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NEWTON-BASED DISTRIBUTED VOLTAGE CONTROL METHOD FOR HIGH PENETRATION OF PV GENERATION

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ABSTRACT

Due to the integration of distributed photovoltaic (PV) and access of electronic devices, power systems are suffering from serious voltage problems, and therefore they require greater flexibility. By using appropriate methods, a PV cluster can autonomously regulate reactive power output in a distributed manner to improve voltage profile. In this paper, a distributed Newton-based reactive power control method to realize distributed voltage control for high penetration of PV generation is introduced, which can fast respond to voltage mismatch and address the robustness issues of (de-)centralized approaches against communication delay and noises. The proposed distributed control scheme for PV clusters can coordinate PV to provide voltage regulation in a more efficient, reliable and flexible way than existing decentralized methods.

Keywords: PV cluster, distribution network, voltage control, distributed control, Newton method

1. INTRODUCTION

Solar energy, as one of the most prominent renewable resources, is recognized as a feasible alternative. However, high penetration of distributed energy resources may induce severe disturbance due to its intrinsic randomness and possible structural weakness of the grid, and present significant challenges and opportunities for the operation of distribution networks. Furthermore, PV generation integrates grid by power electronics inverter, which can decrease system stability relatively for shortage of regulation ability.

Especially, high variability of PV generation results in unexpected voltage fluctuations, at time-scales much faster than the existing voltage control devices, such as on-load tap changers and capacitor banks [1],[2]. Besides, the installation of large amount of distributed PV generation increases the complexity of the voltage control problem, and requires voltage control methods to increase response speed. Thankfully, the reactive power regulation ability of PV inverter gives rise to unprecedented capability of fast voltage regulation to meet voltage security limits.

The distributed calculation techniques can improve respond speed to some extent by mitigating the computation burden [3]. Based on this concept, the multiple agent system (MAS) based methods have been developed aiming to solve the optimal reactive power control problem. Several distributed optimization algorithms have been proposed to solve the problem using information exchange among neighboring buses. For instance, alternating direction method of multipliers (ADMM) has been utilized in [4],[5], while a subgradient iterative solver has been developed by [6]. There are two standard approaches used in distributed optimization problems of MAS. One of the approaches uses dual decomposition and subgradient update method [7],[8], while the other makes use of consensus-based schemes [9]. Nevertheless, all these decentralized approaches have an inherent linear convergence, and thus slow control performance. Due to their fast convergence rate, distributed Newton-type methods have been used in network optimization problems. The Hessian matrix can be used to determine a better descent direction if it so happens to also be computable in a distributed manner, and therefore approximate Newton methods exhibit faster convergence relative to their corresponding first order methods. This paper adapts the curvature estimation technique of the Broyden-Fletcher-Goldfarb-

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Shanno (BFGS) quasi-Newton optimization method for use in distributed settings. This adaption leads to the development of the distributed BFGS method that can be implemented with inverters. The advantages of the proposed method relative to existing methods are that it doesn't require computation of Hessian, and it applies in any scenario in which gradient is distributedly computable irrespectively of the structure of the Hessian matrix.

Besides, most decentralized approaches still require high-quality communication of the measurement and control signals, which is not yet a reality for almost all distribution networks. Therefore, potential communication delay and noises can challenge the optimality and stability for real-time implementation. In this paper, we design a control strategy using local information and neighboring information, i.e., bus voltage magnitude measurements. Once the communication failure occurs, a local control framework using only local information will effectively respond to voltage deviation. Since the bus voltage is more significantly affected by local reactive power, the compromised method can get an acceptable voltage control result. To the best of the authors' knowledge, this paper makes the following contributions:

1) A distributed voltage control method for a high-penetration PV cluster is introduced. Compared to centralized methods, the proposed scheme gives rise to flexibility and reliability.

2) A quasi-Newton method is developed, which has a super-linear convergence rate. The distributed voltage control method has much better performance than those first order algorithms.

3) The proposed method applies to actual conditions in distribution networks: communication failures and delay. It can even guarantee the voltage security under no communication using only local information.

2. POWER FLOW AND OPTIMIZATION MODEL

Assume that a distribution network with *N*+1 buses represented by the set $\mathcal{N}=\{0,1,\cdots,N\}$, and feeders represented by the set $\mathcal{L}=\{(i,j)\}\subset\mathcal{N}\times\mathcal{N}$. For a network with tree-topology, the number of feeders $|\mathcal{L}|=N$. Bus 0 is the point of common coupling (PCC), or the distribution substation. The voltage of bus 0 is taken as reference. For bus *i*, let V_i denotes the voltage magnitude, and p_i, q_i denote the bus active and reactive power injection. For each feeder (i,j), r_{ij} and x_{ij} denote its resistance and reactance, and P_{ij}, Q_{ij} denote the active and reactive power from bus *i* to bus *j*. Besides, the set $\mathcal{N}_j\subset\mathcal{N}$ denotes neighboring bus of bus *j*, which are downstream from the reference bus. In this paper, we adopt the DistFlow equations [10] to model the power flow. Assuming the loss is negligible, we construct a linear approximation of DistFlow, which can be expressed as

$$P_{ij}-\sum_{k\in\mathcal{N}_j}P_{jk}=-p_j \tag{1}$$

$$Q_{ij}-\sum_{k\in\mathcal{N}_j}Q_{jk}=-q_j \tag{2}$$

$$V_i-V_j=r_{ij}P_{ij}+x_{ij}Q_{ij} \tag{3}$$

The total injected reactive power $q_j=q_j^g-q_j^d$, where q_j^g denotes the reactive power of PV inverter at bus *j*, and q_j^d denotes the reactive power load at bus *j*. Therefore, the task of the voltage control is to solve for reactive power of PV inverters for given active power injection and reactive power load. The optimization model can be written as

$$\begin{aligned}&\min_{\mathbf{q}^g}\ \frac{1}{2}\left\|\mathbf{V}-\boldsymbol{\mu}\right\|_2^2\\&\text{s.t. }\underline{q}_j^g\le q_j^g\le\overline{q}_j^g\\&\quad V_0=1,\ \text{and (1)}-\text{(3)}\end{aligned} \tag{4}$$

where bold symbols denote column vector, and $\boldsymbol{\mu}$ is the desired voltage profile. The bounds of q_j^g depends on the capacity and power factor rating of the inverter.

We use matrix $\mathbf{M}^0$ denote the incidence matrix for the distribution network. Let $M_{il}^0=1$ and $M_{jl}^0=$-1 when $j\in\mathcal{N}_i$, and $\mathbf{m}_0^T$ denote the first row of $\mathbf{M}^0$, which corresponds to bus 0, and the rest of matrix (size $N\times N$) is denoted by **M**. As we know, the matrix **M** is invertible. In this way, power flow equations (1)-(3) can be rewritten as

$$\mathbf{MP}=\mathbf{p} \tag{5}$$

$$\mathbf{MQ}=\mathbf{q} \tag{6}$$

$$\mathbf{m}_0+\mathbf{M}^T\mathbf{V}=\mathbf{D}_r\mathbf{P}+\mathbf{D}_x\mathbf{Q} \tag{7}$$

where $\mathbf{D}_r$ is a diagonal matrix composed of r_{ij}, and $\mathbf{D}_x$ is a diagonal matrix composed of x_{ij}. Substituting (5)-(6) into (7) yields

$$\mathbf{M}^T\mathbf{V}=\mathbf{D}_r\mathbf{M}^{-1}\mathbf{p}+\mathbf{D}_x\mathbf{M}^{-1}\mathbf{q}-\mathbf{m}_0 \tag{8}$$

or equivalently,

$$\begin{aligned}\mathbf{V} &= \mathbf{Rp} + \mathbf{Xq} - \mathbf{M}^{-T}\mathbf{m}_0 \\ &= \mathbf{Rp} + \mathbf{Xq}^g - \mathbf{Xq}^d - \mathbf{M}^{-T}\mathbf{m}_0 \\ &= \mathbf{Xq}^g + \bar{\mathbf{V}}\end{aligned} \tag{9}$$

where $\mathbf{R}=\mathbf{M}^{-T}\mathbf{D}_r\mathbf{M}^{-1}$, $\mathbf{X}=\mathbf{M}^{-T}\mathbf{D}_x\mathbf{M}^{-1}$, and we denote the voltage profile under no control by $\bar{\mathbf{V}} = \mathbf{Rp} - \mathbf{Xq}^d - \mathbf{M}^{-T}\mathbf{m}_0$.

It can be proved that $\mathbf{R}$ and $\mathbf{X}$ are positive definite. In fact, the inverse of $\mathbf{X}$ satisfies

$$\mathbf{B} = \mathbf{X}^{-1}=\mathbf{M}\mathbf{D}_x^{-1}\mathbf{M}^T \tag{10}$$

It is our familiar matrix used in DC power flow. Therefore, we have the following linearized power flow model:

$$\mathbf{q}^g = \mathbf{B}(\mathbf{V} - \bar{\mathbf{V}}) \tag{11}$$

Let $\tilde{\mathbf{V}} = \boldsymbol{\mu} - \bar{\mathbf{V}}$, the optimization model becomes

$$\begin{aligned}&\min_{\mathbf{q}^g} \ f(\mathbf{q}^g) = \frac{1}{2}\left\|\mathbf{Xq}^g - \tilde{\mathbf{V}}\right\|_2^2 \\ &\text{s.t. } \underline{\mathbf{q}}^g \le \mathbf{q}^g \le \bar{\mathbf{q}}^g\end{aligned} \tag{12}$$

It is a box-constrained quadratic program problem, which can be easily solved by using centralized method. In this paper, we should consider how to solve the problem in a distributed manner, and exhibit a fast convergent at the meantime.

Although the power flow model is derived for tree-topology networks, it can also be generalized to meshed networks. Because the matrix **B** coincides with the fast-decoupled power flow model for transmission network analysis, the model holds for general distribution networks [11].

3. DISTRIBUTED QUASI-NEWTON METHOD

Calculating the gradient of the objective of (12) yields

$$\mathbf{g} = \mathbf{X}(\mathbf{Xq}^g - \tilde{\mathbf{V}}) = \mathbf{X}(\mathbf{V} - \boldsymbol{\mu}) \tag{13}$$

where $\tilde{\mathbf{X}}$ is a matrix only preserving diagonal entries and neighboring elements of bus *i* in row *i*. As we know, bus voltage is more significantly affected by local and neighboring reaction power injection compared to those elsewhere, so the approximation is acceptable. Therefore, the gradient of bus *i* and be obtained using local and neighboring voltage information. This motivates us to adopt the gradient projection method to solve the problem in a distributed manner. However, the gradient-based method suffers from slow convergence. In this section, we will introduce our control framework by solving the problem (12) using distributed BFGS method.

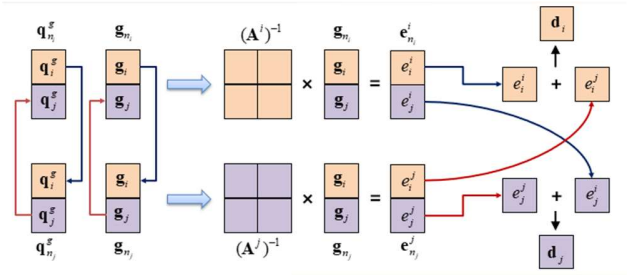

Fig 1 Information exchange process between neighbors

Newton methods inherently have second-order convergence, and hence are applied to distributed optimization. For (12), the Newton iteration formula is

$$\mathbf{q}^g(k+1) = \mathbf{q}^g(k) - \varepsilon \cdot \mathbf{H}(k)^{-1} \cdot \mathbf{g}(k) \tag{14}$$

where **H** is Hessian matrix, ε is step size, and $\bullet(k)$ is the iterative value at iteration *k*. However, the inversion of Hessian matrix definitely requires global information, which cannot be realized in a distributed manner. To tackle this, we use an approximate matrix **A** to substitute for Hessian matrix, and the iterative formula become

$$\mathbf{q}^g(k+1) = \mathbf{q}^g(k) - \varepsilon \cdot \mathbf{A}(k)^{-1} \cdot \mathbf{g}(k) = \mathbf{q}^g(k) - \varepsilon \cdot \mathbf{d}(k) \tag{15}$$

The matrix **A** should be a definite positive symmetric matrix, and it should satisfy the secant condition:

$$\mathbf{A}(k+1)\mathbf{u}(k) = \mathbf{r}(k) \tag{16}$$

where

$$\mathbf{u}(k) = \mathbf{q}^g(k+1) - \mathbf{q}^g(k) \tag{17}$$

$$\mathbf{r}(k) = \mathbf{g}(k+1) - \mathbf{g}(k) \tag{18}$$

Using difference to represent differential, we realize approximating Hessian matrix by **A**. But it should be noted that, the solution of (16) is not unique.

Therefore, we use Gaussian differential entropy to ensure matrix **A** as close as the result at last iteration:

$$\begin{aligned}&\mathbf{A}(k+1) = \arg\min_{\mathbf{Z}} \ \mathrm{tr}[\mathbf{A}(k)^{-1}\mathbf{Z}] - \lg\left|\mathbf{A}(k)^{-1}\mathbf{Z}\right| - N \\ &s.t.\ \mathbf{Zu}(k) = \mathbf{r}(k),\ Z \succeq \mathbf{0}\end{aligned} \tag{19}$$

This way we can obtain unique solution of **A**, the iteration formula is

$$\mathbf{A}(k+1) = \mathbf{A}(k) + \frac{\mathbf{r}(k)\mathbf{r}(k)^T}{\mathbf{r}(k)^T\mathbf{u}(k)} - \frac{\mathbf{A}(k)\mathbf{u}(k)\mathbf{u}(k)^T\mathbf{A}(k)}{\mathbf{u}(k)^T\mathbf{A}(k)\mathbf{u}(k)} \tag{20}$$

Thus, $\mathbf{A}(k+1)$ can be computed using the previous approximation matrix $\mathbf{A}(k)$ as well as the variable $\mathbf{u}(k)$ and gradient $\mathbf{r}(k)$.

However, it is obvious that the distributed implementation of (20) is difficult because neither $\mathbf{A}(k)$ nor $\mathbf{A}(k)^{-1}$ have a sparsity pattern to permit local evaluation of descent direction, and the inner product $\mathbf{r}(k)^T\mathbf{u}(k)$ requires global information. Next,

we resolve the issue by modifying the iteration in a distributed manner.

First, we define the *i*th block of a vector $\mathbf{z} \in \mathbb{R}^N$ is denoted as $\mathbf{z}_i \in \mathbb{R}$, and $\mathbf{z}_{n_i} \in \mathbb{R}^{m_i}$ denotes the components in n_i (n_i is the bus set of neighboring bus of *i*, and m_i is the number of neighboring buses of *i*). Likewise, define $\mathbf{Z}_{n_i} \in \mathbb{R}^{m_i \times m_i}$ to be the m_i rows and columns of $\mathbf{Z} \in \mathbb{R}^{N \times N}$ corresponding to buses in n_i.

Define the diagonal normalization matrix $\mathbf{D} \in \mathbb{R}^{N \times N}$ whose *i*th entry is m_i^{-1} and a small regularization parameter $\gamma > 0$, and define the modified neighborhood variable and gradient variation:

$$\tilde{\mathbf{u}}_{n_i}(k) = \mathbf{D}_{n_i}[\mathbf{q}^g_{n_i}(k+1) - \mathbf{q}^g_{n_i}(k)] \tag{21}$$

$$\tilde{\mathbf{r}}_{n_i}(k) = \mathbf{g}_{n_i}(k+1) - \mathbf{g}_{n_i}(k) - \gamma \tilde{\mathbf{u}}_{n_i}(k) \tag{22}$$

Because the modified neighborhood variable and gradient variation use only information bus *i* can locally access through neighbors, we can compute a local Hessian approximation $\mathbf{A}^i(k) \in \mathbb{R}^{m_i \times m_i}$ as follows:

$$\mathbf{A}^i(k+1) = \mathbf{A}^i(k) + \frac{\tilde{\mathbf{r}}_{n_i}(k)\tilde{\mathbf{r}}_{n_i}(k)^T}{\tilde{\mathbf{r}}_{n_i}(k)^T \tilde{\mathbf{u}}_{n_i}(k)} - \frac{\mathbf{A}^i(k)\tilde{\mathbf{u}}_{n_i}(k)\tilde{\mathbf{u}}_{n_i}(k)^T \mathbf{A}^i(k)}{\tilde{\mathbf{u}}_{n_i}(k)^T \mathbf{A}^i(k)\tilde{\mathbf{u}}_{n_i}(k)} + \gamma \mathbf{I} \tag{23}$$

The matrices $\mathbf{A}^i(k)$ along with an additional regularization parameter $\Gamma > 0$ are used by bus *i* to compute the neighborhood descent direction $\mathbf{e}^i_{n_i}(k) \in \mathbb{R}^{m_i}$ as

$$\mathbf{e}^i_{n_i}(k) = -(\mathbf{A}^i(k)^{-1} + \Gamma \mathbf{D}_{n_i})\mathbf{g}_{n_i}(k) \tag{24}$$

which contains components for variables of bus *i* itself and all neighbors. Likewise, neighboring buses $j \in n_i$ contain a descent component of the form $e^j_i(k)$. Therefore, the local descent $d_i(k)$ is given by the sum of the components for all neighbors:

$$d_i(k) = \sum_{j \in n_i} e^j_i(k) \tag{25}$$

Substituting (25) into (15) yields

$$q^g_i(k+1) = q^g_i(k) - \varepsilon \cdot d_i(k) \tag{26}$$

Obviously, the iteration can be implemented only using local and neighboring voltage information, according to (13), (21)-(26). The information exchange process between neighboring buses is shown in Figure 1.

Considering the possible communication interrupt during control process, the method should allow for local control schemes using only local voltage. It turns out that the voltage norm in (12) needs to be weighted by **B**:

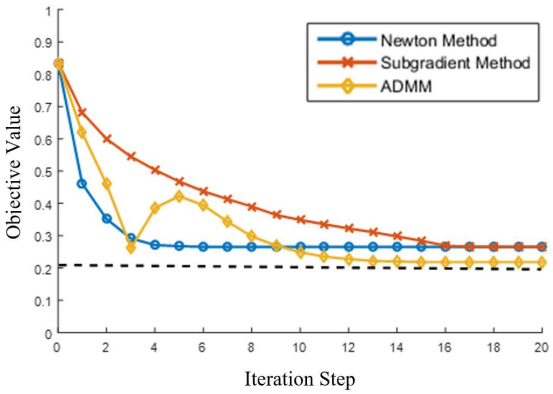

Fig 2 Convergence speed comparison of three methods in 33 buses system

$$f(\mathbf{q}^g) = \frac{1}{2}(\mathbf{X}\mathbf{q}^g - \tilde{\mathbf{V}})^T \mathbf{B}(\mathbf{X}\mathbf{q}^g - \tilde{\mathbf{V}}) \tag{27}$$

If every PV has unlimited reactive power capability, the optimal solutions to both objectives coincide. This implies that under abundant reactive power resources, the optimal solution to the weighted (27) has the potential to closely approximate the minimum of (12). Calculating the gradient of the objective of (27) yields

$$\mathbf{g} = \mathbf{X}\mathbf{q}^g - \tilde{\mathbf{V}} = \mathbf{V} - \boldsymbol{\mu} \tag{28}$$

In this way, the voltage control can be implemented in a fully distributed manner – using local information. Even if the communication fully breaks down, the proposed method still can ensure voltage security of the system with a sub-optimal solution, which provides higher reliability and flexibility.

4. NUMERICAL TESTS

In this section, we present numerical test results to demonstrate the effectiveness of the quasi-Newton voltage control method. To better compare various algorithms, the desired voltage magnitude μ_i is set to be unit in p.u. for every bus. We compare the convergence, optimality and control performance of three method: the proposed quasi-Newton method, ADMM and projected subgradient method. We adopt modified 33, 69 and 123 buses standard test systems for analysis. 4 cases of PV injection are carried out in each system. The solution of ACOPF is taken as the actual optimal value. V_0 is fixed at 1, and the bounds of bus voltage are set to be [0.95, 1.05] in p.u. value. All numerical tests are implemented in MATLAB, and we use MATPOWER to solve for the actual power flow and obtain the voltage results as feedback for iteration. In this way, the actual bus voltage magnitude, instead of the one obtained by the DistFlow model, is used for

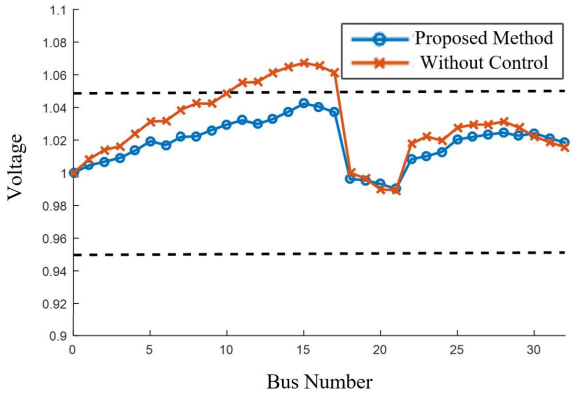

Fig 3 Voltage profiles under the proposed control method and no control

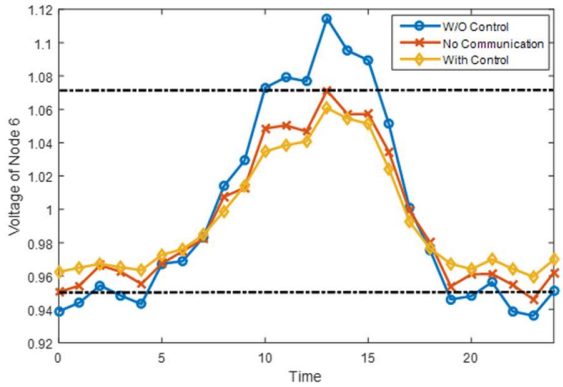

Fig 4 Voltage variation curve of bus 6 in 33 buses system in 24 hours

updating the voltage control commands and numerical performance comparisons.

The convergence speed comparison of three methods in the first case of 33 buses system is shown in Fig 2. The blue curve represents the iteration results under the proposed method, the yellow curve represents the iteration results under ADMM, and the red curve represents the iteration results under subgradient method. It can be concluded that the convergence rate under the proposed method is obviously faster than those under the other two methods

Definitely, the most important index to evaluate the effectiveness of the method is the voltage control performance. Fig 3 compares the bus voltage profiles by using the proposed control method and no control under the second case of 33 buses system. We can find that the voltage profile under the proposed method is significantly better than that without control, and the control method ensure the voltage security of all the buses.

The required iteration steps for different methods to converge to the solution under all the cases in different test systems are shown in Table 1 to Table 3. It can be concluded that, the iteration steps required to reach convergence under the proposed method is significantly less than the other two methods in all cases and test systems. In fact, the quasi-Newton method can make iteration step decrease one order of magnitude compared to the other two methods.

Table 1 Iteration steps for different methods in 33 buses test system

Case 1	Newton	Subgradient	ADMM
1	6	16	16
2	6	18	17
3	7	22	24
4	8	24	24

Table 2 Iteration steps for different methods in 69 buses test system

Case 2	Newton	Subgradient	ADMM
1	9	45	41
2	10	51	55
3	12	56	58
4	13	66	66

Table 3 Iteration steps for different methods in 123 buses test system

Case 1	Newton	Subgradient	ADMM
1	15	98	134
2	17	103	141
3	18	106	139
4	20	112	152

The above analysis mainly focuses on the voltage control performance under good peer-to-peer (P2P) communication conditions. However, communication failures are unavoidable in actual distribution networks. Therefore, it is necessary to verify the control performance under no communication. In previous sections, we introduce the compromised local control strategy. Using an actual daily variation curve of irradiation intensity as input, we compared the control performance under complete communication, local control and no control. Fig 4 shows the voltage variation curve of bus 6 in 33 buses system in 24 hours under different conditions. From the figure, it can be found that the voltage magnitude of bus 6 will exceed the security bounds at some points. By using the proposed method with complete communication, the bus voltage is maintained within the range. Even if the communication is interrupted, the local strategy also can ensure the security by operating the system at a sub-optimal point.

Although high variability of PV generation results in unexpected voltage fluctuation, the reactive power regulation ability of PV inverter gives rise to unprecedented capability of fast voltage regulation to

meet voltage security limits. The numerical tests demonstrate that, by using the proposed method, a huge amount of PV generation can be effectively organized and controlled to provide corresponding support and exert “friendly” effect on power systems.

5. CONCLUSION

This paper proposes a distributed Newton-based reactive power control method to realize distributed voltage control for distribution networks with high penetration of PV generation, which can fast respond to voltage mismatch and address the robustness issues of (de-)centralized approaches against communication failures. The proposed distributed control scheme for PV clusters can coordinate PV to provide voltage regulation in a more efficient, reliable and flexible way than existing decentralized methods. Compare to subgradient method and ADMM, the proposed method can make iteration step decrease one order of magnitude, which can realize online optimization and online voltage control. Even if the communication system breaks down, the local strategy also can ensure the voltage security of the system.

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