the fractional-order coefficient.

3.2 Model for ultra-capacitors

The fractional order model for the ultracapacitor is shown in Fig.2.

$$\begin{array}{c} CPE \\ R_{S} \\ P_{+} \\ R_{C} \end{array}$$

Fig 2 Fractional order model of the ultracapacitor The impedances of CPE and Warburg-like element can be expressed by:

$$Z_{CPE} = \frac{1}{CS^{\alpha_1}} \tag{6}$$

$$Z_{W} = \frac{1}{WS^{\alpha_{2}}} \tag{7}$$

3.3 Model Identification

Based on circuit analysis, the transfer function of the fractional-order models for the lithium-ion battery and the ultracapacitor can be written as:

$$Y(t) / I(t) = (R_0 + R_1 + R_2 + R_1 C_1 (R_0 + R_2) \mathfrak{D}^{\alpha_1} + R_2 C_2 (R_0 + R_1) \mathfrak{D}^{\alpha_2} + R_0 R_1 R_2 C_1 C_2 \mathfrak{D}^{\alpha_1 + \alpha_2}) / (1 + R_1 C_1 \mathfrak{D}^{\alpha_1} + R_2 C_2 \mathfrak{D}^{\alpha_2} + R_1 R_2 C_1 C_2 \mathfrak{D}^{\alpha_1 + \alpha_2})$$
(8)

$$U_{0}(t) / I(t) = (R_{0} + R_{1} + R_{2} + R_{1}C_{1}(R_{0} + R_{2})\mathfrak{D}^{\alpha_{1}} + R_{2}C_{2}(R_{0} + R_{1})\mathfrak{D}^{\alpha_{2}} + R_{0}R_{1}R_{2}C_{1}C_{2}\mathfrak{D}^{\alpha_{1}+\alpha_{2}})$$
(9)
$$/(1 + R_{1}C_{1}\mathfrak{D}^{\alpha_{1}} + R_{2}C_{2}\mathfrak{D}^{\alpha_{2}} + R_{1}R_{2}C_{1}C_{2}\mathfrak{D}^{\alpha_{1}+\alpha_{2}})$$

Noting that $Y(t) = OCV(t)-U_0(t)$ in Eq.(8). Then the time-domain parameter identification algorithm such as the particle swarm optimization (PSO) can be used for the parameter identification of the fractional-order models.

4. FRAMEWORK OF SOC ESTIMATION

The SOC is a critical parameter in the energy management system. The SOC at time k can be expressed as z[k] which can be expressed as follows:

$$z k = z k_0 - \frac{\eta}{Q} \sum_{t=k_0+1}^{k} l[t]$$
 (10)

where $z[k_0]$ represents the SOC at time k_0 , Q represents the rated capacity of the energy storage device, η represents the coulomb efficiency and the current is defined as I (positive for discharge).

Based on the fractional-order models, the modelbased estimation framework for the SOC estimation of the battery and ultracapacitor can be applied. In this work, the EKF is employed for the state estimation for both the battery and the ultracapacitor hybrid system. Based on Eq.(10) and the fractional-order models, the general state-space model can be written as:

$$x_k = f(x_{k-1}, u_{k-1}, w_{k-1})$$
 (11)

$$\mathbf{y}_k = h(\mathbf{x}_k, \mathbf{u}_k, \mathbf{v}_k) \tag{12}$$

where w_k and v_k are independent Gaussian noise processes having covariance matrices \sum_w and \sum_v .

The main steps of the EKF are as follows:

Step 1: Initialization. For k = 0, set:

$$\hat{x}_{0}^{+} = E[x_{0}]$$
 (13)

$$\Sigma_{x,0}^{+} = E[(x_{0} - \hat{x}_{0}^{+})(x_{0} - \hat{x}_{0}^{+})^{T}]$$
(14)

Step 2: For k = 1,2,..., calculate:

(a) State-prediction time update:

$$\hat{x}_{k}^{-} = f(x_{k-1}, u_{k-1}, w_{k-1})$$
 (15)

(b) Error-covariance time update:

$$\Sigma_{x,k}^{-} = E[(x_k - \hat{x}_k^{-})(x_k - \hat{x}_k^{-})^{T}]$$
(16)

(c) Measurement time update:

$$\hat{y}_{k} = h(\hat{x}_{k}^{-}, u_{k}, v_{k})$$
 (17)

(d) Estimator gain update:

$$L_{k} = E[(x_{k} - \hat{x}_{k}^{-})(y_{k} - \hat{y}_{k})] / E[(y_{k} - \hat{y}_{k})(y_{k} - \hat{y}_{k})^{T}]$$
(18)

(e) State-prediction measurement update:

$$\hat{x}_{k}^{+} = \hat{x}_{k}^{-} + L_{k}(y_{k} - \hat{y}_{k})$$
 (19)

(f) Error-covariance meas. update:

$$\boldsymbol{\Sigma}_{k}^{+} = \boldsymbol{\Sigma}_{k}^{-} - \boldsymbol{L} \boldsymbol{\Sigma}_{k} \boldsymbol{L}_{k}^{T}$$

$$(20)$$

5. RESULTS ANALYSIS AND DISCUSSION

In order to verify the proposed fractional-order models for the hybrid system and the SOC estimation algorithm, experiments under dynamic load profiles are presented in this section. The commercial lithium iron phosphate and the Maxwell ultracapacitor are employed in the tests.







Fig 4 (a) Prediction of the SOC. (b) Prediction error of the SOC.

The identification results of the fractional-order model parameters are shown in Table. 1. The prediction errors of the battery and ultracapacitor are shown in Fig. 3 (a) and (b). Fig 3 (a) shows the prediction of the voltage based on the presented fractional-order models. Fig 3 (b) shows the voltage prediction error of the hybrid energy storage system. The SOC and the SOC estimation error of the hybrid system are shown in Fig 4 (a) and (b). The results show that the proposed method can track the real value with fast convergence, and the SOC error is within 5%.

Battery FOM	Values
R ₀	6.6 mΩ
<i>R</i> ₁	41 mΩ
<i>R</i> ₂	69 mΩ
<i>C</i> ₁	460 F
C ₂	1900 F
α ₁	0.62
α ₂	0.41
Ultracapacitor FOM	Values
Rs	0.96 mΩ
С	816 F
R _c	2.5 mΩ
W	3023 F
α1	0.9874
α ₂	0.9877

Table. 1. Parameters of the fractional-order models

6. CONCLUSIONS

This paper presents a fractional-order model-based state-of-charge estimation algorithm for the lithium-ion battery and ultracapacitor hybrid system. First, the fractional-order models for the lithium-ion battery and ultracapacitor, and the fractional order calculus are introduced. The models can approximate the dynamic behavior of the lithium-ion battery and ultracapacitor accurately. Second, a closed-loop EKF method is proposed for state-of-charge estimation of the hybrid system. Finally, experiments under dynamic load profiles are used to verify the proposed method. The experimental results indicate that the proposed method can obtain robust estimation results for the hybrid energy storage system.

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