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ANALYTICAL AND NUMERICAL STUDY ON SOLIDIFICATION OF COMPOSITE PHASE CHANGE MATERIAL IN A SPHERE

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ABSTRACT

Numerical and computational analyses of interface position during inward solidification of composite phase change materials (PCM) in spherical container were explored in this study. The applications of methods such as perturbation, strained coordinates and (improved) quasi-steady solution on spherical inward solidification were investigated and compared with the results of numerical simulation. The solidification positions of porous composite PCM with different porosities solved by strained coordinates and (improved) quasi-steady methods were compared when the Stefan number was 0.1. The complete solidification time was found to be rapidly shortened as the porosity decreased.

Keywords: cold storage, metal foam, inward solidification, sphere, analytical solution

NONMENCLATURE

Abbreviations	
PCM	phase change material
MF	metal foam
Symbols	
T	temperature, K
Ste	Stefan number ($=c_p(T_f-T_0)/L$)
c_p	specific heat ($J\cdot kg^{-1}\cdot K^{-1}$)
L	latent heat of solidification (J/kg)
R	radial direction of sphere (m)
p	dimensionless constant
r	normalized radial position
Greek symbols	
α	thermal diffusivity (m^2/s)
λ	thermal conductivity (W/(K·m))
ρ	density (kg/m^2)
δ	normalized solidification position
τ	normalized time
Ψ	independent variable
ε	porosity of metal foam
Subscripts	
e	effective
f	freezing
s	quasi-steady approximation
m	improved quasi-steady analysis

1. INTRODUCTION

Energy storage technology contributes to sufficient utilization and balance of energy supply and demand [1]. A spherical unit in the ice thermal storage system belongs to a general class of moving phase-change boundary problems that were first introduced by Josef Stefan around 1890. For more than 100 years, the transient phase change problems in spherical containers have been studied by numerical analysis, simulation and experiments.

The numerical solutions of the inward solidification in a sphere or cylinder were obtained by Tao [2] in graphical form, with the thermal conductivity and heat capacity of solid phase assumed to be constants. Pedroso and Domoto presented the perturbation solutions that was valid for outward and partial inward solidification in a sphere [3], which was improved and resolved by the

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strained coordinates method proposed in the latter study [4]. Riley et al. [5] presented an analytical study on the freezing of a sphere/cylinder by a two-region analysis to accommodate the point of singularity near the center, which compared well with the numerical solutions for small Stefan numbers. Shih et al. [6] and Hill et al. [7] developed a numerical model for dealing with the freezing processes for saturated liquid in spherical containers with constant heat transfer coefficient and radiation at the container surface. The application of the perturbation expansions method in phase change heat transfer was sufficiently reviewed by Aziz et al. [8], including the solutions of 1D, 2D and 3D problems in Cartesian, cylindrical and spherical coordinates. Sui et al. [9] proposed an improved quasi-steady solution based on the ratio of the heat flux at phase interface to that at spherical surface, whose error was one tenth of the error of the quasi-steady solution.

Ismail et al. [10] presented a numerical solution for the solidification of water in a spherical enclosure under convective boundary conditions. They studied the effects of parameters including initial PCM temperature, cooling temperature, wall material and the diameter of the sphere on complete solidification time and solidification rate. Bilir et al. [11] numerically investigated the solidification of PCM in a cylindrical/spherical container under the third boundary conditions, whose initial temperature was not the solidification temperature. The correlations expressing the dimensionless complete freezing time in terms of Stefan Number, Biot Number and Superheat Parameter were derived. Eames et al. [12] established an experimental system to test the melting and freezing processes of different sphere diameters and heat transfer fluid temperatures. They derived semi-empirical equations to predict the mass fraction at any time. Shi et al. [13] experimentally determined the role of metal foam on the solidification process of water in ice ball, indicating that the metal foam can enhance the heat transfer during the phase change process.

Previous literatures showed that analytical solutions for inward solidification of metal-foam composite PCM were rarely studied. This paper therefore focused on comparison of numerical solutions with several analytical solutions. The solidification processes of PCM embedded in metal foam with different porosities were studied by available analytical models.

2. ANALYTICAL MODEL

Consider a one-dimensional spherical form of radius R_0 shown in Fig. 1. The temperature of the liquid in the sphere was assumed originally at the freezing temperature T_f. Suddenly, at the time t=0, the temperature at the fixed wall R=R_0 dropped to a lower temperature T_0 and kept constant. Then the liquid in the sphere started to freeze inwardly. Assuming that all thermophysical properties were uniform and constant, the governing equation within the solid phase was the transient heat conduction equation as follows:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}=\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial T}{\partial R}\right) \quad R_f<R<R_0 \tag{1}$$

The boundary conditions can be written as:

$$T=T_0 \;\text{ at }\; R=R_0;\; T=T_f \;\text{ at }\; R=R_f \tag{2}$$

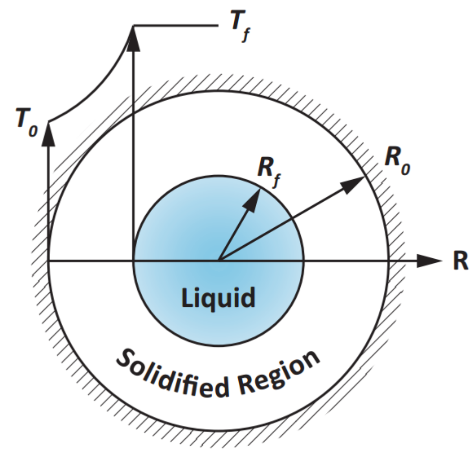

Fig 1 The Diagram of physical model

Energy balance in the solid-liquid interface was described as:

$$\frac{\partial T}{\partial R}=\frac{\rho L}{\lambda}\frac{dR_f}{dt} \quad R=R_f \tag{3}$$

The initial condition was specified as:

$$R_f=R_0 \;\text{ at }\; t=0 \tag{4}$$

Define the dimensionless physical quantities, Ste, radial position, r, solidification position, δ, and time, τ, as:

$$Ste=\frac{c_p\left(T_f-T_0\right)}{L},\quad r=\frac{R}{R_0},\quad \delta=\frac{R_f}{R_0},\quad \tau=\frac{\alpha t}{R_0^2}. \tag{5}$$

According to the strained coordinates method proposed by Pedroso and Domoto [4], the normalized solidification position and freezing time can be yielded in terms of a new independent variable Ψ:

$$\delta=\psi-Ste\frac{1-\psi}{6\psi}+Ste^2\frac{(22\psi-3)(1-\psi)}{360\psi^3} \tag{6}$$

$$\tau^*=\frac{2\psi^3-3\psi^2+1}{6}+Ste\frac{(1-\psi)^2}{3}-Ste^2\frac{(1-\psi)^2}{180\psi^2} \tag{7}$$

where $\tau^*=\tau \cdot Ste$ for the convenience of derivation. In order to make full use of the first three terms of the above equations, the Shanks transformation [14] was separately applied to extend the application range of the solution.

As the solidification process going on, the frozen region must be cooled to lower temperature than the freezing point, so that the liquid surrounded by the solid phase can be continued freezing. By neglecting the conducting heat loss in the solid phase, the quasi-steady equations can be solved to approximate the exact solution that was unavailable for sphere.

Neglecting the transient term on the left side of the governing Eq.(1), the quasi-steady governing equation can be obtained as follow:

$$\frac{\partial}{\partial R}\left(R^2\frac{\partial T}{\partial R}\right)=0 \tag{8}$$

Combined with the boundary and initial conditions, Eqs. (2-4), the solidification position of the quasi-steady approximation can be derived as:

$$\frac{\delta^3}{3}-\frac{\delta^2}{2}+\frac{1}{6}=Ste\cdot\tau_s \tag{9}$$

In order to improve the quasi-steady approximation, an additional term was introduced to approach the effect of undercooling in the frozen region, which was the heat flux ratio defined by:

$$\left(\frac{\partial T}{\partial R}\right)_{R=R_f}\Bigg/\left(\frac{\partial T}{\partial R}\right)_{R=R_0}=e^{-p^2} \tag{10}$$

where p was determined by the Neumann's solution of solidification in a plate, when it was also assumed valid for the sphere:

$$p\sqrt{\pi}\exp\left(p^2\right)\mathrm{erf}\left(p\right)=Ste \tag{11}$$

Considering the imposed condition Eq. (10) and the same boundary conditions, the Eq. (8) can be solved to obtain the temperature distribution u_m. Substitute u_m into Eq. (3) and integrate Eq. (3) with the initial condition, Eq. (4), to obtain the solidification position of improved quasi-steady analysis:

$$\frac{\delta^3}{3}-\frac{\delta^2}{2}+\frac{1}{6}=\frac{2Ste\cdot\tau_m}{1+e^{p^2}} \tag{12}$$

The p value for the spherical case was actually different from that for the plate as defined in Eq. (11), which can be transformed by [9]:

$$p'=\left(3+\sqrt{3}\right)\frac{p}{3}=1.577p \tag{13}$$

The metal foam was uniformly saturated in a spherical filled with pure water to generate a composite phase-change spherical. Assuming local thermal equilibrium at the interface of solid and liquid phases, the effective physical parameters of composite PCM were determined by:

$$\rho_e=\rho_{MF}\left(1-\varepsilon\right)+\rho_{PCM}\varepsilon \tag{14}$$

$$c_{p,e}=c_{p,MF}\left(1-\varepsilon\right)+c_{p,PCM}\varepsilon \tag{15}$$

$$\lambda_e=\lambda_{MF}\frac{1-\varepsilon}{2.71}+\lambda_{PCM}\varepsilon \tag{16}$$

where ρ, c_p and λ defined the density, specific heat and thermal conductivity, ε represented the porosity of metal foam. To obtain the solidification process of composite PCM in a sphere, the effective thermal diffusivity α_e was transformed to the thermal diffusivity of pure water according to:

$$\frac{\alpha_e}{\alpha_{PCM}}=\frac{\lambda_e}{\lambda_{PCM}}\frac{c_{p,PCM}}{c_{p,e}}\frac{\rho_{PCM}}{\rho_e} \tag{17}$$

The dimensionless parameters τ_e and p_e can now be yielded:

$$\tau_e=\frac{\alpha_e t}{R_0^2}=\frac{\alpha_e}{\alpha_{PCM}}\tau \tag{18}$$

$$p_e=\frac{\delta}{2\sqrt{\tau_e}}=\sqrt{\frac{\tau}{\tau_e}}p=\sqrt{\frac{\alpha_{PCM}}{\alpha_e}}p \tag{19}$$

Substituting the effective parameters determined by Eqs. (18-19) into the previous solutions to solidification of pure water in a sphere, the solidification position of composite-PCM in a sphere can be solved.

3. RESULTS AND DISCUSSION

Fig. 2 demonstrated the solutions of dimensionless solidification position obtained by four analytical methods (lines) and a numerical simulation (points) [2, 3, 4, 9] as a function of dimensionless time. Among them, the perturbation solution was obviously divergent when the solidification was about to be completed.

It can be seen that as solidification progressing, the error of the quasi-steady solution gradually increased. The complete solidification time was 7.5% less than the strain coordinate solution. The error of complete solidification time between the improved quasi-steady solution and the strain coordinate solution was reduced to 1.7%, in a reasonable range to be accepted.

Fig. 3 depicted the solidification positions of composite PCM with different porosities obtained by the strained coordinates and (improved) quasi-steady methods. It can be observed that the complete solidification time with a porosity of 0.98 was reduced by half compared to that of the pure PCM. The complete solidification time with a porosity of 0.9 was only 23% of that for the pure PCM. The embedded metal foam increased the thermal conductivity of the composite PCM and accelerated the heat transfer into the center. Besides, as Fig. 3 shown, as the porosity decreased, the improved quasi-steady solution got closer to the quasi-steady solution, and its error compared with the strain coordinate solution gradually increased.

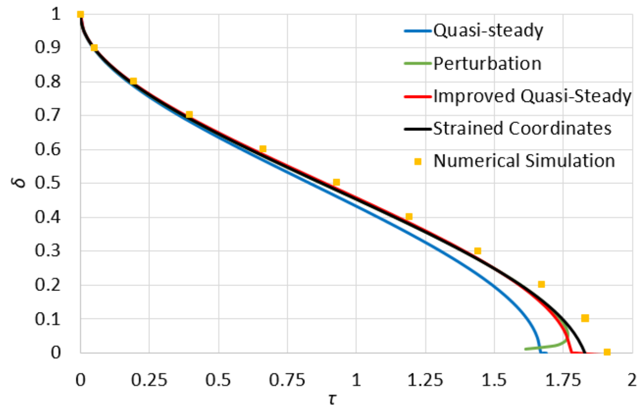

Fig 2 Comparison of the spherical inward solidification positions solved by different methods [2,3,4,9] when *Ste*=0.1

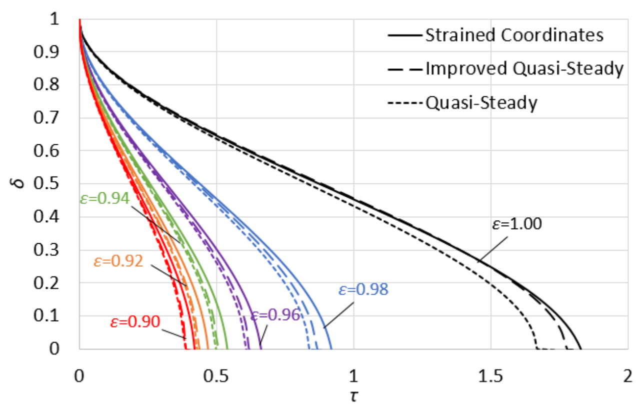

Fig 3 Comparison of solidification position solutions of composite PCM with different porosities when *Ste*=0.1

4. CONCLUSION

In the present work, the analytical solutions obtained by perturbation, strained coordinates and (improved) quasi-steady methods were compared and discussed. The four analytical models were validated by comparing with numerical results in literature. The analytical solutions to the solidification in pure PCM were extended to composite PCM (water saturating metal foam) and the effect of foam porosity on the solidification process was also explored. The results showed that the complete solidification time was greatly shortened as the porosity reduced from 1.0 to 0.9.

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