

# FORCE CHAIN NETWORK ANALYSIS OF GRANULAR MATERIAL BASED ON THE COMMUNITY DETECTION METHOD

Luming Cha<sup>1\*</sup>, Qihong Feng<sup>2</sup>, Sen Wang<sup>3</sup>, Tian Xia

1 China University of petroleum (East China), Qingdao, China

2 China University of petroleum (East China), Qingdao, China

3 China University of petroleum (East China), Qingdao, China

3 China University of petroleum (East China), Qingdao, China

## ABSTRACT

In the last decade, the behavior of granular materials has great significance for chemical engineering and petroleum engineering. This multiple process can affect how a granular material responds when exposed to external perturbations. Traditional studies mainly focus on the macroscopic performance of granular materials by using experimental methods, which tend to be implicitly agnostic to granular structures, however, there are few reports on the mechanism of the mechanism such as the mechanism of rupture and evolution. To address this challenge, we treat granular materials as spatially-embedded networks in which the particles (nodes) are connected by weighted edges. We employ community detection network analysis method to find sets of closely connected particle clusters and characteristic chain-like structure that is reminiscent of force chains based on the discrete element method. A group of tightly connected communities and branching chain structures are founded and the three-dimensional properties of granular materials under different fluid conditions are studied. The results show that the granular contact network exhibit a very clear community structure and network analysis method provides new viewpoints to underlying and generate a new perspective of describing granular material problem compared with traditional methods. The distribution of force chain is consistent with an exponential distribution and the high-pressure force chain network exhibit compact instead of branching structure communities. And granular matter displays traits of self-organization forming complex force network which arrange in

response to applied load or compression and more specifically force network rearranges prominently when the applied pressure is disturbed. These results provide new tools for considering and reducing descriptions for the structure of force chain network.

**Keywords:** granular material; force chain network structure; community detection method; numerical simulation

## NONMENCLATURE

<i>Abbreviations</i>	
DEM	Discrete Element Method
LBM	Lattice Boltzmann Method
<i>Symbols</i>	
$W$	weighted adjacent matrix
$w_{ij}$	Contact force between node $i$ & $j$
$Q$	modularity
$N_c$	Number of communities
$l_c$	Edges of communities
$k$	Strength of weight
$d$	Total number of degrees

## 1. INTRODUCTION

The behavior of granular materials under fluid action has great significance for chemical engineering and petroleum engineering [1]. However, scholars mainly focuses on the macroscopic performance of granular materials, but there are few reports on the mechanism of microscopic mechanism such as the mechanism of rupture and evolution.

Vishal et al. (2018) [2] investigated the coupled fluid-solid interaction at the pore scale by the digital rock physics technology. They observed the pore structure behavior at different fluid pressure and states. The interaction behavior of pore structure and solid particles under different fluid pressures was analyzed in numerical simulation and laboratory experiments. Zhang et al. (2017) [3] have introduced an efficient Discrete Element Lattice Boltzmann Method (DEM-LBM) to simulate the macroscopic behaviors of multiphase system including particle-fluid and particle-particle interaction. Particularly, for the particle-fluid interact problems, most scholars have done quite a lot of work based in indoor experiments and numerical simulation method. However, it's quite complicated to relate macroscopic behavior to physical process at the granular scale because it's impossible to measure contact forces and particle movement inside the particulate material, especially the fluid impact, in most experimental implementations.

Whereas, the concept of force chain of granular material (see Fig 1) arises in recent years and it's the chains of particles which contact to each other transferring forces from one particle to another [4]. And this force transmission constructs the inhomogeneous stress distribution in a granular packing. Prior studies have investigated the influence of network topology of force chain from laboratory experiments, such as acoustic propagation and spatial patterns distribution in 2D contact network of the granular material.

Whereas, it's still unclear about how to identify the force network automatically and quantify its properties and how it affects the bulk properties of the whole system. Although various analysis methods of the force chain and experiments, 2D granular convection using photoblastic disks, have been conducted, little is known about the relationship between the evolution of the force chain and the rearrangement of particles caused by particle migration and movement [5]. Thus, it's necessary to yield a deeper understanding of how they impact the mechanical properties of granular system [6]. The granular force chain network exhibits a very clear community or branch-like structure which means the interaction of the pairwise particles potentially determined and govern basic behaviors and structures of granular material. Hidalgo *et al.* [6] found that the acoustic transmits radially along the force chain network by experiments, which indicates the chain-like structures inside the network influence the property of the granular material.

Recently several studies have made contributes to the structure using various methods based on the network science approaches and have already had successes in describing this system. Therefore, we use the network analysis approach based on the community detection method to investigate the bulk property and the imbalance of system rupture of the granular material by quantifying force chain system in three-dimensional. It's also essential to note that the network-based representation and analysis method can provide insightful behavior descriptions of granular materials with different additional complexities. More details about the community detection method will be discussed in the next section.

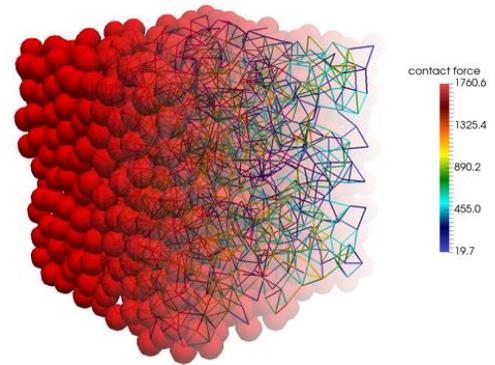


Fig 1 A sample packing of granular particles and the force chain network

## 2. Community Detection Method

Community detection method is a common issue in network science to locate clusters of nodes and edges which the connections between them are closer than those in the other communities [7]. In particular, based on the physics, mathematics and data analysis, network analysis method is an interdisciplinary method which has been successfully in many areas. In another word, the promising method consists in decomposing the force network into several sub-units or communities, which are sets of highly inter-connected nodes and edges [8]. Thus, to obtain more details about the granular material and quantify the organization of the system, we need to investigate the nodes (or edges) within each communities separately, instead of the whole network system.

In graphical language, the position of each particle represents as a node, and we represent the individual inter-particle contact as the edge of which the weight is determined by the magnitude of the force at the contact point. Hence, a force-weighted network as an  $N \times N$

weighted adjacent matrix  $W$ . The element  $w_{ij}$  in the matrix is the normal contact force between particle  $i$  and particle  $j$  that zero for particles not in contact. If two particles are not in contact with each other, then  $W_{ij}=0$ . The adjacent matrix  $W$  of force chain network associated with an undirected network is symmetric.

$$W_{ij} = \begin{cases} w_{i,j}, & \text{there is a contact force} \\ 0, & \text{otherwise} \end{cases}$$

Accordingly the modularity  $Q$  of a network is proposed,

$$Q = \sum_{i,j} [W_{ij} - \gamma P_{ij}] \delta(c_i, c_j)$$

Where particle  $i$  is assigned to community  $c_i$  while particle  $j$  is assigned to community  $c_j$  and if  $c_i=c_j$ , then, zero otherwise. The parameter  $\gamma$  is a resolution value and one can change the size of the communities by tuning  $\gamma$ . And  $P_{ij}$  is the expected weight of the edge which contact particle  $i$  and particle  $j$  under certain null model which has great influence on maximizing modularity optimization.

The most common null model for modularity optimization is the so-called Newman-Girvan model,

$$P_{ij} = \frac{k_i k_j}{2m}$$

$$k_i = \sum_j w_{ij}$$

$$m = \frac{1}{2} \sum_{i,j} w_{ij}$$

Here  $k_i$  is the strength of weight which connected with particle  $i$ .

And the only contribution to the sum of the modularity results from the particle vertex pairs belonging to the same cluster or community, therefore we can integrate these contributions and rewrite the sum of modularity of the system,

$$Q = \sum_{c=1}^{N_c} \left[ \frac{l_c}{m} - \left( \frac{d_c}{2m} \right)^2 \right]$$

$N_c$  is the number of the communities,  $l_c$  is the total number of edges of community  $c$  and  $d_c$  is total number of degrees of the particle vertices of  $c$ .

Correspondingly the maximum of modularity can be described as follow [9]:

$$Q_{\max} = \max \kappa \left\{ \sum_{c=1}^{N_c} \left[ \frac{l_c}{m} - \left( \frac{d_c}{2m} \right)^2 \right] \right\}$$

$$= \frac{1}{m} \max \kappa \left\{ \sum_{c=1}^{N_c} \left[ l_c - \left( \frac{d_c}{2m} \right)^2 \right] \right\}$$

$$- \frac{1}{m} \min \kappa \left\{ - \sum_{c=1}^{N_c} \left[ l_c - \left( \frac{d_c}{2m} \right)^2 \right] \right\}$$

Here  $\max \kappa$  and  $\min \kappa$  represents the maximum and minimum of the whole possible graph partitions  $\kappa$  respectively.

### 3. RESULTS AND DISCUSSION

The structure of each community in a given granular packing system which is divided by the network analysis method provided an important view, however, scholars pay more and more attention in how such structure evolves when the granular material experiences external perturbations [10]. Additionally, the community structure reveal more insight information about the granular of which the behavior can be reflected by the mesoscale network features.

In Fig.2, it shows the variation of communities in the granular material and for proper understanding and visualization, the network system is converted into granular ensemble with particle center as node, using the same color for the same community and for different communities we mark it with different numbers as showed in Fig.2 for different views.

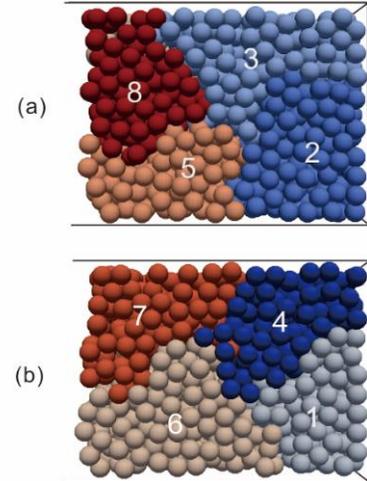


Fig 2 the packing system is divided into different communities based on the community detection method (different colors are applied for different communities) (a) from x-axis positive direction (b) from x-axis negative direction

#### 3.1 Pressure influence

In this simulation, the particles are generated by a packing of 1700 densely spheres (radius 0.01cm) in 3D within simulation domain by using the discrete element method. And under the driven pressure, fluid progressively move towards the granular bed in the negative z direction that interact with each other in a Hertzian-like manner where the damping force are proportional to the relative normal and tangential overlap [11]. This simulation ends after the whole network deformation is achieved and no appreciable change in the state is observed. And in this work we only consider the static particle force chain networks instead of dynamic and unsteady networks.

The basic simulation structure and particle distribution at different times ( $t=0.1s, 0.4s, 1.0s$ ) can be seen as Fig.3 (a). And Fig.3 (b) shows the contact force structure which corresponding to the particle distribution and the color of the each line or edge represents the magnitude of contact force. The heterogeneity and the quasilinear pathways of the force chain network in the granular packing system is clear.

The common way of characterizing community structure is to investigate the community size distribution of the granular ensemble. Thus, the complementary cumulative probability distribution function (CCDF) of community size ( $s$ ) is considered and displayed in Fig.4 at different pressure stages (these data are calculated from several critical states). As usually observed, in each case the cumulative curves display linear decay for community sizes up to 200. However, compared with different pressure stages, the above linear distribution suggests that there is no obvious characteristic community size in the network.

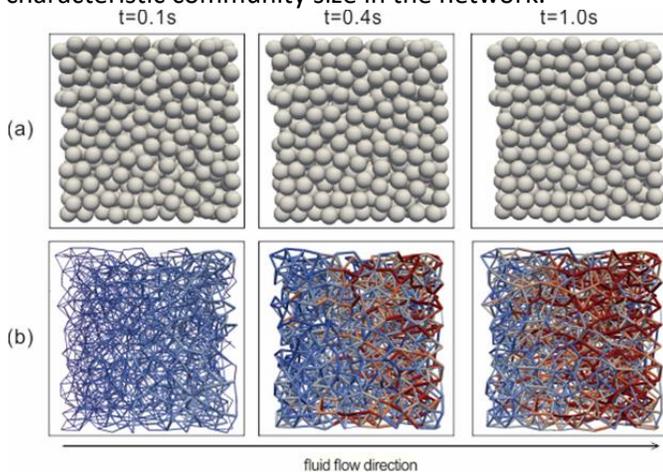


Fig 3 A 3-D granular packing, compressed by fluid in the negative z-direction (other boundaries are periodic)  
 (a) particles distribution in granular ensemble at different times. (b) The force chain structure among the

packing system. The color of the each line or edge represents the magnitude of contact force, with the light one (red line) represents to the large force while the dark (blue line) corresponding to small forces.

Fig 5 shows the evolution of the community size. At low fluid pressure, small communities dominate, while as the fluid pressure increases, there is typically a large communities near the bottom and a group of smaller and weaker communities at the top area. The various size of communities capture how the fluid pressure pattern changes according to the redistribution of contact forces by compression. In the compression process, the communities become more compact, but there also distinct difference, especially in community size.

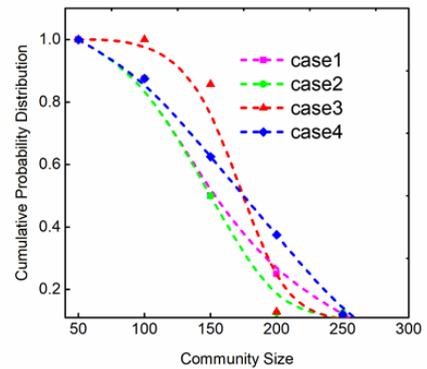


Fig 4 Complementary Cumulative probability distribution function (CCDF) of the community size

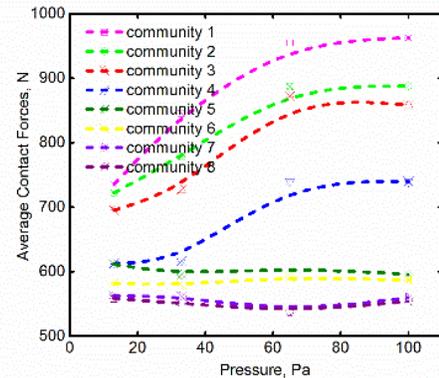


Fig 5 comparison of average contact force of different communities as a function of fluid pressure

### 3.2 Fluid viscosity influence

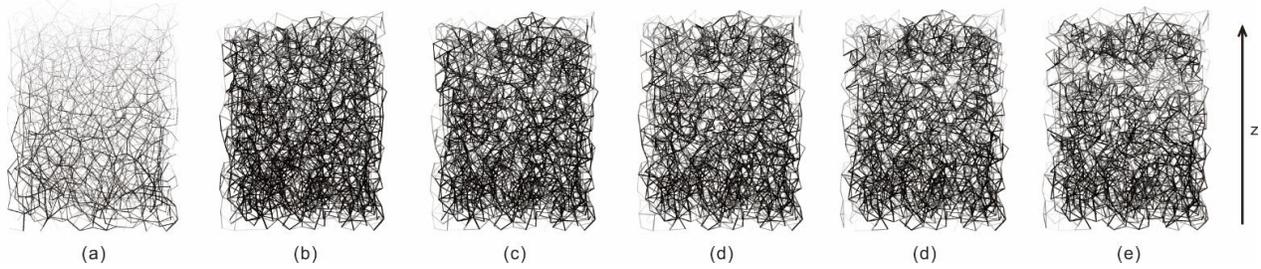


Fig 6 Representative spatial distribution of the major force chain network for the granular material at different fluid kinematic viscosity ( $\nu$ ): (a) before collapse; (b)  $\nu=0.005\text{m}^2/\text{s}$ ; (c)  $\nu=0.0075\text{m}^2/\text{s}$ ; (d)  $\nu=0.01\text{m}^2/\text{s}$ ; (e)  $\nu=0.012\text{m}^2/\text{s}$ ; (f)  $\nu=0.015\text{m}^2/\text{s}$ ;

To understand the effect of fluid, we consider the structure changes of force chain network under different fluid conditions. The evolution of the major force chain network for the granular is analyzed (see Fig 6). For small fluid kinematic viscosity (Fig 6 (b)), we observe a dense spatial distribution of the major force chain network. Remarkably, the intensity of the force chain is affected by the fluid kinematic viscosity and we notice that the dense of the network increases as the fluid viscosity decreases.

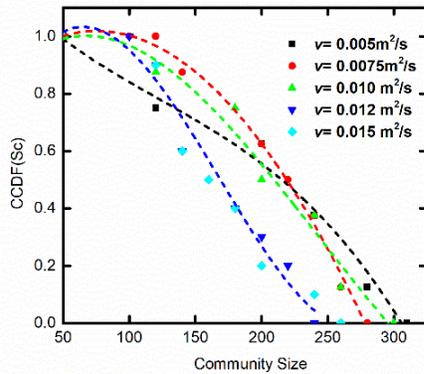


Fig 7 complementary cumulative distribution (CCDF) associated with the community size ( $S$ ) under different fluid kinematic viscosities ( $\nu=0.005\text{m}^2/\text{s}$ ,  $\nu=0.0075\text{m}^2/\text{s}$ ,  $\nu=0.010\text{m}^2/\text{s}$ ,  $\nu=0.012\text{m}^2/\text{s}$ ,  $\nu=0.015\text{m}^2/\text{s}$ ).

In Fig 7, we consider the complementary cumulative distribution function (CCDF) of community size as a function of fluid kinematic viscosity (the kinematic viscosity is from  $0.005\text{m}^2/\text{s}$ ,  $0.0075\text{m}^2/\text{s}$ ,  $0.010\text{m}^2/\text{s}$ ,  $0.012\text{m}^2/\text{s}$ ,  $0.015\text{m}^2/\text{s}$ ). Here we qualitatively explain these results. For small fluid kinematic viscosity, nodes (particles) are trapped in a strong force chain network. The configuration of communities contains more large ones compared with higher fluid kinematic viscosity. Indeed, kinematic viscosity is sensitive to the number and size of communities.

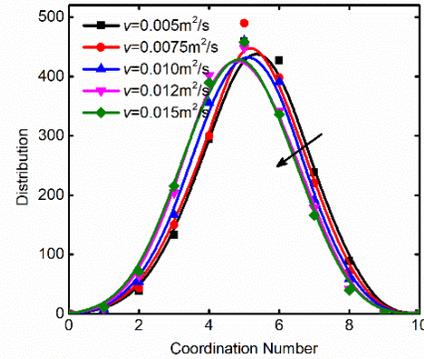


Fig 8 the particle distribution of the monodisperse system according to the coordination number

For identifying the complex structure of the contact network, the coordination number of the monodisperse particle system is analyzed. Fig 8 displays the evaluation of the particle distribution under various fluid kinematic viscosity based on the coordination number. Basically, the coordination number is the determined quantify for mechanical stability and understand the structure of the granular system [12][13]. It can be observed that coordinate number at each fluid condition exhibits normal distribution and the average coordination number slightly declines as the fluid viscosity increases, which means the lower viscosity fluid system has a more stable chainlike structures and a more compact force system.

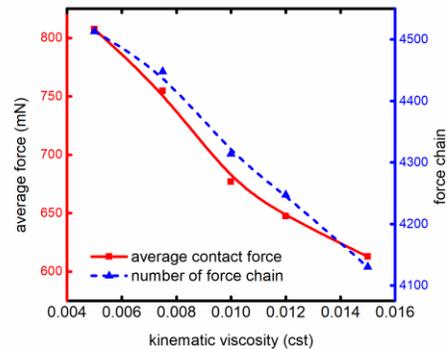


Fig 9 the evolution of contact force and the number of force chain as a function of kinematic viscosity

The average force and the number of the force chain among the system as a function of kinematic viscosity are analyzed (as showed in Fig 9). Clearly, both of the average force of the communities and the number of force chain decline more or less linearly as the kinematic viscosity increases. The average force drops 40% while the force chain structure declines by almost 10% as the kinematic viscosity increases from 0.005m<sup>2</sup>/s to 0.015 m<sup>2</sup>/s, which means the viscosity changes results in the triggering of the potential material's stability. In addition, communities are more compact at low viscosity and it shows the same results as we described before. It indicates that the fluid properties, especially the viscosity, have remarkable effect on the behavior of granular structures and integrity.

#### 4. CONCLUSION

Based on DEM simulation, we use community detection method to characterize the force chain network predicting the bulk property of the compacted granular material. In particular, the evolution and spatial distribution of the community system are analyzed over a range of pressure and fluid kinematic viscosity. The result shows that, as in 3D system, there's transition in community size and higher-pressure system exhibits compact rather than branching communities, which is in contrast with the fluid kinematic viscosity. Additionally, for the monodisperse system, lower fluid viscosity system has a more stable and a more compact force structure according the coordination number and the average force.

This work provides the further insight of the jammed granular material according to the force chain network and it helps to understand the complicated chain-like structures.

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