Applied Energy Symposium 2019: Low carbon cities and urban energy systems
October 16-18, 2019, Xiamen, China
Paper ID: 0155

NUMERICAL SIMULATION OF GROUND HEAT EXCHANGERS BASED ON MODEL ORDER REDUCTION METHOD

Zhendi Ma[1] Qiongxiang Kong[1*] Peifen Wang[2] Siyan Liu[1] Liwen Jin[1]

1 Group of Building Environment and Sustainability Technology, School of Human Settlements and Civil Engineering, Xi'an Jiaotong University, Xi'an 710049,China.

2 School of Mathematics and Statistics, Xi'an Jiaotong University, Xi'an, 710049, China.
[*]Corresponding author: qxkong@xjtu.edu.cn

ABSTRACT

The model order reduction (MOR) method is to convert a large system into a small system, which can reduce the amount of data calculation and save time and cost while maintaining a certain precision. In this paper, Krylov subspace, balanced truncation and Laguerre orthogonal polynomial MORs are developed to the numerical simulation of the heat transfer characteristics of ground heat exchangers under laminar and turbulent conditions. Firstly, the finite volume method is used to discretize the two-dimensional governing equation into an ordinary differential equation with respect to the pipe length direction. Then three MOR systems are established respectively. Finally, the specific examples are calculated. The results show that for a certain length of ground heat exchangers the total time taken by the balanced truncation method is much larger than that of the direct solution method, the Krylov subspace method and the Laguerre orthogonal polynomial method. The reasonable orders of the reduced order systems by the Krylov subspace method and the Laguerre orthogonal polynomial method are both 20. At this time, their solving time is all less than 11.3% (laminar flow) and 6.2% (turbulent flow) of the direct solution method. And the relative errors under the above conditions are all less than 10^{-7}. As the length of the calculated pipe increases, the efficiency of the two MOR methods is more prominent.

Keywords: model order reduction (MOR); ground heat exchanger; Krylov subspace; Laguerre orthogonal polynomial; balanced truncation

NONMENCLATURE

Abbreviations	
MOR	Model order reduction
KS	Krylov subspace
LP	Laguerre orthogonal polynomial
BT	Balanced truncation
DS	Direction solution
GSHP	Ground source heat pump
Symbols	
T	Temperature
w	Axial velocity
ρ	Fluid density
c_p	Specific heat capacity
λ	Thermal conductivity
λ_t	Turbulent thermal conductivity,
w_m	Average velocity
R	Pipe radius
η	Dynamic viscosity coefficient
η_t	Turbulent dynamic viscosity coefficient
p	Pressure
Nu	Nusselt number
l_m	Mixing length
T_w	Wall temperature

1. INTRODUCTION

The ground heat exchanger is the key to the safe, reliable and economic operation of the ground source heat pump system (GSHP). It will be very complicated,

Selection and peer-review under responsibility of the scientific committee of CUE2019

large in scale, long in time, high in cost, and limited by seasonal regions to determine the parameters of the buried tube heat exchanger. The numerical simulation method will reduce the cost and time-consuming. It is not limited by the experimental conditions and can simulate many working conditions.

In recent years, some scholars have studied the heat transfer characteristics of buried pipes by numerical simulation using commercial software. Li et al. [1] simulated the heat transfer characteristics of buried pipes with a depth of more than 2000 m using Fluent. What's more, Some scholars have used self-programming methods to conduct research related to buried pipes. Both Kuzmic et al. [2] and Guan et al. [3] simplified the U-shaped buried tube heat transfer problem by the equivalent diameter method and used the finite volume method to discretize, and used FORTRAN language programming to calculate.

However, the buried depth of the shallow buried pipe is 200-300 m and that of the medium-deep GSHP system is 2000-3000 m. Due to the large buried depth and many buried pipes in a GSHP system, the calculation area will be large. No matter what numerical calculation method is adopted, under the premise of ensuring accuracy, a large system will be formed after the control equation is discretized. Therefore, it will take up a lot of computational resources and take a long time to solve the system. Thus, a fast calculation method is needed to solve such problems.

At present, the main fast calculation methods include parallel computing methods and MOR methods. The parallel computing method requires a multi-core computer, which requires high performance on the computer, while the MOR does not. The MOR is to convert a large system into a small system for calculation, which can reduce the calculation amount of data and reduce the difficulty of analysis as well as save time and cost while maintaining a certain precision [4]. The theoretical research on the MOR [5] for continuous linear time-invariant systems has been relatively mature, including the Laguerre orthogonal polynomial model reduction method (LP-MOR) in the time domain and the Krylov subspace model reduction method (KS-MOR) in the frequency domain, balanced truncation model reduction method (BT-MOR), and proper orthogonal decomposition model reduction method (POD-MOR). These methods have been successfully applied to many engineering simulation fields such as large-scale integrated circuit problems [6], control systems [7], and fluid mechanical systems [8]. The MOR is also applied to the heat transfer calculation of the building envelope. Gao et al. [9] used the BT-MOR to simulate the heat transfer of the two-dimensional envelope structure. Our group [10] applied the KS-MOR and BT-MOR methods to simulate the dynamic heat transfer of one-dimensional single-layer and multi-wall and two-dimensional thermal bridges. The results show the advantages of KS-MOR method used to calculate the heat transfer of building envelopes.

The LP-MOR can be directly developed in the time domain and can maintain the coefficients in the original system output variable expansion. What's more, the results calculated by LP-MOR are independent of input variables. Due to the advantages of LP-MOR and the gap in the simulation calculation of the buried pipe based on the MOR method, KS-MOR, BT-MOR and LP-MOR were used to simulate the convective heat transfer of laminar or turbulent flow in buried pipes under extracting heat in winter and rejecting heat in summer. The appropriate order of the reduced order system and the effects of different MOR methods are analyzed.

2. ESTABLISHMENT OF THE ORIGINAL SYSTEM

2.1 The governing equation

For the flow in the buried pipe, a two-dimension fully developed convective heat transfer in the pipe model was established as shown in the Figure 1. The temperature governing equation is as followed:

$$\rho c_p w \frac{\partial T}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}[r(\lambda + \lambda_t)\frac{\partial T}{\partial r})] \tag{1}$$

Where ρ is the fluid density, kg/m^3; c_p is the specific heat capacity of fluid, J/(kg · K); w is the fluid velocity in the axial direction, m/s, which is relevant to the distance to the symmetry axis r; T is the fluid temperature, K; λ is the thermal conductivity, W/(m · K); λ_t is the turbulent thermal conductivity, W/(m · K)。

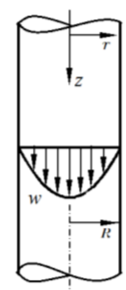

Fig 1 The coordinate system for heat transfer in pipes

However, under the turbulent flow condition, the axial velocity w needs to be obtained by solving the momentum governing equation:

$$\frac{dp}{dz}=\frac{1}{r}\frac{\partial}{\partial r}[r(\eta+\eta_t)\frac{\partial w}{\partial r})] \tag{3}$$

where η is the dynamic viscosity coefficient and η_t is the turbulent dynamic viscosity coefficient. The axial velocity w can be obtained by solving the dimensionless form of Equation (3).

The turbulent dynamic viscosity coefficient η_t can be acquired by the mixing length theory as Equation (4).

$$\eta_t=\rho l_m^2\left|\frac{\partial w}{\partial r}\right| \tag{4}$$

where l_m is the mixing length, which can be calculated as the literature[11].

The turbulent thermal conductivity λ_t can be calculated by Equation (5).

$$\lambda_t=\frac{\eta_t}{\sigma}c_p,\quad \sigma = 0.9 - 1 \tag{5}$$

Since η_t is related to w, the momentum governing equation needs to be solved by the iteration.

In order to focus on the calculation method, the boundary conditions of the pipe wall of the buried pipe are simply treated as a constant temperature first. Therefore, the boundary conditions are:

$$r=0,\frac{\partial T}{\partial r}=0;r=R,T=T_W \tag{6}$$

where T_w is the temperature of the pipe wall.

2.2 The establishment of the state equation and output equation

Many problems in the engineering field can be regarded as an input and output system. The relationship between the state variables and the input variables is usually expressed by first-order differential equations or differential equations, called state equations. The relationship between output and state variables and input variables is generally an algebraic equation called the output equation.

Based on the heat transfer characteristics of the fluid heat transfer governing equation (Equation (1)) in the buried pipe, in the radial direction (represented by r) and the tube length direction (axial direction denoted by z), Equation (1) was discretized in the radial direction to a set of ordinary differential equations (Equation (7a))by the finite volume method, which is the state equation. The state equation and output equation are as followed:

$$\begin{cases}\dfrac{dT(z)}{dz}=AT(z)+Bu(z) & (a)\\ y(z)=CT(z)+Du(z) & (b)\end{cases} \tag{7}$$

where $T(z)=[T_1\quad T_2\quad \cdots\quad T_{n-1}\quad T_n]^T\in R^{n\times1}$, $u(z)=T_W\in R$, $B=[0\quad 0\quad \cdots\quad 0\quad a_{n,E}]^T\in R^{n\times1}$,

$$A=\begin{pmatrix}-a_{1,E} & a_{1,E} & & & \\ a_{2,W} & -a_{2,P} & a_{2,E} & & \\ & \ddots & \ddots & \ddots & \\ & & a_{n-1,W} & -a_{n-1,P} & a_{n-1,E}\\ & & & a_{n,W} & -a_{n,P}\end{pmatrix}\in R^{n\times n},$$

$a_{i,E}=\dfrac{2[\lambda_{i,e}+(\lambda_t)_{i,e}]r_{i,e}}{(\rho c_p)_i w_i(r_{i,e}^2-r_{i,w}^2)(\delta r)_i}$, $a_{i,W}=\dfrac{2[\lambda_{i,w}+(\lambda_t)_{i,w}]r_{i,w}}{(\rho c_p)_i w_i(r_{i,e}^2-r_{i,w}^2)(\delta r)_{i-1}}$,

$a_{i,P}=a_{i,E}+a_{i,W}$.

The coefficient matrix C and D are decided by the output variables needed.

3. ESTABLISHMENT OF THE REDUCED-ORDER SYSTEM

For the original system of the buried tube heat transfer problem, the Equation (7) can be directly solved for each n-dimensional variable in each section of the pipe, which is referred to as the direct solution (DS) method. Besides, the MOR can be used to reduce the n-dimensional variable to the r-dimensional variable in order to obtain the reduced order system. Then it can be solved by the numerical method. The reduced order system can be expressed as Equation (8):

$$\begin{cases}\dfrac{d\tilde{T}(z)}{dz}=\tilde{A}\tilde{T}(z)+\tilde{B}u(z) & (a)\\ \tilde{y}(z)=\tilde{C}\tilde{T}(z)+\tilde{D}u(z) & (b)\end{cases} \tag{8}$$

Where $\tilde{T}(z)\in\mathbb{R}^m,\tilde{A}\in\mathbb{R}^{m\times m},\tilde{B}\in\mathbb{R}^m,\tilde{C}\in\mathbb{R}^{k\times m},\tilde{D}\in\mathbb{R}^k$, $m\ll n$, $T(z)\approx V\tilde{T}(z),\tilde{y}(z)\approx y(z),\tilde{A}=V^TAV,\tilde{B}=V^TB,\tilde{C}=CV$, $\tilde{D}=D$. The m is the order of the reduced order system and k is the amount of the output variable. Thus, the original system can be estimated by the reduced order system. Three MOR methods of KS-MOR, BT-MOR and LP-MOR will be discussed.

3.1 The Krylov subspace method

The KS-MOR method is a common and well-founded projection method. The basic process is to construct the Krylov subspace of the system according to the transfer function of the original system. Then the base vector of the Krylov subspace is performed standard orthogonal processing by the Arnoldi process, thereby an orthogonal projection matrix is obtained to the construct the reduced order system [12].

3.2 The balanced truncation method

The BT-MOR method is easy to find the error between the original and the reduced order system, and can maintain the stability of the original system [4]. At first, the controllable and the observable Gram matrices of the original system need to be obtained by solving two

Lyapunov equations. Then the balanced transformation matrix is constructed by the two matrices above to acquire the balanced system. Finally, the state variables corresponding to the smaller Hankel singular value of the diagonal elements in the balanced Gram matrix are truncated. Therefore the reduced order system which is approximate to the original system can be obtained.

3.3 The Laguerre orthogonal polynomial method

The LP-MOR method is to performed the series expansion on the state variables and input variables of the original system in the space formed by the orthogonal polynomial from the time domain. Then the linear equations with the expansion coefficients of the state variables as the independent variables are constructed by using the linear independence of the Laguerre orthogonal polynomials. The expansion coefficients of the state variable are obtained by solving and the orthogonal projection matrix can be constructed. At last, the reduced order system is obtained.

4. RESULTS AND DISCUSSION

4.1 The laminar and turbulent flow under the constant wall temperature

The laminar and turbulent flow model were calculated. According to the literature [13], in the increasing zone of subsurface temperature of the rock-soil, the rock-soil temperature gradient averages 26.2 °C/km, that is, about 1 °C per 40 m, so the rock-soil temperature could be thought to have no change in the length of 40 m. The length of the pipe is 40 m. The step length in the axial direction is taken as 0.01 m. The diameters of the two models are 0.02 m and 0.04 m, respectively. The flow rates are 0.05m/s and 1.2 m/s, respectively. In the radial direction, the number of internal nodes is divided into 2000. That is, the order of the original system is 2000. When extracting heat , the rock temperature of the is 70 - 90 °C [13] at 2 - 3 km underground, so the wall temperature is T_w = 80 °C and the inlet water temperature is T_{in} = 20 °C. When injecting heat , the T_w is 17 °C and the T_{in} is 37 °C.

First the problem was solved by the DS method. In order to verify the accuracy of this calculation method, the Nusselt number (Nu) was calculated. The Nu obtained by the DS method and the experience Nusselt number from the literature [11] and calculated by the Gnielinski formula [14] are shown in Table 1. It is also shown the relative error of the two kinds of Nu. Then KS-MOR, LP-MOR and BT-MOR methods were used to solve the problem. In order to find a suitable reduced order, the cases of the order of 5, 10, 20, 40, 50, 100, 500 were solved. The results show that the regular patterns of the relative error and total time consumption for calculation of the reduced order systems with different orders for laminar and turbulent flow are the similar. Thus, Figure 2 and 3 show the results of the turbulent model.

Table 1 the Nusselt number

Model		Laminar	Turbulent
Calculated	winter	3.813	391.0
Nu	summer	3.824	337.1
Experience	winter	3.657	388.6
Nu	summer	3.657	338.0
Relative	winter	4.28%	0.61%
error	summer	4.58%	0.24%

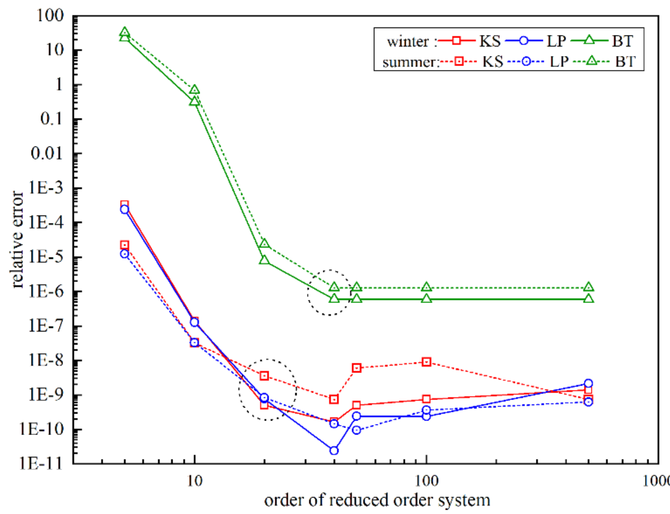

Fig 2 The relative errors of turbulent model under different orders of MOR systems

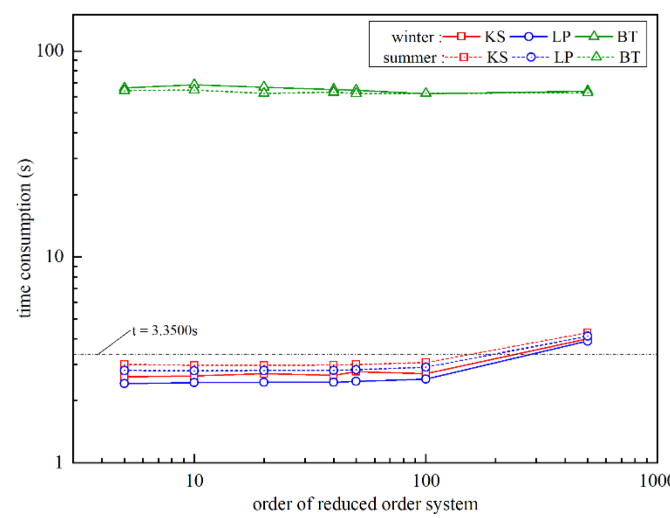

Fig 3 The total time consumption of calculation of turbulent model under different orders of MOR systems

It can be seen from Figure 2 that the relative errors of the KS-MOR, LP-MOR and BT-MOR methods decrease with the increase of the order, and remain basically unchanged after a certain order. The relative errors of LP-MOR and KS-MOR are stable and less than 10^{-8} after the order of 20 while that of BT-MOR is stable at 10^{-6} after the order of 40. For the laminar flow, the orders when the relative error become stable are 20 for the KS-MOR and LP-MOR and 50 for the BT-MOR. Moreover, the total time consumption of calculation increases with the increase of the order as shown in Figure 3. When the order of the reduced-order system is 500, the total computation time of the three MOR methods is greater than that of the DS method (3.3500s).

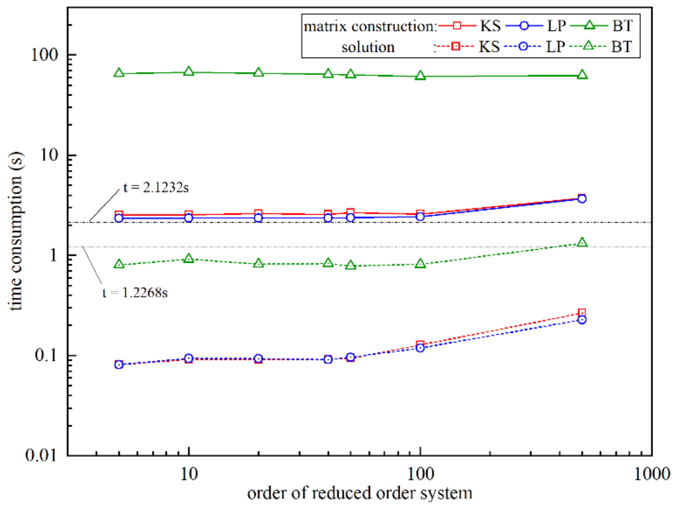

Fig 4 The time consumption for matrix construction and solution of turbulent model when extracting heat

In order to further analyze the time-consuming composition of the calculation, Figure 4 indicates the time consumption of matrix construction and solution in winter for the turbulent flow. Combined with Figure 3 and 4, it suggests that the total time consumption of the BT-MOR is much larger than that of other MOR methods and DS method, because this method needs to solve two large Lyapunov equations when constructing the matrices, which is time consuming. What's more, the inverse matrix calculation is needed in the solution process, so the solution time is also larger.

The total time consumption for calculation and matrix construction of the DS method in winter for turbulent flow are 3.3500s and 2.1232s, respectively. The time consumptions for the solution of the KS-MOR and LP-MOR methods are less than that of the DS method as Figure 4 shown. When the order of the reduced order system is 20, they are 6.0% and 6.2% of that of the DS method, respectively. While for the laminar flow, they are 11.3% and 10.9%, respectively.

Combining the relative error and calculation time of the above several calculation conditions, the BT-MOR method is not suitable for solving the heat transfer problem of the short single-tube buried pipe. It is more suitable to take the order of the reduced order system as 20 for both the KS-MOR and LP-MOR methods.

4.2 The effect of the pipe length on the time consumption for the calculation

The orders of reduced order system for the KS-MOR and LP-MOR in the laminar flow and the turbulent flow are both 20, and that of the BT-MOR is 50 and 40 under two kinds of flow, respectively.

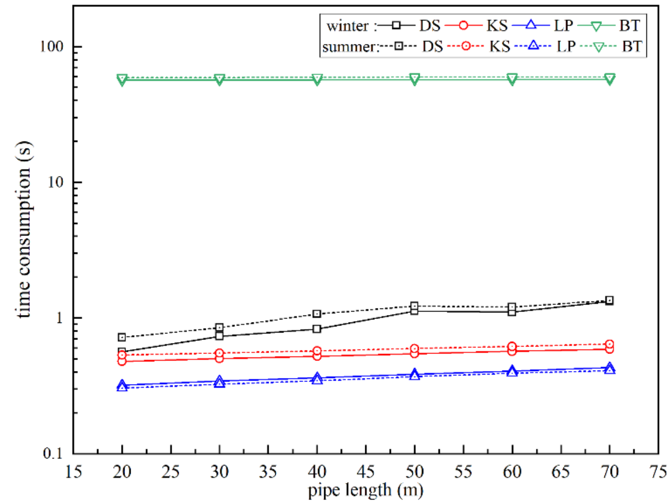

Fig 5 The trend of total time consumption for calculation under laminar flow with increasing pipe length

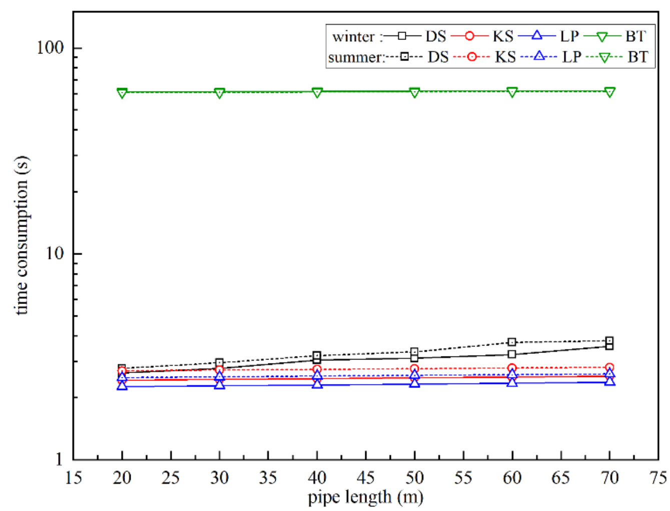

Fig 6 The trend of total time consumption for calculation under turbulent flow with increasing pipe length

Figure 5 and 6 demonstrate that the DS method has a large increase in the total time consumption as the length of the pipe increases, while the increase trend of the three MOR methods is small. Besides the relative errors of KS-MOR, LP -MOR and BT-MOR can be kept below 10^{-7}, 10^{-7}, 10^{-6}, respectively. Because the order of the original system is several hundred times larger than that of the reduced order system, the longer the pipe, the faster the solution time increases.

Since the process of constructing a matrix by the BT method is time consuming, when the calculation area is large, the total computational time is less than that of the DS method. Therefore, this reveals that the MOR method can greatly reduce the time consumption for calculation and improve the calculation efficiency when maintaining accuracy for the GSHP system with large calculation area.

5. CONCLUSION

In this paper, the convective heat transfer problems of the laminar or turbulent fluid in the buried pipe under the conditions of heat extraction in winter and heat injection in summer were calculated by Krylov subspace, balanced truncation and Laguerre orthogonal polynomial three MOR methods. The relative error and time consumption for calculation of different MOR methods and DS method under different conditions are compared. The conclusion can be obtained as followed:

1. The relative error between the three MOR and the DS methods can be less than 10^{-5} under a certain reduced order. However, for the BT-MOR method, the total time consumption is much larger than that of the DS and other MOR methods. Compared with the other two MOR methods, it is not suitable for solving the single-tube, short-length buried pipe heat transfer problem.

2. Combining the relative error and the computational time, it is suitable for the KS-MOR and the LP-MOR methods to take 20 as the order of the reduced order system under laminar and turbulent flow. The time consumption for solution of the two methods is less than 11.3% (laminar flow) and 6.2% (turbulent flow) of that of the DS method and the relative error is less than 10^{-7}. It is appropriate for the BT-MOR method to select 50 as and 40 as the order under laminar flow and turbulent flow, respectively.

3. As the calculation area increases, the total time consumption for calculation of the direct DS increases proportionally, while the KS-MOR and the LP-MOR methods increase slowly, and the relative error can still be guaranteed to be less than 10^{-7} (The order of the reduced order system is 20). Therefore, the larger the calculation area, the more efficient the two MOR methods .

ACKNOWLEDGEMENT

The research was supported by National Key R&D Program for the 13th - Five - Year Plan of China (2018YFF0300304 in 2018YFF0300300).

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