A RECEDING HORIZON ECO-DRIVING STRATEGY FOR ELECTRIC VEHICLES CONSIDERING TRAFFIC FLOW AND SIGNAL INFORMATION

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ABSTRACT

A real-time eco-driving strategy for electric vehicles is presented with the objectives of minimizing the travel time and energy consumption. The strategy is developed following a receding horizon approach such that it can take real-time traffic flow information in terms of speed limits into account when planning the vehicle’s speed profile. The problem is tackled by dividing it into two sub-problems, a travel time minimization problem and an energy consumption minimization problem. Analytic solutions to the travel time minimization problem is provided to facilitate real-time application. Simulation is done over a 10 km road to demonstrate the strategy’s effectiveness.

Keywords: Eco-driving, speed advisory, receding horizon optimization, electric vehicles, traffic flow, traffic signal

NONMENCLATURE

<table>
<thead>
<tr>
<th>Abbreviations</th>
<th>Time minimization problem</th>
<th>Energy minimization problem</th>
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<tr>
<td>Symbols</td>
<td>$\bar{x}$</td>
<td>$\underline{x}$</td>
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<tr>
<td></td>
<td>Index of road sections</td>
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1. INTRODUCTION

Although electric vehicles (EVs) have made their way to road applications, the limited mile range per single charge is still the main bottleneck limiting their widespread adoption. Eco-driving that reduces energy consumption per distance travelled is one of the key techniques having the potential of extending the driving range of EVs [1].

Either as an offline speed advisory system or online real time driver assistance speed planner, the eco-driving system’s main function is to optimize speed profile of the vehicle over a distance covering one or several traffic lights ahead of it [2]. Most of the published works on eco-driving focus on light duty fossil fuel powered vehicles with a few of them investigated heavy duty city transient buses [3] and electric vehicles [1].

From methodology perspective, researchers have tried to apply rule as well as optimization based methods to reduce fuel consumption of road vehicles by means of reducing the average speed, sharp acceleration, and waiting times at traffic lights [2].

However, these studies all assumed that the speed limit is constant throughout the trip or at least in a road segment between two traffic lights. This is not always true for highly populated urban cities. In a real-world scenario, the speed limit of the vehicle is affected by both legal speed limit that is constant and the traffic flow density, which is time varying. Under such circumstances, the speed optimized by the reported eco-driving systems might no longer be optimal or even feasible in real-time applications. To tackle the speed advisory problem under changing speed limits, this paper proposes a receding horizon optimization approach for EVs aiming at
minimizing both the energy consumption and trip time of the vehicle.

2. MODELLING OF THE ECO-DRIVING SYSTEM

The general form of the eco-driving system presented is graphically illustrated by Fig 1, in which the vehicle has to adhere to speed limits that are different in different sections within a segment of the road (the road covering the complete distance between traffic lights).

Under the situation that the speed limit over a road segment is constant, which might be true for low traffic density roads, the speed planning in eco-driving only needs to plan the acceleration of the vehicle according to a single speed limit. However, when the speed limit within a road segment changes over distance, the speed planning problem becomes much more complex.

In the following, the eco-driving system proposed is divided into two sub-problems, namely the travel time minimization problem (TMP) and energy consumption minimization problem (EMP).

2.1 TMP formulation

The TMP is solved by a receding horizon approach such that any updates on the traffic flows that affect the speed limits from V2I and V2V communications can be incorporated to get a more realistic speed profile. In particular, the predicting horizon is chosen to be two adjacent sections that have different speed limits. In TMP, the vehicle passes the two sections with maximum possible speed such that the travel time can be minimized.

There are two cases, as shown in Fig 2, in which the red solid line is the speed limit and the blue dashed line is one possible speed profile. In case 1, the speed limit of the second section is higher and the otherwise is true for case 2. For case 1, the vehicle can first accelerate to the lower speed limit if time allows, travel at the speed limit for the rest of section 1, and then accelerates again during the second section. The following optimization problem needs to be solved for case 1.

\[
\min_{a_1, \Delta t_1, a_2, \Delta t_2} t_1 = \frac{d_1 + \frac{1}{2} a_1 \Delta t_1^2}{v_0 + a_1 \Delta t_1} + \frac{d_2 + \frac{1}{2} a_2 \Delta t_2^2}{v_0 + a_1 \Delta t_1 + a_2 \Delta t_2},
\]

(1)

where \(a_1\) and \(a_2\) are accelerations, \(\Delta t_1\) and \(\Delta t_2\) are the acceleration time, and \(d_1\) and \(d_2\) are the distances of the first and second road sections, respectively. \(v_0\) is the initial speed of the vehicle. Subject to the following constraints.

\[
a \leq a_1, a_2 \leq \bar{a},
\]

(2)

\[
v_1 \leq v_0 + a_1 \Delta t_1 \leq \bar{v}_1,
\]

(3)

\[
v_2 \leq v_0 + a_1 \Delta t_1 + a_2 \Delta t_2 \leq \bar{v}_2.
\]

(4)

From here and onwards, subscripts 1 and 2 denotes the index of road sections.

The explicit solution of the TEM under such a case is given by Algorithm 1.

**Algorithm 1: Case 1 solution**

**Input:** \(\bar{v}_1, \bar{v}_2, \bar{a}, v_0\).

Set \(k := 1\).

**while** \(k \leq 2\) **do**

Set \(\bar{v} = \bar{v}_k\), \(d = d_k\), \(v_0 = v_{k-1}\).

if \(\frac{\bar{v} - v_0}{d} > d\) **then**

the vehicle keeps accelerating to \(v_x = \sqrt{2ad} + v_0\) with \(\Delta t_k = t_k = \frac{v_x - v_0}{a}\)

else

the vehicle accelerates to \(\bar{v}\) then cruise at \(\bar{v}\):

\[\Delta t_k = \frac{\bar{v} - v_0}{a}, \quad t_k = \Delta t_k + \frac{d - \bar{v}^2 + v_0^2}{\bar{a}}\]

**end**

\(k = k + 1\).

**end**

For the second case, the general time minimization problem can be formulated as

\[
\min_{a_1, \Delta t_1, a_2, \Delta t_2} t_2 = \frac{d_1 + \frac{1}{2} a_1 \Delta t_1^2}{v_0 + a_1 \Delta t_1} + \frac{d_2 + \frac{1}{2} a_2 \Delta t_2^2}{v_0 + a_1 \Delta t_1 + a_2 \Delta t_2} + \frac{d_1 + \frac{1}{2} a_1 \Delta t_1^2 - \frac{1}{2} a_2 \Delta t_2^2 - a_1 \Delta t_1 \Delta t_2 - v_0 \Delta t_2}{v_0 + a_1 \Delta t_1}
\]

(5)

Subject to constraints

\[
a \leq a_1, a_2, a_3 \leq \bar{a},
\]

(6)

\[
v_1 \leq v_0 + a_1 \Delta t_1 \leq \bar{v}_1,
\]

(7)

\[
v_2 \leq v_0 + a_1 \Delta t_1 + a_2 \Delta t_2 \leq \bar{v}_2,
\]

(8)
Analytic solution to the second case can be found following Algorithm 2.

**Algorithm 2: Case 2 solution**

**Input:** \(\bar{v}_1, \bar{v}_2, \bar{a}, a_0, v_0\).

Set \(s_1 = \frac{\bar{v}_1^2 - v_0^2}{2a}, s_2 = \frac{\bar{v}_2^2 - \bar{v}_1^2}{2a}, s_3 = \frac{\bar{v}_2^2 - v_2^2}{2a}\).

- **if** \(s_1 + s_2 \leq d_1\) **then**
  \[- \frac{v_0}{a} + \frac{v_0 - \bar{v}_1}{a} + \Delta t_1 = \bar{v}_1,\]
  \[- \frac{v_1}{a} + \bar{v}_1 + \Delta t_1 + \Delta t_2 = \bar{v}_2;\]
  \[- \frac{v_2}{a} + \frac{\bar{v}_2 - v_2}{a} + \Delta t_2 = \bar{v}_2;\]

- **else if** \(s_3 \geq d_1 + d_2\) **then**
  \[- \frac{v_0}{a} + \frac{v_0 - \bar{v}_1}{a} = \bar{v}_1,\]
  \[- \frac{v_1}{a} + \frac{\bar{v}_1 - v_1}{a} = \bar{v}_2;\]

- **else if** \(d_1 \leq s_3 < d_1 + d_2\) **then**
  \[- \frac{v_0}{a} + \frac{v_0 - \bar{v}_1}{a} = \bar{v}_1,\]
  \[- \frac{v_1}{a} + \frac{v_1 - \bar{v}_2}{a} + \bar{v}_2 = \bar{v}_2;\]

- **else**
  The vehicle first accelerates to \(v_x\) then decelerates to \(v_2\) over the first section.

**end**

With the help of the above formulations, the receding horizon travel time minimization problem is solved as shown in Algorithm 3.

**Algorithm 3: Receding horizon travel time minimization**

Set \(k := 1\) and \(t = 0\).

**while** \(k \leq \text{number of road sections}\) **do**

- **if** \(\bar{v}_1 < \bar{v}_2\) **then**
  **if** \(t_1 = t_1\)**
  **else**
  Solve the Case 1 TEM problem
  \(t_1 = t_1 + \Delta t_2\)

- **else**
  Solve the Case 2 TEM problem
  \(t_1 = t_1 + \Delta t_2\)

**end**

**end**

Notice that by following the receding horizon method, the updated road speed limits received by the vehicle through V2I and/or V2V communications can be readily incorporated, allowing an effective real-time implementation of the presented strategy. It is however noted that the above TMP formulated does not take into consideration of the passenger ride comfort because of two reasons. Firstly, the ride comfort can be accounted for by setting a relatively small max acceleration limit. Secondly, once the minimum travel time of a road segment is found, it is usually the case that the vehicle will not be able to pass the traffic light with this minimum time. The minimum pass time at the traffic signal obtained by (10) is therefore usually longer, allowing ride comfort to be considered by the MEP.

\[
t_{\text{pass}} = \begin{cases} t_1, & \text{if } t_1 \in [t_g, t_r); \\ t_g, & \text{if } t_g \in [t_r, t_g); \end{cases}
\]

where \(t_r\) and \(t_g\) are the time until the traffic light turns to red and green, respectively.

### 2.2 MEP formulation

Once the time to pass the traffic light ahead of the vehicle \(t_{\text{pass}}\) is determined, the MEP then optimizes the speed profile of the vehicle over \([0, t_{\text{pass}}]\) to minimize energy consumption by solving the following problem.

\[
\min_v \int_0^{t_{\text{pass}}} f(t) v(t) dt \\
\text{s.t.} \quad a \leq a \leq \bar{a} \\
\quad v \leq v \leq \bar{v}
\]

\[
\int_0^{t_{\text{pass}}} v(t) dt = D
\]

where

\[f_i = ma + f_s + f_r + f_a\]
\[f_s = \mu mg \cos(\alpha) + mg \sin(\alpha)\]
\[f_r = \frac{1}{2} \rho_s C_d A_f v^2\]

are tractive force, resistance forces, and aerodynamic drag of the vehicle, respectively. \(m\) is the vehicle mass properly adjusted to account for rotational parts. \(\mu\) is the rolling friction coefficient, \(\alpha\) is the road slope, \(\rho_s\) is air density, \(C_d\) and \(A_f\) are air drag coefficient and the frontal cross area of the vehicle, respectively.

### 3. SIMULATION

The same traffic light configuration used in [4] is used here. All the traffic lights are represented by 40 s of green and 60 s of red periods. In terms of distance, they are away from the vehicle at \(d_1 = 1500\) m, \(d_2 = 3000\) m, \(d_3 = 6850\) m and \(d_4 = 10000\) m. Thus, the TEM is solved over a total distance of 10 km.
The parameters of the vehicle used in simulations are given in Table 1.

We first solve the TEM problem by Algorithms 1 and 3 to find the minimum passing time of the vehicle at the four traffic lights ahead of it.

With an initial traveling speed of $v_0 = 50$ km/h, and the speed limits over the road given in Fig. 3, the minimum passing times $t_{pass}$ of the vehicle at the four traffic lights are, respectively, 100 s, 220 s, 480 s, and 640 s.

With these pass times determined, the MEP is then solved, which resulted in the speed profile of the vehicle over the simulated road shown in Fig. 3, in which the optimized speed profile by the method presented in this study and that obtained by a heuristic method presented in [4] are shown for a quick comparison.

It can be seen that the method presented in [4] and many similar research studies works very nicely if the speed limit over a road segment is constant. However, because [4] did not considered speed limit variation over a segment of the road, it can be seen in Fig. 3 that the resulting speed profile violates the speed limit over the section [3270, 5000] m of the road. The proposed method, on the hand, is able to find the optimal speed profile over the 10 km travel distance considered without violating any speed restrictions.

With the optimized speed profile shown in Fig. 3, Fig. 4 shows how the vehicle passed through all the traffic lights without stopping. In particular, the vehicle passes the first traffic light right before it turns to red, passes the second, third and fourth traffic lights right after the lights turn green.

ACKNOWLEDGEMENT
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REFERENCE

Table 1: Vehicle parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>vehicle mass (kg)</td>
<td>1500</td>
</tr>
<tr>
<td>vehicle frontal area (m²)</td>
<td>2.35</td>
</tr>
<tr>
<td>gravitational acceleration (m/s²)</td>
<td>9.8</td>
</tr>
<tr>
<td>air density (kg/m³)</td>
<td>1.02</td>
</tr>
<tr>
<td>air drag coefficient</td>
<td>0.3</td>
</tr>
<tr>
<td>rolling resistance coefficient</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Fig. 3 Optimized speed profile

Fig. 4 Vehicle passing traffic lights