

COMPREHENSIVE VULNERABILITY ASSESSMENT METHOD OF SHIPBOARD POWER SYSTEM

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ABSTRACT

Aiming at the demand of comprehensive vulnerability assessment in shipboard power system security defense, the paper proposed a vulnerability status description model of shipboard power system. The structure and physical properties of shipboard power network were fully analyzed. The product space which was called shipboard power network comprehensive vulnerability index set was established by degrees, betweenness, maximum connection sub-diagram scale, reliability indexes. Secondly, the normalized index sets were compactness and the P norm on the compactness sets were continuous were proved in this paper. The norm of vulnerability index was vulnerability output equation of shipboard power system. Using the norm on the product space, multi-scale integrated shipboard power network vulnerability norm was proposed, and the comprehensive structure performance evaluation of the shipboard power system network was formed. The index describes the change tendency of the shipboard power network comprehensive vulnerability. Finally, the test on a certain type of shipboard power network demonstrates the validity of the model.

Keywords: shipboard power system, transfer equation, output equation, comprehensive vulnerability, product space, norm.

NONMENCLATURE

Abbreviations

Symbols

P_s^P	multiscale comprehensive vulnerability norm of shipboard power network
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1. INTRODUCTION

With the development and application of the concept of integrated power system of ships^[1], the network forms of shipboard power system become more flexible and diverse. The topological structure of shipboard power network is becoming more and more complex. The vulnerability of shipboard power network is an important aspect of power system security.

The vulnerability of power system was first proposed by professor Fouad and his students in 1994^[2], including component level and system level. System-level vulnerability is only revealed when the power network fails, and it shows whether the power network has the ability to maintain the stability and normal power supply of the system. Component-level vulnerability refers to the weak links existing in the power system. Identification of weak links is to analyze and order the degree of influence caused by component faults in the system.

At present, the research on power system vulnerability assessment mainly focuses on the identification of weak links in the system. Looking for the weak links in the power network evaluates the system vulnerability from a "micro" perspective. From the perspective of the model of topology structure, it can be divided into unauthorized power network vulnerability assessment model^[3-5] and authorized power network vulnerability assessment model^[6-8]. From the

perspective of the identification method of important nodes by complex network theory, it can be divided into social network analysis method and system science analysis method. Due to the requirements of combat mission and mission, the continuity of system power supply is required. The continuity of power supply can be reflected by the fragility of the system. For the ship power system, in addition to identifying the weak links of the ship power system, it is necessary to put forward the quantitative evaluation index of the overall fragility of the ship power system.

Quantitative analysis of the overall vulnerability of the land interconnection power system has been carried out by scholars. For example, literature [9] puts forward the vulnerability assessment method and index of complex power system based on risk theory, and reflects the security state of power grid with low voltage risk, overload risk, comprehensive risk and other indicators. Literature [10] also proposes a system vulnerability assessment method based on risk theory for the safety of power system voltage and HVDC transmission system.

This paper quantitatively evaluates the comprehensive vulnerability of the whole shipboard power system network, which provides a theoretical basis for the realization of vulnerability assessment of shipboard power system from a "macroscopic" perspective. The fragile state space of ship power system is described by establishing the state model of ship power system vulnerability. Based on the deep analysis of each vulnerability evaluation index, the comprehensive vulnerability index set of naval power network is formed by the product space composed of degree, betweenness, maximum connection sub-diagram scale and reliability index set. The norm of each vulnerability index vector is used as the vulnerability output equation of naval power system. Based on the norm of product space, the correlation of different vulnerability evaluation indexes is realized, and the evaluation index of the change trend of the comprehensive vulnerability degree of shipboard power system network is put forward, so as to realize the comprehensive evaluation of the overall vulnerability degree of shipboard power system network from different scales.

2. THEORY

2.1 Vulnerability state space model of ship power system

In order to establish the vulnerability state space model of ship power system, it is necessary to first

determine the appropriate vulnerability indexes. Based on the in-depth analysis of each vulnerability index, the following four indexes are selected in this paper to form the comprehensive vulnerability assessment index set of ship power network: Degree index; Betweenness index; The maximum connection sub-diagram scale; reliability.

The set family of vulnerability indicators of naval power system is Ω :

$$\Omega = \{I_1, I_2, I_3, I_4\} \quad (1)$$

Where, I_1, I_2, I_3, I_4 respectively represents degree, betweenness, maximum connection sub-diagram scale and reliability index.

Degree and betweenness are efficiency indexes; maximum connection sub-diagram scale and reliability are cost index. The relative value $x_k(v_{kj}(i))$ of vulnerability index value of each node in the i th state of naval power system can be obtained.

Then, $b_1(i)$ is the relative value vector of node degree index under the i th state of shipboard power system. $b_2(i)$ is the relative value vector of the node betweenness index under the i th state of the shipboard power system. $b_3(i)$ is the relative value vector of maximum connection sub-diagram scale under the i th state of the naval power system. $b_4(i)$ is the relative value vector of node reliability index under the i th state of naval power system.

The vulnerability state space model of shipboard power system can be expressed as:

$$\begin{aligned} & (b_1(i+1), b_2(i+1), b_3(i+1), b_4(i+1)) \\ & = F\{b_1(i), b_2(i), b_3(i), b_4(i)\} \end{aligned} \quad (2)$$

$$\begin{aligned} & (Y_1(i), Y_2(i), Y_3(i), Y_4(i)) \\ & = G\{b_1(i), b_2(i), b_3(i), b_4(i)\} \end{aligned} \quad (3)$$

The transition equation (2) and the output equation (3) constitute the state space model of the vulnerability of the shipboard power system.

2.2 Functional analysis of vulnerability indicator set

\tilde{B}_1 is the degree index set of shipboard power network; \tilde{B}_2 is the betweenness index set of shipboard power network interfaces; \tilde{B}_3 is the maximum connection sub-diagram scale of the shipboard power network; \tilde{B}_4 is the reliability index set of shipboard power network. \tilde{B}_j ($j=1,2,3,4$) is a subset of w in m -dimensional Euclidean space R^m and satisfies the following theorem:

Theorem 1: $\tilde{B}_j \subset R^m$, ($j=1,2,3,4$) is a compact

set.

Proof: For any member $b_j = \begin{pmatrix} b_{j1} \\ b_{j2} \\ \vdots \\ b_{jm} \end{pmatrix}$ of set

\tilde{B}_j , because of $b_{jk} \in [0,1]$, $k=1,2,\dots,m$.

So, $\exists M > 0$, such that $\|b_j\| \leq M$. That subset \tilde{B}_j of R^m is bounded.

For any sequence $\{b_n\}$ of \tilde{B}_j , when $b_n \rightarrow b_0$, exists subsequence of $\{b_n\}$, $b_{n_k} \rightarrow b_0 \in \tilde{B}_j$. Then set \tilde{B}_j is a closed set.

Any bounded closed subset in a finite dimensional linear normed space is a compact set, so $\tilde{B}_j \subset R^m$ is a compact set. Prove that the end.

Continuous function on compact set, namely continuous functional has properties similar to continuous function on closed interval:

Theorem 2^[11]: Let X be the distance space, A is compact set in X ; $f: A \rightarrow R$ is a continuous function, then $f(x)$ reaches upper and lower bounds on A . that is, $f(x)$ takes its maximum and minimum on A .

In fact, it can be proved that the p -norm defined on a compact set is a continuous function:

Theorem 3: The p -norm ($p=1,2,3,\dots$) on a compact set $\tilde{B}_j \subset R^m$, ($j=1,2,3,4$) is a continuous function.

Proof: $f: \tilde{B}_j \rightarrow R$, let $f(x) = \|x\|_p$, $\forall x \in \tilde{B}_j$

Because of $\|x+y\| \leq \|x\| + \|y\|$, $\forall x, y \in \tilde{B}_j$, then $\forall x_0 \in \tilde{B}_j$

$$0 \leq |f(x) - f(x_0)| = \left| \|x\|_p - \|x_0\|_p \right| \leq \|x - x_0\|_p$$

So when $x \rightarrow x_0$, $f(x) \rightarrow f(x_0)$.

$f(x)$ is continuous on the compact set \tilde{B}_j , so the p -norm on a compact set $\tilde{B}_j \subset R^m$ is a continuous function. Prove that the end.

According to theorems 1, 2 and 3, the maximum and minimum values of p -norm on $\tilde{B}_j \subset R^m$ can be obtained on \tilde{B}_j .

3. METHODS

Norm can reflect the difference between any two elements in a linear space. For the shipboard power system network, the distance $\rho(b_j, b_{\min}) = \|b_j - b_{\min}\|$ between the current system and

the system with the lowest vulnerability is measured by using the vulnerability index vector b_j and b_{\min} . The smaller the difference, the lower the fragility of the system. The greater the difference, the higher the fragility of the system. And notice that each of the components of b_{\min} are 0, then $\rho(b_j, b_{\min}) = \|b_j\|$. That is, the distance between the vulnerability indicator vectors b_j and b_{\min} is the norm of the vulnerability indicator vector b_j .

The p -norm ($1 \leq p < \infty$) is used to describe the norm $\|b_j\|_p$ of vulnerability index vector $b_j \in \tilde{B}_j$:

$$\|b_j\|_p = \left(\sum_{k=1}^m |b_{jk}|^p \right)^{1/p} \quad (4)$$

The norm $\|b_j\|_p$ of the vulnerability index vector $b_j \in \tilde{B}_j$ can describe the vulnerability output of the warship power system under this index. At this time, the output equation of the vulnerability output vector of the warship power system is:

$$Y(j) = G\{b_j, j\} = \|b_j\|_p = \left(\sum_{k=1}^m |b_{jk}|^p \right)^{1/p} \quad (5)$$

Note that when $p=1$, the output equation is a linear equation:

$$Y(j) = C(j)b_j = (1 \quad 1 \quad \dots \quad 1) \begin{pmatrix} b_{j1} \\ b_{j2} \\ \vdots \\ b_{jm} \end{pmatrix} \quad (6)$$

$$= \sum_{k=1}^m |b_{jk}| = \|b_j\|_1$$

The comprehensive vulnerability index set $B = \tilde{B}_1 \times \tilde{B}_2 \times \tilde{B}_3 \times \tilde{B}_4$ of shipboard power network is a product space composed of degree index set \tilde{B}_1 , betweenness index set \tilde{B}_2 , maximum connection sub-diagram scale index set \tilde{B}_3 and reliability index set \tilde{B}_4 . That is:

$$B = \{b = (b_1, b_2, b_3, b_4) : b_j \in \tilde{B}_j\} \quad (7)$$

After going through the above four vulnerability indicators, the one-dimensional output vector of the vulnerability output equation of ship power system under each indicator can be obtained from equation (6), namely the p -norm of the vulnerability indicator vector. Then the vector r of the total relative value of the vulnerability value of the whole ship power system:

$$r = (\|b_1\|_p, \|b_2\|_p, \|b_3\|_p, \|b_4\|_p) \quad (8)$$

Generally speaking, the influence of different vulnerability indexes on the overall vulnerability of ship power system is different. In order to describe the difference and correlation among the indicators, the influence coefficient $\beta_j, j=1,2,3,4$ of the four vulnerability indicators of degree, betweenness, the scale of the maximum connected subgraph and reliability on the overall vulnerability of the ship power system was defined and expressed as vector β :

$$\beta = (\beta_1 \ \beta_2 \ \beta_3 \ \beta_4) \quad (9)$$

Consider the dot product $\beta \otimes r$ of vectors β and r :

$$\begin{aligned} \tilde{r} &= \beta \otimes r \\ &= (\beta_1 \cdot \|b_1\|_p, \beta_2 \cdot \|b_2\|_p, \beta_3 \cdot \|b_3\|_p, \beta_4 \cdot \|b_4\|_p) \end{aligned} \quad (10)$$

Where, \tilde{r} is the total relative value vector of the vulnerability value of naval power system with the influence coefficient taken into account.

In order to correlate the results of vulnerability evaluation indexes to form the evaluation results of the comprehensive vulnerability of ship power system, the multiscale comprehensive vulnerability norm P_s^p of ship power network is defined in this paper by using the norm of product space:

$$\begin{aligned} P_s^p &= \|\tilde{r}\|_p = \left(\sum_{j=1}^4 (\beta_j \cdot \|b_j\|_p)^p \right)^{1/p} \\ &= \left\{ \sum_{j=1}^4 \left(\beta_j \cdot \left(\sum_{k=1}^m |b_{jk}|^p \right)^{1/p} \right)^p \right\}^{1/p} \\ &= \left\{ \sum_{j=1}^4 \beta_j^p \cdot \left(\sum_{k=1}^m |b_{jk}|^p \right) \right\}^{1/p} \end{aligned} \quad (11)$$

The multiscale comprehensive vulnerability norm P_s^p of shipboard power network comprehensively measures the change trend of system comprehensive vulnerability from four scales of degree, betweenness, the scale of the maximum connected subgraph and reliability. The change of P_s^p value from small to large reflects the change trend of the comprehensive vulnerability of the system from low to high, which can be used as an evaluation index to measure the comprehensive vulnerability of the system.

In equation (11), the multiscale comprehensive vulnerability norm P_s^p of the ship power network can only compare the differences in the comprehensive vulnerability of different ship power system networks under the same scale. When the size of the shipboard

power network is different, that is, the number of included nodes is different, this index has some limitations. In order to eliminate the influence of system size on evaluation index, the improved multiscale comprehensive vulnerability norm \tilde{P}_s^p of shipboard power network is proposed:

$$\begin{aligned} \tilde{P}_s^p &= \left\{ \sum_{j=1}^4 \frac{1}{m} (\beta_j \cdot \|b_j\|_p)^p \right\}^{1/p} \\ &= \left\{ \sum_{j=1}^4 \frac{1}{m} \left(\beta_j \cdot \left(\sum_{k=1}^m |b_{jk}|^p \right)^{1/p} \right)^p \right\}^{1/p} \\ &= \left\{ \sum_{j=1}^4 \frac{1}{m} \beta_j^p \cdot \left(\sum_{k=1}^m |b_{jk}|^p \right) \right\}^{1/p} \end{aligned} \quad (12)$$

Where, m is the number of nodes in the ship power network.

In particular, if it is not possible to determine the influence coefficient of each vulnerability index on the overall vulnerability of the ship power system, it can be:

$$\beta = (\beta_1 \ \beta_2 \ \beta_3 \ \beta_4) = (1 \ 1 \ 1 \ 1) \quad (13)$$

At this time, the multiscale comprehensive vulnerability norm P_s^p of the shipboard power network can be written as:

$$\begin{aligned} P_s^p &= \|\tilde{r}\|_p = \left(\sum_{j=1}^4 (\|b_j\|_p)^p \right)^{1/p} \\ &= \left\{ \sum_{j=1}^4 \left(\sum_{k=1}^m |b_{jk}|^p \right) \right\}^{1/p} \end{aligned} \quad (14)$$

The multiscale comprehensive vulnerability norm \tilde{P}_s^p of the improved ship power network can be written as:

$$\begin{aligned} \tilde{P}_s^p &= \left\{ \sum_{j=1}^4 \frac{1}{m} (\|b_j\|_p)^p \right\}^{1/p} \\ &= \left\{ \sum_{j=1}^4 \frac{1}{m} \left(\sum_{k=1}^m |b_{jk}|^p \right) \right\}^{1/p} \end{aligned} \quad (15)$$

4. CALCULATION

The model verification is carried out with a ship as an example. The equivalent network model of ship power system is established according to the method in literature [12]. Figure 1 is the structure diagram of a certain type of ship power network, and figure 2 is the equivalent topology model corresponding to the power network.

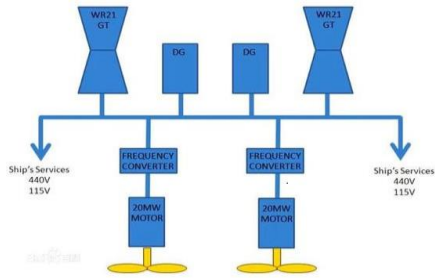


Fig.1 Sketch map of the shipboard power network

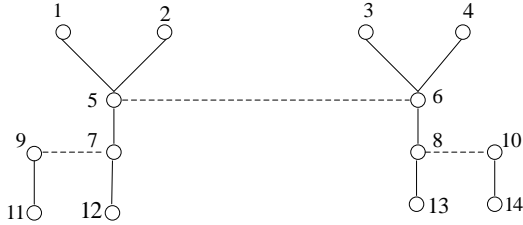


Fig.2 Topological graph of the shipboard power network

Node 1 and node 4 in figure 2 are two sets of firewood generator; node 2 and node 3 are two sets of combustion-generating units; node 5 and 6 are distribution board; nodes 7 and 8 are transformers; nodes 9 and 10 are auxiliary generators; Nodes 11, 12, 13 and 14 are load.

The reliability of a certain type of ship's combustion-generating unit is poor. If two sets of combustion-generating units are replaced with firewood generating units, the reliability index of components will change. In this section, the comprehensive vulnerability of the system after the reliability of some components changes is studied by using the comprehensive vulnerability measurement index of ship's power network.

The topology of the whole shipboard power network is not changed after the two sets of fuel generators are replaced with fuel generators, but the reliability of some nodes is changed. The situation in which two combustion generators are replaced with fuel generators is called mode 2, and the initial mode is called mode 1. According to the average failure rate, the reliability of each node of a certain type of ship power network under mode1 and 2 can be calculated, as shown in table 1 and 2.

Tab.1 The results of reliability index for the shipboard power network

No.	reliability	No.	reliability
1	0.3679	8	0.7945
2	0.7189	9	0.7189
3	0.7189	10	0.7189
4	0.3679	11	0.9999
5	0.9048	12	0.9999

6	0.9048	13	0.9999
7	0.7945	14	0.9999

Tab.2 The results of reliability index for the shipboard power network mode 2

No.	reliability	No.	reliability
1	0.7189	8	0.7945
2	0.7189	9	0.7189
3	0.7189	10	0.7189
4	0.7189	11	0.9999
5	0.9048	12	0.9999
6	0.9048	13	0.9999
7	0.7945	14	0.9999

5. RESULTS

In order to analyze the changes of system vulnerability before and after the reliability changes of some components in a certain type of ship power network, the multiscale comprehensive vulnerability norm \tilde{P}_s^p of the improved ship power network under two modes of a certain type of ship power network is calculated, as shown in table 3.

Tab.3 Modified Multiscale norm of comprehensive vulnerability degree

p	mode1	mode2
$p = 1$	0.9366	0.8838
$p = 2$	0.2023	0.1931
$p = 3$	0.1306	0.1267
$p = 4$	0.1079	0.1059
$p = 5$	0.0974	0.0964
$p = 6$	0.0916	0.0910
$p = 7$	0.0879	0.0876
$p = 8$	0.0854	0.0852
$p = 9$	0.0836	0.0835
$p = 10$	0.0822	0.0821

It can be seen from table 4 that, no matter what p value is taken, the modified multi-scale comprehensive vulnerability norm value in mode 2 is lower than that in mode 1. It is noted that when the p value changes from 1 to 3, the improved multiscale comprehensive vulnerability norm in mode 1 is higher than that in mode 2; when $p = 4$, the degree of improvement is no more than 2%; when $p > 5$, the difference is smaller. So, p usually ranges from 1 to 3. The smaller the multi-scale comprehensive vulnerability norm is, the lower the comprehensive vulnerability of the ship power network system will be. Therefore, after the two gas turbines with low

reliability in the ship power network are replaced by the diesel engines with high reliability, the system vulnerability will be reduced, which is also consistent with the actual situation. The validity of improving the multiscale integrated vulnerability norm to measure the difference of system vulnerability is demonstrated.

6. CONCLUSIONS

The shipboard power network has its own characteristics, so it is necessary to evaluate the comprehensive vulnerability of the whole ship power system network from the macro level. In this paper the state model of vulnerability of warship power system is established from two aspects of network topology and reliability.

In order to evaluate the vulnerability of the whole ship power network, the functional properties of the vulnerability index set are analyzed, and the compactness of each normalized evaluation index set is proved; in functional analysis, norm, as the mapping of elements to real Numbers in the linear space, can reflect the difference between any two elements in the linear space. Based on the norm of product space in functional theory, the multiscale comprehensive vulnerability norm of ship power network is proposed, and the evaluation results of each vulnerability index are correlated to form a comprehensive evaluation of the comprehensive vulnerability of ship power system. In the index of multiscale comprehensive vulnerability norm, the change of the index value from small to large reflects the change trend of the comprehensive vulnerability degree of ship power network from low to high. In order to eliminate the limitation brought by the scale of the system to the index, an improved multiscale comprehensive vulnerability norm index of the shipboard power network is introduced. This method can evaluate the vulnerability of ship power network from a new perspective, and with the improvement of the performance of ship power network, it can analyze the vulnerability of ship power network from a broader dimension.

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