# THE SHORT-TERM VOLTAGE ROBUST REACTIVE POWER OPTIMIZATION OF POWER SYSTEM CONSIDERING THE UNCERTAIN OUTPUT OF DISTRIBUTED PHOTOVOLTAIC

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#### ABSTRACT

The uncertainty of distributed photovoltaic (PV) output brings difficulties to reactive power optimization of power system. A robust reactive power optimization model for power system considering the uncertain output of PV is proposed. The probabilistic model of PV output is used to solve the problem that the robust optimization solution is too conservative. The effectiveness of the proposed model is verified by an example.

**Keywords:** distributed photovoltaic; reactive power optimization; robust optimization; uncertainty

# 1. INTRODUCTION

Reactive power optimization is an important means to ensure the safety and economy of power system operation and improve the power quality of users<sup>[1]</sup>. Because the constraints of reactive power optimization have the constraints of power flow equation, the problem of reactive power optimization is actually a nonlinear problem of higher order, which is also one of the difficulties of reactive power optimization. As a kind of clean and renewable energy, solar energy has great development value, and photovoltaic power generation will be a very important mode of electricity production in the future<sup>[2]</sup>. However, the uncertainty of distributed photovoltaic (PV) output makes the reactive power optimization problem become a high-order nonlinear uncertainty problem.

At present, there are two main methods to solve the problem, based on the stochastic power flow algorithm, a reactive power optimization model is established in [3-4], based on the probability distribution of distributed PV output. This approach relies heavily on the probability distribution of distributed PV output. Since reliable

probability distribution requires a large number of sample data, this method takes a long time and is not conducive to short-term reactive power optimization. In [5], the robust optimization method is used to transform the uncertain high-order linear problem into a determined second-order cone optimization model. The selection of the uncertain set is based on the mediumand long-term distributed PV output prediction, which can solve the medium and long-term reactive power optimization problem, but it is difficult to solve the shortterm reactive power optimization problem, and the robust optimization method is conservative and economical.

In this paper, the advantages of stochastic power flow method and robust optimization method are combined to select the uncertainty set through the probability distribution model of distributed photovoltaic output. The set has low sensitivity to the accuracy of probability distribution, and is suitable for short-term reactive power robust optimization, and balances the robustness and economy. Then, this paper establishes a reactive power optimization model with the aim of improving voltage quality, and uses the Newton iterative method to simplify the model, which transforms the reactive power optimization problem into a linear problem, which greatly reduces the calculation time. Then the paper uses the robust duality theorem to transform the above-mentioned optimization model with uncertain variables into a completely determined optimization model. Finally, the effectiveness of the model is verified by an example.

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# 2. REACTIVE POWER OPTIMIZATION MODEL CONSIDERING PV GRID-CONNECTED

## 2.1 PV output uncertainty model

The output power of distributed PV is determined by factors such as light intensity, PV array area and photoelectric conversion efficiency. The calculation formula is shown in (1).

$$P_{G} = IS\eta \tag{1}$$

Where, *I* is light intensity; *S* is the area of the PV array; and  $\eta$  is the photoelectric conversion efficiency.

It can be seen from (1) that after the installation of photovoltaic power supply, the output of distributed photovoltaic power is only related to light intensity. The light intensity is affected by cloud constraints and other factors, showing obvious uncertainty. According to statistics, in a certain period of time, the light intensity can be approximately considered as obeying Beta distribution. The probability density functions are:

$$f(I) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \left(\frac{I}{I_c}\right)^{\alpha - 1} \left(1 - \frac{I}{I_c}\right)^{\beta - 1}$$
(2)

Where,  $I_C$  is the maximum light intensity, and  $\alpha$  and  $\beta$  are the shape parameters of the Beta distribution, respectively.

$$\alpha = \mu \left[\frac{\mu(1-\mu)}{\sigma^2} - 1\right] \tag{3}$$

$$\beta = (1 - \mu) [\frac{\mu (1 - \mu)}{\sigma^2} - 1]$$
(4)

Where,  $\mu$  and  $\sigma$  sigma are the expected value and variance of light intensity in a certain time period.

## 2.2 Power system model with distributed photovoltaic

Distributed PV is connected to the grid through inverter, and its power factor is about -0.95~0.95. This means that its reactive power output is much smaller than that of active power, so the power system model including distributed power supply can ignore its reactive power output. The power system model with distributed photovoltaic is shown in figure 1.





In Figure 1, the  $Z_k$  is the circuit admittance, the  $U_k$  is the voltage of k node, the  $P_{L,k}+jQ_k$  is load power of k node, the  $P_{G,k}+jQ_{G,k}$  is the k-node distributed power supply power. If there is no distributed PV supply on the node, the distributed PV supply power is 0 here.

## 2.3 Robust reactive power optimization

#### 2.3.1 Uncertain set

Unlike stochastic programming, which uses probability distribution to describe uncertainty, robust reactive power scheduling takes all possible implementations of the uncertainty under consideration into a set, which is called the uncertainty set. The selection of uncertain sets plays an important role in robust reactive power scheduling. When the selection of the uncertain set is too small, the reactive power scheduling strategy will be difficult to meet the robustness requirements, and the selection of the uncertain set is too large, the reactive power scheduling result will be too conservative. In this paper, the probabilistic model of solar radiation intensity is used to select uncertain sets at different confidence levels, which can give consideration to both robustness and conservatism according to actual needs.

First, the upper and lower limits of the illumination intensity at the set confidence level are selected according to the illumination intensity probability model. In order to obtain the upper and lower limits of the illumination intensity easily, the upper and lower limits of the light intensity are set to be symmetric with respect to the expected value of the light intensity  $\mu$ .

$$\int_{I_{l}}^{I_{u}} f(I)dI = \int_{\mu-a}^{\mu+a} f(I)dI = P_{set}$$
(5)

Where,  $I_u$  is the upper limit of illumination intensity;  $I_l$  is the lower limit of illumination intensity;  $P_{set}$  is the set confidence level.

After obtaining the confidence interval of illumination intensity, the uncertain interval of illumination intensity is substituted into equation (1) to obtain the uncertain range of each distributed PV output. The box-type uncertainty set of distributed PV output is shown in equation (6).

$$\left\{P_G\left|\left(P_{\mu}\right)_j + \left(\delta\right)_j, \left|\left(\delta\right)_j\right| \le \left(\omega\right)_j, \omega_j = a\eta_j S_j, j \in n\right\}\right\}$$
(6)

Where,  $P_G$  is the column vector constituted by the actual output of each distributed PV;  $P_{\mu}$  is the column vector formed by the expected output value of each distributed PV; ()<sub>j</sub> represents the *j*th element of the vector;  $S_j$  and  $\eta_j$ respectively represent the photoelectric conversion efficiency and PV area of the *j*th PV;

2.3.2 Traditional reactive power scheduling model

## (1) objective function

The goal of reactive power scheduling is mainly to improve voltage quality and reduce network loss. This paper mainly discusses improving voltage quality. The objective functions are shown in equation (7).

$$\min f_1 = \sum_{i \in n} w_i \left| U_i - U_N \right| \tag{7}$$

Where,  $w_i$  is the weight of each node's voltage quality;  $U_N$  is the rated voltage;

(2) constraints

The constraint conditions mainly include two parts: state variable constraint and decision variable constraint. The decision variable selected in this paper is the reactive power output of the reactive power compensation device installed at each node of the power network. The selected nodes are all p-q nodes except the power node, so the state variable is the voltage amplitude and phase Angle of each node (except the power node).Decision variables and state variables should satisfy the power flow equation, as shown in equations (8) and (9).The decision variable shall satisfy the output constraint of the reactive power device, as shown in equation (10). The state variable shall satisfy the node voltage constraint, as shown in equation (11).

$$P_{G,i} - P_{L,i} = U_i \sum_{j \in i}^{j} U_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij})$$
(8)

$$Q_{C,i} - Q_{L,i} = U_i \sum_{j \in i} U_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij})$$
(9)

$$Q_{C\min} \le Q_C \le Q_{C\max} \tag{10}$$

$$U_{L\min} \le U_i \le U_{L\max} \tag{11}$$

Where,  $Q_{C,i}$  Is the output of the reactive power compensation device, and is the decision variable.

# 2.3.3 Simplification of traditional models

The traditional reactive power scheduling model is a complex high-order nonlinear problem, which is quite different from the general robust optimization model (especially equations (8) and (9)), which makes it difficult to solve the reactive power optimization problem with robust optimization method. Therefore, when using robust optimization method to solve the reactive power optimization and transformation should be made to the traditional reactive power optimization model.

(1) Benchmark value determination. According to the expected value of distributed photovoltaic power output and the current state of each reactive power compensation device, the optimal power flow algorithm is used to solve the expected value meeting the target reactive power output.

(3) Constraint treatment of power flow equation. Equations (7) and (8) are high-order nonlinear equations, which are difficult to solve. Since reactive power optimization mainly focuses on variation, Newton power flow method can be used here to transform higher-order nonlinear equations into linear equations about variation. According to Newton iteration method, the power flow equation can be approximated as the equation shown in (12).

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = -J \begin{bmatrix} \Delta \theta \\ \Delta U / \overline{U} \end{bmatrix}$$
(12)

As the voltage amplitude change of the node of state variable is caused by the uncertainty of the output of distributed photovoltaic and the reactive power regulating device, it is required to change into equation (13) from equation (12).

$$\begin{bmatrix} \Delta \theta \\ \Delta U / \overline{U} \end{bmatrix} = -J^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$
(13)

Since the emphasis of reactive power optimization is to consider the change of voltage amplitude, and reactive power scheduling has little influence on the change of node phase Angle, and the uncertain output of distributed photovoltaic with large influence on phase Angle is also very small in a short time, so the change of phase Angle can also be ignored. Here, only the equation related to voltage is considered.

$$\Delta U = \overline{M} \Delta P + \overline{L} \Delta Q \tag{14}$$

Where,  $\Delta P$ ,  $\Delta Q$  and  $\Delta U$  are the deviations of the distributed photovoltaic output, the reactive device output, and the voltage relative to the respective reference quantities solved in equation (1);  $\overline{M}$  and  $\overline{L}$  are to multiply each element of M and L by the corresponding element of  $\overline{U}$ .

## 2.3.4 Voltage robust reactive power optimization model

By transforming equation (14), the constraint can be changed into an equation about node voltage deviation, as shown in equation (15).

$$|U - U_N| = |\Delta U + \overline{U} - U_N|$$
  
=  $|\overline{M}\Delta P + \overline{L}\Delta Q + \overline{U} - U_N|$  (15)

The solution under this condition is very sensitive to the change of the equation. When the  $\Delta P$  is uncertain, it is difficult to find a solution that meets the requirements and is immune to the change of the  $\Delta P$ . Here, (15) is substituted into the objective function in the form of penalty function, and the inequality constraint (16) is added to reduce the sensitivity of (17) to the solution.

$$\min f_{1} = W^{T} x + \lambda (|M\Delta P + L\Delta Q + U - U_{N}| - x)$$
$$= W^{T} x + \lambda t \qquad (16)$$
$$t = |\overline{M}\Delta P + \overline{L}\Delta Q + \overline{U} - U_{N}| - x$$

$$\left|\overline{M}\Delta P + \overline{L}\Delta Q + \overline{U} - U_{N}\right| - x - t \le 0$$
(17)

Where, x is the absolute value vector of voltage

deviation; *W* is the voltage weight vector of each node;  $\lambda$  is the penalty factor.

In order to facilitate the calculation, inequality (17) needs to be transformed into inequality (18) and inequality (19) without absolute value.

$$\overline{M}\Delta P + \overline{L}\Delta Q + \overline{U} - U_N - x - t \le 0$$
(18)

$$-\overline{M}\Delta P - \overline{L}\Delta Q - \overline{U} + U_N - x - t \le 0$$
<sup>(19)</sup>

The new inequality constraint (22) - (23) can be obtained by substituting the new variable into the inequality constraint (10) - (11).

$$x \le \left| U_{\max} - U_N \right| \tag{20}$$

$$Q_{c\min i} \le \Delta Q \le Q_{c\max} \tag{21}$$

$$x \ge 0 \tag{22}$$

From the robust duality theorem, the equivalent form of the robust model under the box set can be obtained, as shown in equation (23). In this form, the inequality constraint no longer contains uncertain parameters, and the problem is transformed from an uncertain problem to a definite one.

$$\min f_{1} = \overline{W}^{T} x + \lambda t$$

$$s.t. \overline{M} \omega y_{1} + \overline{L} \Delta Q + \overline{U} - U_{N} - x - t \leq 0$$

$$\overline{M} \omega y_{1} - \overline{L} \Delta Q - \overline{U} + U_{N} - x - t \leq 0$$

$$y_{1} \geq 1, y_{2} \geq 1$$

$$(20) - (22)$$

$$(23)$$

## 3. NUMERICAL EXAMPLE

In order to verify the effectiveness of the model in this paper, the robust optimization analysis is carried out on the distribution network model with distributed pv as shown in Figure 1. In Figure 1, n=5 is taken, and other specific parameters are shown in literature [6]. Voltage deviation of each node is not allowed to exceed 3%, and each node is connected to a 1MW photovoltaic power station. Within a certain period of time, take the output of distributed PV as  $P_G$  is 0.7, and the set of uncertain variables is:

$$\left\{ P_G | (P_\mu)_j + (\delta)_j | (\delta)_j | \le 114, j \in \{1, 2, 3, 4, 5\} \right\}$$
 unit: kw

In order to study the effect of robust optimization method, the optimization results of robust optimization method and stochastic power flow method under two extreme scenarios (maximum and minimum output of distributed photovoltaic) were compared. Figure 2 shows the voltage optimization results of the robust reactive power optimization scheme and the optimization scheme without considering uncertain variables in two extreme scenarios. As can be seen from the figure, robust optimization can ensure that the node voltage still meets the voltage constraint in extreme scenarios, while conventional optimization schemes cannot achieve this effect.



Fig.2 Voltage optimization results in extreme scenarios

# 4. CONCLUSIONS

Based on the stochastic power flow optimization method and the traditional robust optimization method, this paper proposes a robust optimization method combining the advantages of both. This method has good robustness, can still get better optimization results in extreme scenarios, and can effectively solve the problem of short-term voltage reactive power optimization under large-scale photovoltaic grid-connected condition. However, due to the use of partial simplification in the modeling process, the calculation accuracy is reduced, and how to improve the accuracy still needs further study.

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