

# THE ADAPTIVE COMPREHENSIVE VULNERABILITY ASSESSMENT MODEL OF SHIPBOARD POWER SYSTEM UNDER SEQUENTIAL RELATION

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## ABSTRACT

Aiming at the deficiencies of node vulnerability assessment based on complex network theory, adaptive model based on multi-attribute to assess node comprehensive vulnerability of shipboard power network is proposed. First, the structural vulnerability and physical vulnerability is considered. Then, the multi-attribute methods were applied in adaptive comprehensive evaluation model to obtain reasonable weights allocation plan of two kind indices. The adaptive comprehensive vulnerability assessment model of shipboard power network consider both the topological structure of the network and the physical property of the components. Thirdly, prove the maximal element theorem under quasi-order to verify the reasonability of maximizing the evaluation function. Finally, the comprehensive vulnerability index is compared to traditional structural vulnerability index. The test on a ring shaped shipboard power network demonstrates the validity of the model.

**Keywords:** shipboard power network, comprehensive vulnerability, multi-attribute, order relation, adaptive, weight

## NONMENCLATURE

<i>Abbreviations</i>	
PTDFs	power transmission distribution factors
<i>Symbols</i>	
$\mu$	coefficient matrix
$w$	weight matrix

## 1. INTRODUCTION

Most of the traditional vulnerability studies analyze the impact of network topology on system vulnerability<sup>[1]</sup>. The research methods of power system vulnerability assessment mainly include: Power system vulnerability assessment based on complex network theory; Power system vulnerability assessment based on probability theory; Power system vulnerability assessment based on system analysis; Power system vulnerability assessment based on system brittleness theory; Power system vulnerability assessment based on intelligent algorithm. Among them, complex network theory is the most significant one to describe the vulnerability of power system.

From the view of the model of topological structure, it can be divided into two models: unauthorized power network vulnerability assessment model and authorized power network vulnerability assessment model.

In the authorized power network model, most of the weight factors consider the electrical characteristics of nodes or lines<sup>[2-6]</sup>. For example, the reactance of the circuit is selected as the weight parameter in literature [2]. Literature [3] used electric medium as weight factor. Literature [4] adopts quasi-steady state distribution factor to establish an improved branch weight model; Literature [5-6] combines electrical distance, PTDFs (power transmission distribution factors) and line transmission capacity limit with vulnerability indicators in complex network theory. The above entitlement network model has played a certain role in correcting the vulnerability of land power system in a more reasonable way. As for the ship power system, the power equipment is in the special environment of the ship cabin, and the

influence of the inherent reliability factors of the equipment components on the vulnerability of the whole power system is also worthy of attention. Therefore, in addition to the influence factors of topology, the reliability of equipment in the ship power system also affects the vulnerability of the whole network.

In literature [7], the reliability parameters of power system components are taken into consideration when evaluating the vulnerability degree of power system, but the unit parameter vulnerability strength proposed in this paper is only the product of simple reliability weight and medium. Literature [8] proposed a method for identifying weak links based on operational reliability.

Although the influence of reliability on the structural vulnerability of power systems has been considered in the above literatures, the reasonable and effective superposition of these two types of characteristics has not been deeply studied. Literature [9] identified the vulnerability of nodes in complex power systems by using utility risk entropy. Literature [10] proposed a fuzzy comprehensive evaluation method of power grid node vulnerability based on utility risk entropy weight.

The comprehensive assessment of power system vulnerability needs to collect a number of index information reflecting the power system performance to obtain a comprehensive index. However, there may be conflicts between different types of indicators, which may lead to inconsistent conclusions of evaluation results under different indicators. Therefore, it is the key point of comprehensive assessment of power system vulnerability to determine the weight of each evaluation index reasonably and objectively. In this paper the vulnerability assessment index set of warship power system is formed by considering the network topology and component reliability. On this basis, this paper focuses on how to rationally and objectively obtain the weight distribution scheme of the two types of indicators, and proposes an adaptive comprehensive weight acquisition optimization model for the warship power grid through the combination of different weighting methods using the multi-attribute decision-making method, so as to realize the comprehensive identification of the weak links of the warship power system. The proposed model is verified by an example of a ring ship power system network.

## 2. METHODS

### 2.1 Analysis of the interaction between indicators

As shown in figure 1, this is a simple network topology connection diagram. For node 10 in the

network, its degree value is 6 from the position information and connection mode of the node in the network. It is the node that has the most connection relationship with other nodes in the network, and is also the most important node in the network, namely the vulnerable node. However, from the perspective of system science analysis, the removal of node 10 from the network does not have a great impact on the connectivity level of the network, and it has the same impact on the connectivity level after the removal of other nodes. According to the scale of the maximum connected subgraph, node 10 is not the most vulnerable node in the network.

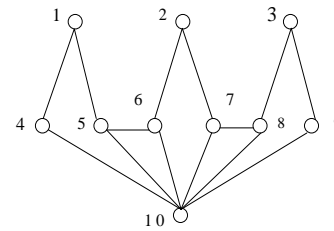


Fig.1 Sketch map of network

Therefore, degree and betweenness are important indexes of nodes in social network analysis. As a node vulnerability index in the system science analysis method, the maximum connected subgraph has an impact on the comprehensive identification of vulnerable links in the power grid. And these two kinds of indicators are measured from the connection relation and position structure of nodes in the network, which is the embodiment of network topology.

On the other hand, compared with the land power grid, the ship power network has its own characteristics, which is composed of a series of electrical equipment (power supply, load, etc.) through circuit breakers and connecting cables to achieve correlation and thus form a whole. If the reliability index of the equipment is very poor and the failure rate is very high, the node of the equipment will become a weak link in the system due to frequent failures, which will lead to the failure of the network. There is a connection path between the nodes of the ship power grid, and the fault of the node will lead to the failure of the connection path. Whether the connection path is effective and reliable -- that is, the influence of the accessibility of the connection path on the vulnerability cannot be ignored. It is necessary to calculate the vulnerability index based on the influence of superposition node equipment reliability in order to make the reflected vulnerability level of power network closer to the actual operation.

### 2.2 Adaptive comprehensive weight acquisition method

In this paper, a set of comprehensive vulnerability assessment indexes for shipboard power network nodes is formed from the aspects of network topology and component reliability. Taking the vulnerability degree of each node of the ship power network as the evaluation object, four kinds of indexes are considered in the comprehensive vulnerability evaluation index of the ship power network node: degree, betweenness, maximum connection sub-diagram scale and reliability. Then the comprehensive vulnerability assessment index  $W_i$  ( $i=1,2,\dots,m$ ) of the node  $i$  can be expressed as:

$$W_i = (w_1, w_2, w_3, w_4) \cdot \begin{pmatrix} W_{1i} \\ W_{2i} \\ W_{3i} \\ W_{4i} \end{pmatrix} \quad (1)$$

$$= w_1 \cdot W_{1i} + w_2 \cdot W_{2i} + w_3 \cdot W_{3i} + w_4 \cdot W_{4i}$$

Where,  $W_{1i}$  is the degree index of the node  $i$ ;  $W_{2i}$  is the betweenness index of the first node  $i$ ;  $W_{3i}$  is the index of the maximum connected sub-graph scale of the node  $i$ ;  $W_{4i}$  is the reliability index of the fourth node  $i$ .

There are many methods to determine the weight of indicators, and different weighting methods give different weight allocation schemes. The quality of the weight allocation scheme can be determined by multi-attribute decision making. Multi-attribute decision making is also known as multi-objective decision making of finite schemes. Its essence is to sort finite alternatives in a certain way by using existing decision information, and the scheme with higher ranking is better than the scheme with lower ranking.

Consider a decision problem with  $m$  schemes and  $n$  attributes. In order to sort the schemes in the scheme set, a preference structure  $(P, I, R)$  on the scheme set  $X$  is established.  $P$  means superior;  $I$  means no difference and;  $R$  means incommensurable.

Definition 1: Additive method with rank  $r$ : if there are  $r$  real valued functions  $V_l(x_i) = \sum_{j=1}^n \omega_j V_{ij}(f_j(x_i), f_j(X))$ ,  $x_i \in X, l=1,2,\dots,r$ , so that for any two schemes  $x_i, x_k \in X$ :

$$\begin{cases} x_i P x_k & , V_l(x_i) > V_l(x_k) \\ x_i I x_k & , V_l(x_i) = V_l(x_k) \\ x_i R x_k & , \text{others} \end{cases} \quad (2)$$

The method is called additive method with rank  $r$ , and  $V_l(x_i)$  is the  $l$  evaluation function of scheme  $x_i$ .

The vulnerability of the nodes of the ship power network is taken as the evaluation object and each vulnerability evaluation index of the nodes is taken as its attribute. Then  $m$  evaluation objects and the decision

matrix of  $n$  indicators is  $R = (r_{ij})_{m \times n}$ .

If there are  $v$  kinds weighting methods,  $w_{kj}$ , ( $k=1,2,\dots,v$ ) represents the weight of the index  $j$  obtained by the weighting method  $k$ , then the weight matrix is:

$$W = \begin{pmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ w_{v1} & w_{v2} & \cdots & w_{vn} \end{pmatrix} \quad (3)$$

Synthesize  $v$  kinds weighting methods, set  $\mu_{kj} \in [0,1]$ , ( $k=1,2,\dots,v; j=1,2,\dots,n$ ) as the participation coefficient, then the participation coefficient matrix  $\mu$  is:

$$\mu = \begin{pmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \mu_{v1} & \mu_{v2} & \cdots & \mu_{vn} \end{pmatrix} \quad (4)$$

The comprehensive weight of the  $j$ th index  $w_{sj}$ :

$$w_{sj} = \sum_{k=1}^v \mu_{kj} w_{kj} \quad (5)$$

The participation coefficient  $\mu_{kj}$  of each index in formula (5) is the key to obtain a reasonable comprehensive weight. By establishing a preference structure on the scheme set, the multi-attribute decision making method can sort the schemes in the scheme set. The top ranked option is better than the bottom ranked option. In order to make the comprehensive weight scheme ranking better than other schemes, the following adaptive comprehensive weight optimization model can be established by using the multi-attribute decision-making method:

$$\begin{cases} \max V_l(x_i) \\ s.t. \begin{cases} 0 \leq \mu_{kj} \leq 1, k=1,2,\dots,v; j=1,2,\dots,n \\ \sum_{j=1}^n [\sum_{k=1}^v \mu_{kj} w_{kj}] = 1 \\ \sum_{k=1}^v \mu_{kj} = 1 \end{cases} \end{cases} \quad (6)$$

Where, the objective function  $V_l(x_i)$  is the evaluation function of the  $i$ th scheme in the multi-attribute evaluation method. If the weighted sum method is used, then  $V_l(x_i) = \sum_{j=1}^n \omega_j w_{ij}$ . Where  $\omega_j$  is the initial weight of each index;  $w_{ij}$  represents the weight of the  $j$ th index obtained by the  $i$ th weighting method.

In the constraint condition, each participation coefficient in the participation coefficient matrix  $\mu$

satisfies  $\mu_{kj} \in [0,1]$ ; and  $\sum_{k=1}^v \mu_{kj} w_{kj}$  is the comprehensive weight of the  $j$ th index, according to the normalization, the sum of the comprehensive weight of  $n$  indicators is 1, i.e.  $\sum_{j=1}^n [\sum_{k=1}^v \mu_{kj} w_{kj}] = 1$ ; the sum of the participation coefficients of  $v$  kinds weighting methods for each index is 1, i.e.  $\sum_{k=1}^v \mu_{kj} = 1, j = 1, 2, \dots, n$ .

By solving the above optimization model, the comprehensive weight of each index can be obtained, and the comprehensive vulnerability assessment index of each node can be obtained by using the comprehensive weight  $W_i$ :

$$W_i = \sum_{j=1}^n w_{ij} b_{ij} \quad (7)$$

$b_{i1}$  is the degree index after normalization of the  $i$ th node;  $b_{i2}$  is the median index after normalization of the  $i$ th node.  $b_{i3}$  is the scale index of the largest connected subgraph after the normalization of the  $i$ th node.  $b_{i4}$  is the normalized reliability index of the  $i$ th node. As  $b_{ij}$  in the comprehensive vulnerability assessment index is the result after normalization, the index value of node comprehensive vulnerability assessment meets the requirement  $W_i \in [0,1]$ . The change of  $W_i$  from 0 to 1 reflects the change of node vulnerability from low to high. The vulnerability degree of the nodes can be quantitatively reflected by the comprehensive vulnerability assessment index of the nodes, so as to realize the comprehensive identification of the weak links in the ship power network.

### 3. THEORY

The core idea of the adaptive comprehensive weight acquisition optimization model proposed in section 2.2 is to maximize the evaluation function under certain constraints by means of combination weighting. From the perspective of multi-attribute decision making, the value of the evaluation function can represent the preference structure  $(P, I, R)$  on the scheme set  $X$ , and the preference relationship provides a theoretical basis for the "priority" ordering of schemes in the scheme set. The preference structure  $(P, I, R)$  mentioned in this paper is a kind of quasi-order relation. If there are maximal elements in the quasi-order meaning, it is reasonable to obtain the objective of the optimization model -- evaluation function to reach the maximum by the adaptive comprehensive weight proposed in section 2.2

from the perspective of completeness.

In nonlinear functional theory, Zorn lemma is a very important order theorem [11]:

The lemma of zorn discusses the maximum element problem under the semi - order relation. Literature [12] proved the lemma of zorn in the sense of quasi-order: Each complete preferred subset  $M$  of non-empty quasi-ordered set  $X$  has an upper bound in  $X$ , so  $X$  has maximal elements. From this lemma, we can obtain theorem 1:

**Theorem 1:** If  $(X, \prec)$  is a quasi-ordered set, and:

(1) If the sequence  $\{x_n\}$  in  $X$  that satisfies

$x_1 \prec x_2 \prec \dots x_n \prec \dots$  is a compact set;

(2)  $X$  satisfies upper pseudo-separable.

Then there must be some maximal element in  $X$ .

**Proof:** if  $M$  is set as the complete preference subset in  $X$ , then by the Zorn Lemma in the sense of quasi-order, it is only necessary to prove that  $M$  has an upper bound in  $X$ .

According to the condition (2),  $X$  satisfies upper pseudo-divisible, and there exists countable set:  $\{x_n\} \subset M$ , s.t.  $\forall x \in M$  and  $x \neq \sup M$ ,  $x_{n_0}$  exists in  $\{x_n\}$  satisfy:  $x \prec x_{n_0}$ .

Construct the sequence  $\{z_n\}$  as follows:  $z_1 = x_1$ ,

$z_2 = \max\{x_1, x_2\}$ , ...,  $z_n = \max\{x_1, x_2, \dots, x_n\}, \dots$ .

Obviously,  $\{z_n\} \subset \{x_n\} \subset M$ , and

$z_1 \prec z_2 \prec \dots \prec z_n \prec \dots$ .

Note that,  $x_n \prec z_n, n = 1, 2, \dots$ .

According to the condition (1),  $\{z_n\}$  is a compact sequence, it can be known that:  $\{z_{n_i}\}$  exists in  $\{z_n\}$  and  $z^* \in X$ , such that  $z_{n_i} \rightarrow z^*$ .

Since  $z_1 \prec z_2 \prec \dots \prec z_n \prec \dots$ , then  $z_n \prec z^*, n = 1, 2, \dots$ .

Then  $\forall x \in M$ , exists  $z^* \in X$ , such that  $x \prec x_{n_0} \prec z_{n_0} \prec z^*$ , i.e.  $x \prec z^*$ . So  $M$  has an upper bound in  $X$ , so there are maximal elements.

In the comprehensive vulnerability assessment of ship power network, the weight scheme of  $n$  evaluation indexes obtained by  $v$  kinds weighting methods is regarded as the quasi-ordered set  $X$ , and satisfies the condition in theorem 1, so there must be maximum element in the scheme set.

### 4. CALCULATION

The model verification is carried out with a ship as an example. The equivalent network model of ship power system is established according to the method in literature [13]. Figure 2 is the structure diagram of a ring

type of ship power network, and figure 3 is the equivalent topology model corresponding to the power network.

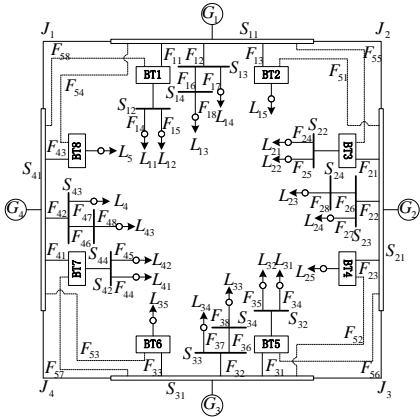


Fig.2 Sketch map of a complex ring shaped

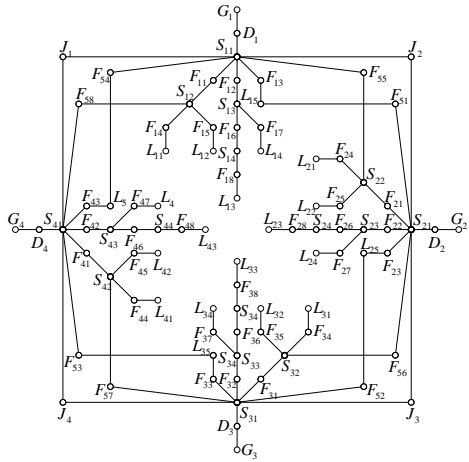


Fig.3 Equivalent model of the network

In order to verify the effectiveness of the adaptive comprehensive weight acquisition model proposed in section 2, entropy method [22] and cloud model [23] were selected as the weight scheme. Entropy weight  $w_{ej}$  and cloud weight  $w_{yj}$  of each node vulnerability index are calculated by entropy value method and cloud model method respectively, as shown in table 1.

Tab. 1 Entropy weights and cloud weights of the indices

Indicators	Degree	Betweenness	Maximum connection sub-diagram scale	Reliability
Entropy weight	0.2002	0.2842	0.3869	0.1287
cloud weight	0.1895	0.3158	0.3702	0.1245

Solve the optimization model (6) with MATLAB, and get:  $\mu_{11} = 0; \mu_{12} = 0.6614; \mu_{13} = 1; \mu_{14} = 1$ .

The calculation results of the comprehensive weight are shown in table 2.

Tab. 2 Comprehensive weights of the indices

Indicators	Degree	Betweenness	Maximum connection sub-	Reliability

			diagram scale	
Comprehensive weight	0.1895	0.2949	0.3869	0.1287

The weighted sum method is adopted to sort the three schemes, and the evaluation function values of the three schemes are obtained. The ordering of the schema set is shown in table 3.

Tab.3 Weights scheme ranking

scheme	Evaluation function value	sort
Entropy method	0.2831	3
Cloud weight method	0.2847	2
Comprehensive weighting method	0.2940	1

As can be seen from table 3, the evaluation function value of the comprehensive weight scheme obtained in this paper is the largest. From the perspective of multi-attribute decision making, the value of the evaluation function represents the priority order of each scheme. Therefore, the comprehensive weight scheme is superior to the entropy method and cloud weight method.

In order to verify the effectiveness of the adaptive comprehensive weight acquisition model proposed in this paper, the comprehensive vulnerability index is compared with the comprehensive vulnerability index under the weight scheme of entropy method. Compared with traditional vulnerability indicators such as the betweenness, degree and the maximum connection sub-diagram scale, the node vulnerability identification results are shown in table 4.

Tab.4 Comparison of vulnerability identification of nodes

sort	Integrated vulnerability indicator nodes	The node of comprehensive vulnerability index under entropy weight	degree node	betweenness node	maximum connection sub-diagram scale node
1	53	53	53	7	53
2	57	57	57	61	57
3	61	61	61	5	61
4	65	65	65	53	65
5	55	14	54	6	14
6	59	22	58	57	22
7	63	30	62	8	30
8	67	38	66	65	38

It can be seen from table 4 that the comprehensive vulnerability index under the weight scheme of entropy method is consistent with the first eight nodes in the ranking result of nodes by the scale index of the maximum connected subgraph. The first four nodes of the ranking of the comprehensive vulnerability index and

the maximum connected subgraph scale index are all the nodes of the main distribution board. The difference is that the nodes in the ranking of the comprehensive vulnerability index proposed in this paper are the nodes of the distribution board. The nodes located in 5-8 in the order of the maximum connected subgraph scale index are feeder cable nodes. The first eight grid components are generator cable nodes and main switchboard nodes when sorted by the index of medium number from large to small. When nodes are sorted by degree index, the first eight grid components are the nodes of main distribution board and distribution board.

## 5. CONCLUSION

In this paper, a set of comprehensive vulnerability assessment indexes for shipboard power network nodes is formed from two aspects of network topology and component reliability. Based on the topological structure and the physical characteristics of each component of the shipboard power network, the multi-attribute decision making method is used to obtain the reasonable weight distribution scheme between the two types of indexes. The existence of maximal elements in the sense of quasi-order is analyzed. From the perspective of completeness, it is proved that the objective of the proposed adaptive comprehensive weight acquisition optimization model -- evaluation function reaches the maximum rationality. The optimization model is obtained by using the adaptive comprehensive weight of the ship power network, so that the weak links in the ship power network can be identified comprehensively, and the one-sidedness of evaluating the vulnerability index of the power network components can be avoided. Finally, the vulnerability assessment model is verified by taking a certain annular ship power grid as an example, and the results are of reference significance.

## ACKNOWLEDGEMENT

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