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APPROXIMATE LINEAR POWER FLOW FOR VSC BASED AC/DC POWER GRID USING LOGARITHMIC TRANSFORM OF VOLTAGE MAGNITUDES

Jinyu Yu[1], Zhigang Li[1,*], Yang Yu[1], Tianyao Ji[1], Q. H. Wu[1], Haibo Zhang[2]

1 School of Electric Power Engineering, South China University of Technology, Guangzhou 510641, China
2 State Key Laboratory of Alternate Electrical Power System with Renewable Energy Sources, North China Electric Power University, Beijing 102206, China.

ABSTRACT

With the development of power system technology, multi-terminal DC transmission based on voltage source converter has gradually become a superior choice for power transmission because of its decoupling control ability. Most studies about AC/DC power flow presented in the literatures use iterative method and nonlinear models. Work on linearization steady state model is still insufficient. An approximate linearized power flow model for VSC based AC/DC grid using logarithmic transform of voltage magnitudes (LTVM) is proposed in this paper. The model of AC/DC system and VSC are established. The logarithm of voltage magnitude is utilized as independent variables of power flow equations. This technique transforms the power flow equations of AC/DC grid from (V, ϑ, U) space to (v, ϑ, u) space, which makes power flow equations a set of linear equations. Simulation of the proposed method and classic iterative algorithm is performed in several test systems. The results demonstrate the validity and accuracy of the proposed method.

Keywords: AC/DC Grid, Linearization, Power Flow, Voltage Source Converter

MAIN NOMENCLATURE

Abbreviations	
VSC	voltage source converter
LTVM	logarithmic transform of voltage magnitudes
NAM	nodal admittance matrix
RMS	root-mean-squared
Symbols	
V	AC voltage magnitude
ϑ	AC voltage phase angle
U	AC voltage
v	modified AC voltage magnitude
u	modified DC voltage magnitude
$G(B)$	real (Image) part of AC NAM
$G'(B')$	NAM containing the impedance of converter station
$P(Q)$	active(reactive) power
G_{dc}	DC Node conductance matrix
P_{dc}	DC Node power injection
z_t	impedance of converter transformer
b_f	susceptance of the shunt filter
z_c	impedance of phase reactor

1. INTRODUCTION

With the development of off-shore wind farm and high voltage bulk capacity transmission, the construction of AC/DC projects is entering a boom. The power network gradually exhibits the phenomenon of AC and DC coexistence. There are two main types of converter in DC projects nowadays: Line Commutated Converter (LCC) and Voltage Source Converter (VSC). Most of the under-construction projects and future ones, especially multi-terminal DC projects, are prefer to VSC because its excellent control capability meets the requirements of modern grid. The power flow calculation of AC/DC hybrid system is a prerequisite for analyzing it.

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The power flow calculation methods of the classic AC/DC grid mainly include unified iterative method and alternative iterative method[1] [2] . They are both iterative methods based on the Newton-Raphson method. The grid state is obtained by solving a set of nonlinear equations. The computing burden is heavy.

In contrast, the linear power flow model provides a lightweight quick analysis approach. For example, the classic DC power flow has been widely used in power system analysis and optimization fields due to its practicability on many practical fields such as sensitivity analysis[3] , contingency analysis[4] , power market clearing[5] , power system planning[6] , etc.

Researchers have also proposed other linear power flow methods. Reference [7] proposed an LPF model that preserves the influence of reactive power. A linear function is used to estimate the trigonometric function of the phase angle, so that the power flow model is linearized with respect to the square of the voltage magnitude and the modified phase angle. Reference [8] employed curve fitting to linearize the nonlinear terms of voltages in the power flow model of a distribution system. But the approximation errors of the active and reactive branch flow are not analyzed. Square of voltage magnitude is used in [9] as independent variable to solve AC/DC grid OPF, and the Taylor series is used to approximate the equations and the VSC losses.

However, due to the different characteristics between AC system and DC system, the equations of the converter are difficult to deal with. Work about linearization of the AC/DC grid is still insufficient. To bridge this gap, this paper applies an approximate linearization method based on Logarithmic Transform of Voltage Magnitudes (LTVM)[10] to the power flow equations of AC/DC grid. The model considers both AC and DC voltages and currents. The loss of VSC is treated equivalently, and the Point of Common Coupling (PCC) connected with different types of VSC are analyzed respectively.

Part 2 of this paper introduces the model of the AC/DC system and the processing of the converter model. The linear power flow model of AC/DC system is established in Part 3. Part 4 shows the simulation results of the model in several test systems. Part 5 summarizes the paper.

2. AC/DC SYSTEM MODEL

AC/DC system is combination of AC network, DC network and converters. The network equations are all based on Kirchhoff's Current Law. While the equations of converter stations are relatively different with network. Their models will be expressed respectively in following parts.

2.1 Network Model

The well-known AC power flow equations, i.e., node power injection equations, can be expressed as follow:

$$\dot{S}_i = \dot{V}_i \sum_{j}^{n} \dot{V}_j^* \left(G_{ij} - \mathrm{j}B_{ij} \right) \quad , \tag{1}$$

where *i* denotes bus *i* of *n* buses. With nodal injection power and voltage of generators known, the buses voltage magnitudes and angles can be solved. They are power flow model of AC grid.

The power flow equations of DC grid have a similar from with ACs' because both are derivations of KCL. The node conductance elements are defined as

$$G_{\mathrm{dc}ij} = \begin{cases} -g_{ij} \ , i \neq j \\ \sum\limits_{k \in \mathrm{TE}(i)}^{n_{\mathrm{dc}}} g_{ik} \ , i = j \end{cases} , \tag{2}$$

where *i* denote DC bus *i* of n_{dc} DC buses, *ij* denote branch connecting buses *i* and *j*. The node power injection equation of DC bus *i* is:

$$P_{\mathrm{dc}i} = U_i \sum_{j}^{n_{\mathrm{dc}}} U_j G_{\mathrm{dc}ij} \quad . \tag{3}$$

With node injection power or DC voltage known at each DC bus, all DC voltages can be solved.

2.2 VSC Model

VSC converter is considered as an appropriate choice for new AC/DC projects because of its flexible and powerful control strategy. A VSC station circuit model is shown in Figure 1. A typical VSC station is composed of a transformer, a filter, a reactor, and the converter valve [11] . The shunt filter is only used in stations with PWM modulation mode, while the Modular Multilevel Converter station generally needs no AC filter. In power flow study, the converter station model can be treated as an AC branch and an active power converter. To simplify the nomenclature, the system side bus, filter bus, and converter valve side bus of the station are denoted as buses s, f and c, respectively.

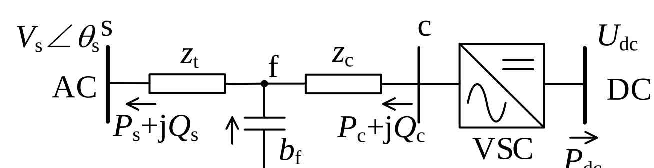

Figure 1 VSC Station Circuit Model

2.2.1 Branches

The branches between bus s, bus f, and bus c have similar construction with generally AC branch, the power injections are as follows:

$$-P_{\mathrm{s}}=V_{\mathrm{s}}\sum_{j=\mathrm{f,s}}V_j\left(G_{\mathrm{s}j}\cos\theta_{\mathrm{s}j}+B_{\mathrm{s}j}\sin\theta_{\mathrm{s}j}\right)\quad, \tag{4}$$

$$-Q_{\mathrm{s}}=V_{\mathrm{s}}\sum_{j=\mathrm{f,s}}V_j\left(G_{\mathrm{s}j}\sin\theta_{\mathrm{s}j}-B_{\mathrm{s}j}\cos\theta_{\mathrm{s}j}\right)\quad, \tag{5}$$

$$\sum_{j=\mathrm{f,s,c}}V_j\left(G_{\mathrm{f}j}\cos\theta_{\mathrm{f}j}+B_{\mathrm{f}j}\sin\theta_{\mathrm{f}j}\right)=0\quad, \tag{6}$$

$$\sum_{j=\mathrm{f,s,c}}V_j\left(G_{\mathrm{f}j}\sin\theta_{\mathrm{f}j}-B_{\mathrm{f}j}\cos\theta_{\mathrm{f}j}\right)=0\quad, \tag{7}$$

$$P_{\mathrm{c}}=V_{\mathrm{c}}\sum_{j=\mathrm{c,f}}V_j\left(G_{\mathrm{c}j}\cos\theta_{\mathrm{c}j}+B_{\mathrm{c}j}\sin\theta_{\mathrm{c}j}\right)\quad, \tag{8}$$

$$Q_{\mathrm{c}}=V_{\mathrm{c}}\sum_{j=\mathrm{c,f}}V_j\left(G_{\mathrm{c}j}\sin\theta_{\mathrm{c}j}-B_{\mathrm{c}j}\cos\theta_{\mathrm{c}j}\right)\quad, \tag{9}$$

where the admittance coefficients G and B are derived from the branch parameters z_{t}, b_{f} and z_{c}, and the formula is as the same with the NAM of generally AC network. The subscript i in converter station equations is omitted to make expression clear. Equations (4)-(9) describe the branches in the converter station.

2.2.2 Valve Loss

The active power transportation formula of VSC is:

$$P_{\mathrm{c}}+P_{\mathrm{dc}}+P_{\mathrm{loss}}=0\quad, \tag{10}$$

where P_{loss} can be expressed as a quadratic function[12] of current flowing through bus c:

$$P_{\mathrm{loss}}=a+b\left|I_{\mathrm{c}}\right|+c\left|I_{\mathrm{c}}\right|^2\quad, \tag{11}$$

$$\left|I_{\mathrm{c}}\right|=\frac{\sqrt{P_{\mathrm{c}}^2+Q_{\mathrm{c}}^2}}{V_{\mathrm{c}}}\quad, \tag{12}$$

where a, b and c are loss parameters decided by characteristics of power electronic components of valve. Equations (11)-(12) are nonlinear because of the square-root term. To simplify the model, $b|I_{\mathrm{c}}|$ is omitted. The rest loss parameters can be equivalently represented by a fixed active power load and a resistance as shown in Figure 2.

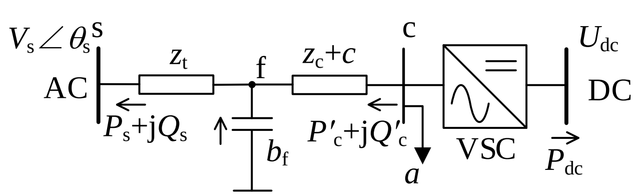

Figure 2 VSC Station Circuit Model with Equivalent Loss

According to the simplified loss model, (10) is rewritten as follow:

$$P'_{\mathrm{c}}+a+P_{\mathrm{dc}}=0\quad, \tag{13}$$

where P'_{c} is

$$P'_{\mathrm{c}}=V_{\mathrm{c}}\sum_{j=\mathrm{c,f}}V_j\left(G'_{\mathrm{c}j}\cos\theta_{\mathrm{c}j}+B'_{\mathrm{c}j}\sin\theta_{\mathrm{c}j}\right)\quad. \tag{14}$$

Under this loss model, all $G_{\mathrm{c}j}$+j$B_{\mathrm{c}j}$ and $G_{\mathrm{f}j}$+j$B_{\mathrm{f}j}$ mentioned above should be replaced by $G'_{\mathrm{c}j}$+j$B'_{\mathrm{c}j}$ and $G'_{\mathrm{f}j}$+j$B'_{\mathrm{f}j}$. These elements of the new NAM take the c parameter into account.

2.2.3 Control Modes

VSC enables independent control strategies of active and reactive power through the converter by controlling the magnitude and phase angle of output voltage[13] . Some researchers also classify these two control aspects as DC side control and AC side control. The active power injection can be modulated in two ways:

a) Constant P_{s}: The active power injection to AC grid at PCC bus is constant.
b) Constant U_{dc}: The DC voltage at DC side of the converter is constant.

The reactive power injection can also be modulated in two ways:

a) Constant Q_{s}: The reactive power injection to AC network at PCC bus is constant.
b) Constant V_{s}: The AC voltage magnitude at PCC bus is constant.

By combining the active and reactive control modes, VSC can be operating in 4 control modes. Each control mode provides 2 set point as boundary conditions of power flow, while the voltage phasor of bus c and both active and reactive power from VSC are unknown. The number of conditions and unknown variables are balance.

The boundary conditions of AC/DC system are consisted of constant power injection from loads and generators, and voltages controlled by VSCs and AC generators. Combining the AC/DC grid model and the boundary conditions, AC and DC voltages can be solved. But the expression of grid model is still a group of nonlinear equations, the linearization method is introduced in the next part of this paper.

3. LINEARIZATION OF POWER FLOW EQUATIONS

The LTVM[10] is employed to do a preprocessing to the state variables. The Taylor first-order expansion is applied to obtain linear equations. The linearization of AC power flow equations, DC power flow equations, and equations of VSC converter stations are formulated respectively in this section.

3.1 Principle of LTVM Method

For normally operating power systems, we have $\vartheta_{ij} \approx 0$ rad, $V_i \approx V_j \approx 1$ p.u. The modified voltage magnitude, symbolized as v_i, is defined as a logarithmic transform of the original voltage magnitude from the V-space into the v-space:

$$v_i = \ln V_i, v_{ij} = v_i - v_j = \ln V_i - \ln V_j \quad . \tag{15}$$

We can obtain

$$V_j \angle \theta_{ij} = V_j e^{\mathrm{j}\theta_{ij}} = \exp\left(v_j + \mathrm{j}\theta_{ij}\right) \quad . \tag{16}$$

There are 3 advantages of using LTVM:

- Logarithm function transforms the multiply terms like V_iV_j into addition ones, which makes the bilinear terms decoupled.
- The voltage is expressed as exponential form, the modified voltage magnitude and phase angle form an additive relationship, which is simpler.
- The nonlinear characteristics of the equation are preserved in the transformation and are independent of the linearization approximation processing.

The Taylor first-order expansion of exponential function is

$$e^v \approx 1 + v\left(v \approx 0\right) \quad . \tag{17}$$

Obviously, $V_i \approx V_j \approx 1$ implies $v_i \approx 0$ and $v_{ij} \approx 0$, therefore

$$\frac{1}{V_i} = e^{-v_i} \approx 1 - v_i \quad , \tag{18}$$

$$V_j \angle \theta_{ij} = e^{v_j + \mathrm{j}\theta_{ij}} \approx 1 + v_j + \mathrm{j}\theta_{ij} \quad , \tag{19}$$

$$\frac{1}{V_i^2} = e^{-2v_i} \approx 1 - 2v_i \quad , \tag{20}$$

$$\frac{V_j \angle \theta_{ij}}{V_i} = e^{-v_{ij} + \mathrm{j}\theta_{ij}} \approx 1 - v_{ij} + \mathrm{j}\theta_{ij} \quad . \tag{21}$$

Besides, the trigonometric functions can also be linearized by Taylor first-order expansion

$$\cos\theta_{ij} \approx 1, \sin\theta_{ij} \approx \theta_{ij} \quad . \tag{22}$$

All approximation formulas mentioned above are helpful to the linearizing processing in the next section.

3.2 Linear Equations of Networks

Both AC and DC networks are described by the node power injection equations. Thus, the manipulation for AC and DC network equations are similar.

Writing the phasors in complex exponential form, real part and imaginary part of (1) are:

$$P_i e^{-\ln V_i} = \mathrm{Re}\left(\sum_j^n e^{\ln V_j + \mathrm{j}\theta_{ij}}\left(G_{ij} - \mathrm{j}B_{ij}\right)\right) \quad , \tag{23}$$

$$Q_i e^{-2\ln V_i} = \mathrm{Im}\left(\sum_j^n e^{\ln V_j - \ln V_i + \mathrm{j}\theta_{ij}}\left(G_{ij} - \mathrm{j}B_{ij}\right)\right) \quad . \tag{24}$$

The left terms in (23) and (24) are divided by $\dot{V}_i$ and $\dot{V}_i^2$ respectively, because this manipulation take a better accuracy for P-equation and Q-equation[10] . Employing the LTVM method and linear approximation formulas yields:

$$\begin{aligned} P_i - \sum_{j=1}^n G_{ij} \approx \left(P_i + G_{ii}\right)v_i + \sum_{j\neq i}^n G_{ij}v_j + \left(\sum_{j\neq i}^n B_{ij}\right)\theta_i \\ - \sum_{j\neq i}^n B_{ij}\theta_j \end{aligned} \quad , \tag{25}$$

$$\begin{aligned} Q_i + \sum_{j=1}^n B_{ij} \approx \left(2Q_i + \sum_{j\neq i}^n B_{ij}\right)v_i - \sum_{j\neq i}^n B_{ij}v_j \\ + \left(\sum_{j\neq i}^n G_{ij}\right)\theta_i - \sum_{j\neq i}^n G_{ij}\theta_j \end{aligned} \quad . \tag{26}$$

The DC network equations can be linearized by the same method as above. The equations after applying the LTVM approximation method are formulated:

$$P_{\mathrm{dc}i} - \sum_j^{n_{\mathrm{dc}}} G_{\mathrm{dc}ij} = \left(P_{\mathrm{dc}i} + G_{\mathrm{dc}ii}\right)u_i + \sum_{j\neq i}^{n_{\mathrm{dc}}} G_{\mathrm{dc}ij}u_j \quad . \tag{27}$$

3.3 Linearization of VSC equations

The essence of converter station is a power conversion component that follows the law of conservation of power. Thus, the key equation of VSC is (13). Because the variables P'_{c} and P_{dc} in it are both unknown before calculation, they need to be eliminated by simultaneous equations:

$$\begin{aligned} V_{\mathrm{c}i} \sum_{j=\mathrm{c}i,\mathrm{f}i} V_j\left(G'_{\mathrm{c}ij}\cos\theta_{\mathrm{c}ij} + B_{\mathrm{c}ij}\sin\theta_{\mathrm{c}ij}\right) \\ + U_i \sum_j^{n_{\mathrm{dc}}} U_j G_{\mathrm{dc}ij} + a_i = 0 \end{aligned} \quad . \tag{28}$$

This equation has similar form to active power injection equations. Applying the LTVM approximation method we obtain:

$$\begin{aligned} -\left(a_i + \sum_j^{n_{\mathrm{dc}}} G_{\mathrm{dc}ij}\right)v_{\mathrm{c}i} + \sum_{j=\mathrm{c}i,\mathrm{f}i} G'_{\mathrm{c}ij}v_j + \sum_j^{n_{\mathrm{dc}}} G_{\mathrm{dc}ij}u_j \\ - a_i u_i - \sum_{j=\mathrm{c}i,\mathrm{f}i} B'_{\mathrm{c}ij}\theta_{ci} + \sum_j^{n_{\mathrm{dc}}} G_{\mathrm{dc}ij} + a_i = 0 \end{aligned} \quad . \tag{29}$$

4. CASE STUDY

4.1 Test System and Error Calculation

The approximate linear power flow algorithm and 2 case studies are demonstrated in MATLAB 2016b. The results of the proposed algorithm are compared with those of the sequential algorithm in MatACDC[14] , a MATLAB tool box, to verify validity and accuracy.

Two modified AC/DC systems are constructed as test cases. The parameters of them are listed as follow.

a) Case24_acdc is a modified IEEE reliability test system[15] [16] . Three AC branches are replaced with DC lines, the corresponding buses are configured with the VSCs. Power injecting or absorbing to the VSCs are set based on the base AC power flow of original system to make sure the modified system is solvable and stable. The branch impedance parameters of converter station are modified based on [17] . The loss parameters are from [12] .The control modes of the three VSCs are presented in Table 1.

Table 1 VSC Control Mode of Test Case case24_vsc

VSC	Active	Reactive
1	Constant P_s	Constant Q_s
2	Constant U_{dc}	Constant Q_s
3	Constant P_s	Constant Q_s

b) Case30_acdc is a modified IEEE 30 bus test system[18] . Five AC branches are replaced with DC lines, and the corresponding buses are configured with the VSCs. The load power of several nodes is adjusted. Impedance parameters and loss parameters are same with case24_acdc. The control modes of the three VSCs are listed in Table 2.

Table 2 VSC Control Mode of Test case case30_vsc

VSC	Active	Reactive
1	Constant U_{dc}	Constant Q_s
2	Constant P_s	Constant Q_s
3	Constant P_s	Constant V_s
4	Constant P_s	Constant V_s

The test systems above are simulated in proposed method and sequential algorithm. To evaluate calculation error, the root-mean-squared (RMS) errors of a solution *x* with respect to the benchmarked solution *x** are defined as follow

$$\Delta x^{\mathrm{RMS}} = \sqrt{\sum_{i=1}^{n}\left(x_i - x_i^*\right)^2 / n} \quad . \tag{30}$$

4.2 Simulation Result

The AC and DC voltages of case24_acdc and case30_acdc are shown in Figure 3 to Figure 6. The RMS error are listed in Table 3. There are few gaps between the proposed linear method and the sequential iterative method, indicating that the AC and DC voltages can be obtained through the proposed method with an acceptable accuracy.

Table 3 RMS Error of Voltage

Case	AC Voltage Error (p.u.)	DC Voltage Error (p.u.)
case24_acdc	1.25E-02	1.29E-03
case30_acdc	2.48E-03	9.54E-05

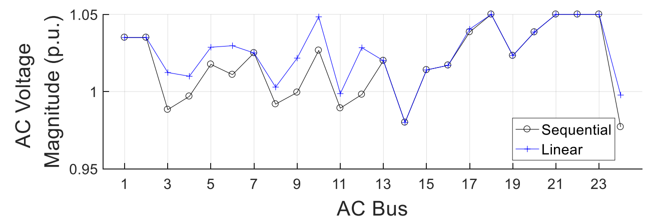

Figure 3 AC Voltage of case24_acdc

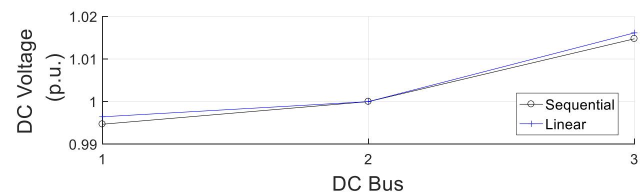

Figure 4 DC Voltage of case24_acdc

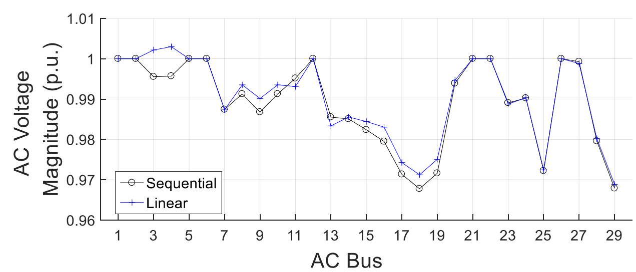

Figure 5 AC Voltage of case30_acdc

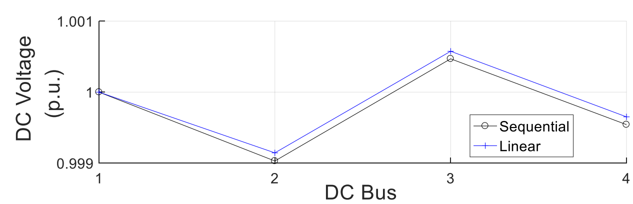

Figure 6 DC Voltage of case30_acdc

The branch flow errors are listed in Table 4. Because the transmission loss and VSC valves power loss is considered in proposed linear method, the branch power results are quite reliable.

Table 4 RMS Relative Error of Branch Power Flow

Case	AC Active Power Flow	AC Reactive Power Flow	DC Power Flow
case24_acdc	0.73%	1.9%	0.10%
case30_acdc	2.8%	7.2%	0.16%

5. CONCLUSIONS

An approximate linear power flow method for AC/DC grid is proposed in this paper. The equations of AC/DC system are linearized via LTVM method. Several test systems are simulated and analyzed under the proposed method and a classical sequential iteration method. The calculation results show the rationality and accuracy of the proposed approach. The advantage of this model comes from its rather precise approximation of the nonlinear terms in the power flow equations.

This model can be used for advance power system analysis and operation where power flow modeling is needed. The estimation model of the power loss of the VSC converter needs some ideal assumptions, while there is still a gap between the assumptions and the actual situation. In the case of known base power flow, further accurate estimation parameters could be used to improve accuracy.

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