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APPLYING BAYESIAN MODEL AVERAGING TO CHARACTERISE URBAN RESIDENTIAL STOCK TURNOVER DYNAMICS

Wei Zhou[1*], Eoghan O'Neill[1], Alice Moncaster[2], David Reiner[1], Peter Guthrie[3]

1 Energy Policy Research Group, University of Cambridge, UK
2 School of Engineering & Innovation, Open University, UK
3 Department of Engineering, University of Cambridge, UK

ABSTRACT

Building stock is a key determinant in building energy modelling and policy analysis. However, official statistics on total floor area of urban residential stock in China only exist up to 2006. Previous studies estimating Chinese urban residential stock size and energy use made various questionable methodological assumptions and only produced deterministic results. This paper presents a Bayesian approach to characterise the stock turnover dynamics and estimate stock size uncertainties. Firstly, a probabilistic dynamic model is developed to describe the building aging and demolition process governed by a hazard function specified by a parametric survival model. Secondly, with each of five candidate parametric survival models, the dynamic model is simulated through Markov Chain Monte Carlo (MCMC) to obtain posterior distributions of model-specific parameters, estimate marginal likelihood, and make predictions on stock size. Finally, Bayesian Model Averaging (BMA) is applied to create a model ensemble that combines the model-specific posterior predictive distributions of the stock evolution pathway in proportion to posterior model probabilities. This Bayesian modelling framework, and its results in the form of probability distributions of annual total stock and age-specific substocks, can provide a solid basis for further modelling and analysis of policy trade-offs across embodied-versus-operational energy consumption and carbon emissions of buildings in the context of sector-wide transition towards decarbonising buildings.

Keywords: building stock, lifetime distribution, Bayesian Model Averaging, Markov Chain Monte Carlo, embodied energy, operational energy

1. INTRODUCTION

China is the world's largest building energy consumer. In 2017, total energy consumption by buildings in China was estimated to reach 963 million tonnes of coal equivalent (Mtoe) [1]. Of this total amount, urban residential buildings accounted for more than one third. Whilst the size of the urban residential stock is critical to the evaluation of stock-level energy consumption, official statistics only exist up to 2006 and this results in an unknown growth trajectory of the stock from 2007 onwards. This necessitates estimating how the urban residential stock has been developing to reach its current status. The stock evolution and expansion are characterised by a turnover process driven by the dynamic interplay between new construction, meeting incremental demand growth as a result of economic growth and rising living standards, existing buildings remaining in use but undergoing an ageing process, and old buildings, which are eventually demolished. As Chinese urban buildings are generally short-lived [2,3], building lifetime is an important factor underlying the dynamics of stock turnover. The short lifetime suggests a high turnover rate, the complexity and uncertainty of which have to be understood when estimating total stock and associated energy and carbon impacts.

A review by Zhou et al. [4] identified three main methodological concerns associated with most previous models estimating Chinese building stock: (i) arbitrary choice of mean and standard deviation parameters of a normal distribution representing building lifetime distribution; (ii) ambiguity associated with existing building stock size and age profile in the initial year of modelling; and (iii) use of per capita floor area data leading to inflated estimates. Whilst Cai et al. [3] and Zhou et al. [4] partially addressed these issues by calibrating building lifetime distribution parameters using statistics on floor area, fundamentally they took a frequentist approach and produced single point estimates of distribution parameters leading to a single profile of building lifetime without characterisation of uncertainty. In this context, model parameters are treated as being fixed. And the calibration is conditional upon the model structure as given, thereby neglecting the model uncertainty.

In contrast, a Bayesian approach, adopted by this study, treats parameters as random variables and derives posterior distributions of parameters by taking account of both prior knowledge about parameter values and the likelihood of observing empirical data given certain parameter values. For a given model, this presents a full picture of the likely parameter space, thus enabling a good understanding of the global shape of the distribution. Such a distribution allows parameter uncertainties to be propagated through to emergent behaviours of model outputs. Moreover, a Bayesian approach allows the model uncertainty to be estimated. Through Bayesian Model Averaging (BMA), predictions made by individual models are combined in proportion to posterior model probabilities.

In particular, from a policy-making perspective, a probabilistic model offers the ability to generate probability distributions over outcomes of policy scenarios. This is important in the context of decarbonising the generally short-lived Chinese buildings, where the operational and embodied energy is likely to create a strategic trade-off. Taking a Bayesian approach, a probabilistic model incorporating building stock turnover, energy and carbon will enable future research investigating the probability that one policy, e.g., extending building lifetime to avoid embodied energy, as compared to another policy, e.g. accelerating stringency of new building design standards, would yield a more favourable outcome of stock-level decarbonisation. This is the overarching objective that motivates this study as an integral part of future research involving a fully-fledged building energy model.

Based on the above considerations, this study applies BMA to develop a probabilistic dynamic model to predict Chinese urban residential stock for the recent historical period of 2006 to 2017. The rest of this paper is organised as follows. Section 2 develops the dynamic model for stock turnover and explains the concepts and implementation of BMA. Section 3 presents key results, including posterior model probabilities and posterior predictive distribution of building stock, and discusses further model applications and policy implications. Section 4 summarises the paper.

2. METHODOLOGY

2.1 Building stock turnover model

Estimating total building stock size requires understanding and modelling the stock turnover, which is characterised by the stock-level dynamics of construction of new buildings as inflow into the stock and demolition of old buildings as outflows from the stock. By the end of a year *t*, the total volume of demolition is the sum of all existing buildings constructed in previous years that have reached the end of their lifetimes in year *t*. The building stock is composed of new buildings constructed in year *t* and those buildings which were previously constructed but have not reached the end of their lifetimes.

Building lifetime is critical to the turn-over dynamics. Despite design lifetime required by building design regulations, often there is a lack of authoritative statistics relating to actual building lifetime data, particularly in developing countries. At a city or even country level, given the huge volume of buildings and significant heterogeneity in terms of their physical characteristics and socio-economic contexts, it is necessary to consider the uncertainties associated with building lifetime. It is unrealistic to assume that a cohort of buildings, i.e. those constructed in a given year, would be in service for exactly the same period and then demolished simultaneously.

This paper proposes to apply the concept of survival analysis [5]. It uses the probability density function (PDF) of a parametric survival model to approximate the likely lifetime distribution profile of a cohort of buildings, so as to recognise and represent the uncertainties associated with factors collectively influencing lifetime of buildings. Thus, in a given year, the proportion of demolished buildings in this cohort of buildings is modelled based on a hazard function. Conceptually, the hazard function represents the conditional probability that a building will expire in year *t*, provided that it has successfully survived to year *t-1*. Mathematically, the hazard function is the ratio of the lifetime PDF to the complement of lifetime cumulative distribution function (CDF).

Applying the above concept, the total stock in year *t* consists of a series of substocks of different ages:

$$Stock_t = \sum_{j=t_0}^{t} substock_t[t-j] \qquad (1)$$

Where $substock_t[t-j]$ represents buildings surviving in year t that are (t-j) years old. For new buildings constructed in year t, they are 0 years old and therefore denoted by $substock_t[0]$.

The aging process undergone by any cohort of buildings is accompanied with annual demolition determined by age-specific hazard rates, *H(age)*. Therefore, the annual total demolition in year *t* is the sum of age-specific demolition of substocks at all ages.

$$Demolition_t = \sum_{j=t_0}^{t} H(t-j) substock_t[t-j] \qquad (2)$$

For a (t-j)-year-old substock in year *t*, its volume is determined by the aging process that it has undergone since it was constructed in year *j*.

$$substock_t[t-j] = \left[\prod_{k=0}^{t-j}\big(1-H(k)\big)\right] substock_j[0] \quad (3)$$

Therefore, equation (1) can be re-written as:

$$Stock_t = \sum_{j=t_0}^{t} substock_t[t-j] = \sum_{j=t_0}^{t}\left\{\left[\prod_{k=0}^{t-j}\big(1-H(k)\big)\right] substock_j[0]\right\} \qquad (4)$$

In above equation (4), the age-specific hazard rate *H(k)* is determined by the parametric survival model chosen. Depending upon the specification, the hazard function of a survival model may or may not have a closed form expression.

2.2 Bayesian modelling

2.2.1 Statistical model

As described by equation (4), the deterministic component of the overall statistical model is the total building stock as the function of unknown parameters θ of a chosen parametric survival model, e.g. Weibull distribution, and the known annual new cohort of buildings constructed over the historical period. This can be denoted by a function $f(\theta, t)$. The probabilistic component of the model is represented by an error term ε_t, which is assumed to be normally distributed with mean zero and unknown variance σ^2, i.e. $\varepsilon_t \sim N(0, \sigma^2)$. $f(\theta, t)$ describes the expectation of modelled building stock. Therefore, in the Bayesian framework, the total stock can be described by the overall probabilistic model as follows:

$$Stock_t = f(\theta, t) + \varepsilon_t \qquad (5)$$

2.2.2 Bayesian model inference

In the context of the statistical model, let *D* represent empirically observed data of total stock y, and annual new buildings x, for the period of 1978 to 2006, i.e. $D = \{(x_i, y_i), i = 1978, 1979, \ldots, 2006\}$. According to Bayes' theorem, the posterior probability density $p(\theta|D)$, given the data D, is calculated as follows:

$$p(\theta|D) = \frac{p(D|\theta)p(\theta)}{p(D)} = \frac{p(D|\theta)p(\theta)}{\int p(D|\theta)p(\theta)d\theta} \qquad (6)$$

where $p(\theta)$ is the prior distribution of θ, representing subjective prior knowledge about θ. $p(D|\theta)$ is the likelihood function, which can be viewed as a function of θ given the empirically observed data *D* which is considered fixed. It represents the likelihood that the given set of empirically observed data *D* is explained by the model with possible parameter values. $p(D)$ is the marginal likelihood, which is an integration of $p(D|\theta)$ over all possible values of θ across its space and therefore is not a function of θ, but a constant. This proportionality constant plays the role of normalizing the posterior density to ensure it integrates to 1. $p(D)$ is also known as model evidence, because it provides evidence for a candidate model, which is critical in selecting and averaging models as discussed later.

The posterior distribution $p(\theta|D)$ fully describes the uncertainty associated with the parameters. Essentially it updates the prior knowledge about the parameters in light of the empirical data. Generally, it is difficult or not possible to analytically express the posterior distribution. The solution is to instead simulate sample draws from the posterior distribution, such that the values of these samples are distributed approximately according to the posterior distribution of the parameters of interest. The samples enable calculation of point estimates of the parameters, such as mean, median, or mode. More importantly, the samples of parameters enable drawing samples from predictive distributions associated with model outputs, e.g. the annual total stock as the high-level emergent behaviour of the dynamic building stock model, thus facilitating policy scenario analysis. Methodologically, this is realised using Markov chain Monte Carlo (MCMC) algorithm.

2.2.3 Posterior predictive distribution

With $p(\theta|D)$, it is possible to make inferences about the total stock for a given year during the period of 2007 to 2017, an unknown observable denoted as $\tilde{y}$, given the known annual new buildings for the same year, denoted as x^*. This leads to the posterior predictive distribution of $\tilde{y}$:

$$\mathrm{p}(\tilde{y}|x^*, D) = \int p(\tilde{y}|x^*, \theta)p(\theta|D)d\theta \qquad (7)$$

This equation suggests that the posterior predictive distribution is derived by marginalising the likelihood function $p(\tilde{y}|x^*, \theta)$ over the entire set of parameters, with each point in the space of parameters weighted according to its posterior probability given the empirically observed data.

2.2.4 Bayesian model averaging (BMA)

The above posterior predictive distribution is conditional upon a choice of model M, i.e. a building stock model employing a particular parametric survival model, e.g. Weibull distribution. The equation can be written more explicitly as:

$$\mathrm{p}(\tilde{y}|x^*, M, D) = \int p(\tilde{y}|x^*, \theta, M)p(\theta|D, M)d\theta \qquad (8)$$

There are multiple choices of parametric survival function, each of which may characterise the dynamics of building stock turnover. Candidates include Weibull, Lognormal, Gamma, etc. Let M_k denote a building stock turnover model using a plausible survival function k specified by parameter vector θ_k, and let $M = \{M_1, M_2, \ldots., M_k\}$ denote the model space under consideration. This creates a model ensemble, which, when making predictions, takes into account the uncertainties associated with not only model-specific parameters but also the models per se. Now, the posterior predictive distribution of total building stock for the period of 2007 to 2017, $\tilde{y}$, is calculated as:

$$p(\tilde{y}|x^*, D) = \sum_{k=1}^{K} p(\tilde{y}|x^*, M_k, D)p(M_k|D) \qquad (9)$$

Where $p(\tilde{y}|x^*, M_k, D)$ is the posterior predictive distribution under model M_k given data D, and $p(M_k|D)$ is the posterior model probability (PMP), which is also referred to as model weight. Hence, the posterior distribution of $\tilde{y}$ predicted by the model ensemble, $p(\tilde{y}|x^*, D)$, is effectively the average of the posterior predictive distribution under each candidate model in the model space, weighted by its PMP.

The PMP of model M_k can be interpreted as the probability of model M_k being the true model predicting $\tilde{y}$, given the observed data *D*, thus reflecting the extent to which M_k fits the observations as compared to other candidate models in the model space. PMP is given by:

$$p(M_k|D) = \frac{p(D|M_k)p(M_k)}{\sum_{j=1}^{K} p(D|M_j)p(M_j)} \qquad (10)$$

Where $p(M_k)$ is the prior probability of model M_k being the true model, allowing the existing prior knowledge about the plausibility of model M_k to be specified explicitly, and $p(D|M_k)$ is the marginal likelihood (or model evidence) of model M_k, which is given by:

$$p(D|M_k) = \int p(D|\theta_k, M_k)p(\theta_k|M_k)\mathrm{d}\theta_k \qquad (11)$$

Where $p(D|\theta_k, M_k)$ is the likelihood of model M_k given observed data *D*, and $p(\theta_k|M_k)$ is the prior probability density of the parameters θ_k under model M_k. In fact, $p(D|M_k)$ is the denominator in the above equation (6) for calculating the posterior probability density of parameters θ_k under model M_k, as given by:

$$p(\theta_k|D, M_k) = \frac{p(D|\theta_k, M_k)p(\theta_k|M_k)}{\int p(D|\theta_k, M_k)p(\theta_k|M_k)\mathrm{d}\theta_k} = \frac{p(D|\theta_k, M_k)p(\theta_k|M_k)}{p(D|M_k)} \qquad (12)$$

Compared with equation (6), the above equation (12) explicitly applies subscript k to reflect that both the priors of model-specific parameters θ_k and the likelihood function of the observed data *D* are conditional on the particular model M_k in the model space.

Based on the above, the posterior mean of $\tilde{y}$, as predicted by the model ensemble, can be calculated as follows:

$$E[\tilde{y}|x^*, D] = \sum_{k=1}^{K} E[\tilde{y}|x^*, M_k, D]p(M_k|D) \qquad (13)$$

Clearly the model ensemble prediction is essentially the average of individual predictions weighted by the likelihood that an individual candidate model is true given the observed data. BMA model ensemble leads to a more spread posterior distribution of y than an individual candidate model does. This avoids the situation where inferences made based on an individual candidate model are overstated and decision-making based on predictions is more risky than expected [6].

2.2.5 Model space

In general, a range of parametric survival distribution functions are available to describe the survival process in various fields [5]. However, literature on survival analysis or lifetime data analysis on buildings is limited. A survey on buildings in the Netherlands found that empirical survival probabilities of buildings were well approximated by Weibull distribution [7]. Miatto et al. [8] tested various PDFs and found that the lognormal distribution offered the best fit to lifespans of buildings in Nagoya and Wakayama, Japan, where buildings were short-lived, with average lifespans shorter than 30 years. Zhou et al [4] applied the Weibull distribution to approximate lifetime uncertainties of Chinese urban residential buildings. From an economic perspective, buildings can be regarded as a type of capital asset, hence building stock can be regarded as capital stock [7]. Hence, a range of PDFs that have been used as a proxy for service lives and retirement/discard patterns of capital stocks may be applied to buildings, such as log-normal, Weibull, Gamma, and so on [7,9,10].

In this paper, the distribution functions used for approximating the lifetime distribution of Chinese urban residential buildings are Weibull, Lognormal, Loglogistic, Gamma and Gumbel distributions. Each distribution characterises the turnover dynamics of the building stock, thereby representing a candidate model M_k in the model space M. The PDFs of these distributions are given in Table 1. Specifying the PDF of a distribution allows the CDF, survival function and hazard function of the distribution to be ascertained.

Table 1: Candidate survival distribution functions

Model	Probability density function	Parameters	Priors
Weibull	$f(x) = \left(\frac{\alpha x^{\alpha-1}}{\lambda^\alpha}\right) e^{(-(\frac{x}{\lambda})^\alpha)}$	Shape $\alpha > 0$ Scale $\lambda > 0$	$\alpha \sim$ uniform(1,10) $\lambda \sim$ uniform(1,100)
Lognormal	$f(x) = \frac{1}{x\sqrt{2\pi}\sigma'} e^{-\frac{1}{2}\left[\frac{lnx-\mu'}{\sigma'}\right]^2}$ $\mu' = ln\left[\frac{\mu^2}{\sqrt{\sigma^2+\mu^2}}\right]$, $\sigma' = \sqrt{\ln\left[1+\left(\frac{\sigma}{\mu}\right)^2\right]}$	Mean $\mu > 0$ Standard deviation $\sigma > 0$	$\mu \sim$ uniform(1,100) $\sigma \sim$ uniform(1,100)
Loglogistic	$f(x) = \frac{e^{\frac{\ln(x)-\mu}{\sigma}}}{\sigma x(1+e^{\frac{\ln(x)-\mu}{\sigma}})^2}$	Scale $\mu > 0$ Shape $\sigma > 0$	$\mu \sim$ uniform(1,100) $\sigma \sim$ uniform(1,100)
Gamma	$f(x) = \frac{1}{\lambda\Gamma(\alpha)} (\frac{x}{\lambda})^{\alpha-1} e^{-\frac{x}{\lambda}}$	Scale $\lambda > 0$ Shape $\alpha > 0$	$\lambda \sim$ uniform(1,100) $\sigma \sim$ uniform(1,100)
Gumbel	$f(x) = \frac{1}{\sigma} e^{-(\frac{x-\mu}{\sigma})} e^{-e^{-(\frac{x-\mu}{\sigma})}}$	Scale $\mu > 0$ Shape $\sigma > 0$	$\mu \sim$ uniform(1,100) $\sigma \sim$ uniform(1,100)

2.2.6 Model priors and model parameter priors

Prior probabilities of models reflect the prior knowledge, or belief, that a specific model is the true model in the domain concerned. Whilst informative priors are expected to benefit model development and improve predictive performance, often non-informative priors have to be used due to little prior knowledge about the relative plausibility of the models considered. As a simple but reasonable neutral choice, it can be assumed that all candidate models in the model space are equally likely a priori. This means applying an uniform distribution over the model space, so that $p(M_j) = \frac{1}{K}, for\ j = 1,2, \ldots, K$. No model is considered more likely a priori than any other one. The consideration is to let the observed data carry all the information. This is the most commonly adopted practice in defining model priors in BMA settings [6]. On this basis, the afore-mentioned 5 distributions are assumed to have the equal prior probability equal to 0.2. This leads to the prior model probabilities being cancelled out and the PMP of a candidate model being proportional to its evidence, i.e. marginal likelihood.

The same consideration is applied to defining prior distributions of model-specific parameters. For any of the five candidate models, there is little prior information about its model-specific parameters. Hence it is straightforward to specify non-informative priors so as to allow the posteriors to be informed by data. As shown in Table 1, the priors of the model-specific parameters are all assumed to be uniformly distributed over their reasonable ranges in the context of generally short lifetimes of urban residential buildings in China.

2.2.7 MCMC and posterior distribution calculation

MCMC is used to simulate the posterior distribution of a model-specific parameters. The principle is to draw values of a parameter vector θ from approximate distribution and then correct those draws to better approximate the target posterior distribution. Sampling is performed iteratively in such a way that at each step of the process it is expected that draws are made from a distribution that becomes closer to the target posterior distribution [11]. The sampling process is sequential and the draws create an ergodic Markov chain, which, after a large number of iteration steps, evolves through the parameter space, becomes stationary and converges to the target posterior distribution. Subsequent model inference can be made based on samples from this process much as based on samples from the target posterior distribution [12].

This study uses the Metropolis-Hastings algorithm, which is well established amongst available MCMC algorithms. At the start of iteration t, a candidate vector θ^* is generated from $\theta^{(t-1)}$ through a proposal distribution $f\left(\theta^* \middle| \theta^{(t-1)}\right)$, which is also known as a jumping distribution. The probability of θ^* being accepted to become $\theta^{(t)}$ is:

$$r = min\left\{1, \frac{p(\theta^*|data)\Big/ f\left(\theta^*\middle|\theta^{(t-1)}\right)}{p\left(\theta^{(t-1)}\middle|data\right)\Big/ f\left(\theta^{(t-1)}\middle|\theta^*\right)}\right\} \qquad (14)$$

The acceptance probability r means that if the result is higher than 1, *r* is set to 1, the candidate θ^* is accepted and the transition from θ^* to $\theta^{(t)}$ is made. Otherwise, if the result is lower than 1, the candidate θ^* is accepted with probability equal to *r* and rejected with probability equal to *1-r*. When accepted, the transition from θ^* to $\theta^{(t)}$ is made. When rejected, no move at iteration *t* is made, hence $\theta^{(t)} = \theta^{(t-1)}$, meaning that the chain is updated using the current value.

The proposal distribution $f(\cdot)$ is chosen to be a random walk proposal, where θ^* is selected by taking a random perturbation ε around the current value $\theta^{(t)}$, i.e. $\theta^* = \theta^{(t)} + \varepsilon$. The random vector ε is drawn independently of $\theta^{(t)}$ and centered on zero. As a common setting, ε is a normal distribution with mean zero and variance set to obtain efficient jumping algorithm [11]. In this regard, this study tunes the algorithm by using adaptive sampling, which generates new candidate parameters with a proposal covariance matrix that is estimated from the covariance matrix of the parameters generated so far, with a scaling factor of 2.42/d, where d is the number of parameters [13].

2.2.8 Marginal likelihood calculation

Generally, the marginal likelihood is not analytically tractable and therefore has to be approximated using numerical methods. Typical Monte Carlo sampling methods include naïve Monte Carlo, Importance Sampling (IS), Harmonic Mean (HM), Generalised HM, and Bridge Sampling. The Naïve Monte Carlo is straightforward and in principle unbiased, but numerically inefficient and unstable if the posterior distribution is peaked relative to the prior method. IS may overcome these issues by having an importance density with fatter tails than the posterior distribution. HM uses the posterior distribution as the importance density. This results in the marginal likelihood being equal to the posterior harmonic mean of the likelihood. Despite its convenience and popularity, HM has been criticized extensively due to numerical instabilities and overestimation of the marginal likelihood. Generalised HM, a more stable version of HM, can be viewed as the reciprocal IS. Thus, for the reason analogous to IS, this method also requires the importance density to be finetuned to avoid unbounded variance. Specifically, it requires importance density to have thinner tails than the posterior distribution. Bridge Sampling is a general

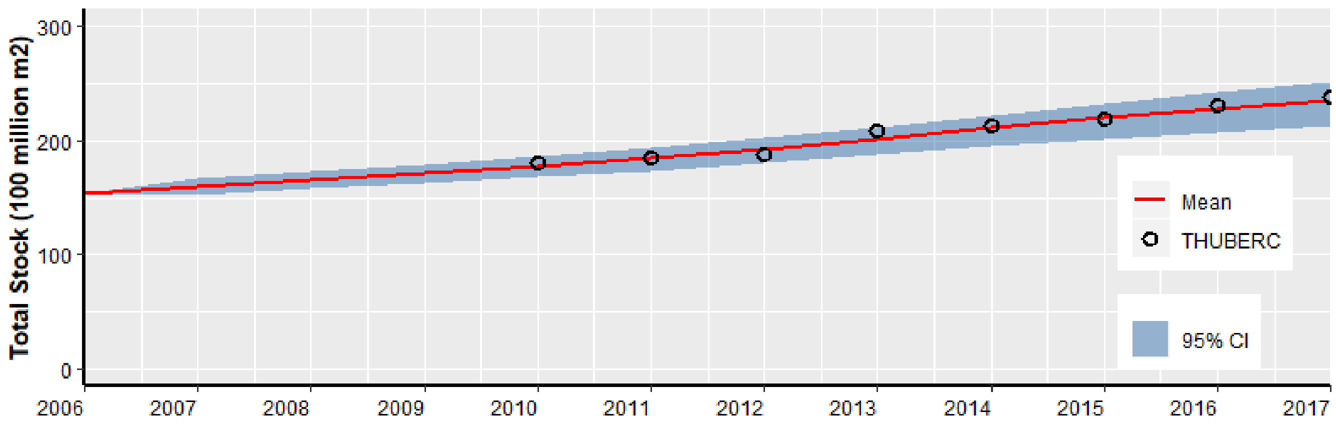

Figure 1: 95% Credible interval of BMA ensemble's posterior prediction of total building stock

case of the afore-mentioned methods. It is more robust to tail behaviours of the proposal distribution (conceptually similar to importance density) relative to posterior distribution and thus avoids large or even infinite variances of estimators [14]. This study uses Bridge Sampling to approximate the marginal likelihood of each of the five candidate models.

3. RESULTS AND DISCUSSION

Based on the methodology elucidated above, the posterior distributions of model-specific parameters of each candidate model, $p(\theta_k|D, M_k)$, were obtained using official statistics on total stock of urban residential buildings up to 2006. The primary data sources included China Statistical Yearbook and MOHURD's Statistical Communique on Urban Housing. Then, the evidence of each candidate model, i.e. the marginal likelihood, was numerically estimated using bridge sampling technique, and the PMP was calculated (Table 2).

Table 2: Prior and posterior probabilities of models

Model	Prior	PMP
Weibull	0.2	0.219
Lognormal	0.2	0.25
Loglogistic	0.2	0.096
Gamma	0.2	0.42
Gumbel	0.2	0.015

With each candidate model, the posterior predictive distribution of total stock over the period of 2007 to 2017, $\tilde{y}$, was obtained through running the probabilistic stock turnover model using the posterior distributions of model-specific parameters, i.e. $p(\theta_k|D, M_k)$, and official statistics on annual new construction from 2007 to 2017. The posterior distribution of $\tilde{y}$ predicted by the BMA model ensemble is the PMP-weighted average of the posterior predictive distribution of $\tilde{y}$ under each candidate model in the model space. Operationally this was obtained by drawing samples from model-specific predictions with probabilities equal to the PMPs and then combining the samples. Figure 1 shows the 95% credible interval of posterior prediction of total stock by the BMA model ensemble. As expected, the total stock size was characterised by a continuously ascending pattern over time. The mean of the credible interval increased by 33% over eight years from 17.7 billion m2 in 2010 to 23.6 billion m2 in 2017. Clearly the line representing the mean of credible interval exhibits a good fit with the estimate by the Annual Report on China Building Energy Efficiency [1], which was developed by Tsinghua University Building Energy Research Centre (THUBERC) and is widely recognised as an authoritative report on the overall situation of building energy in China.

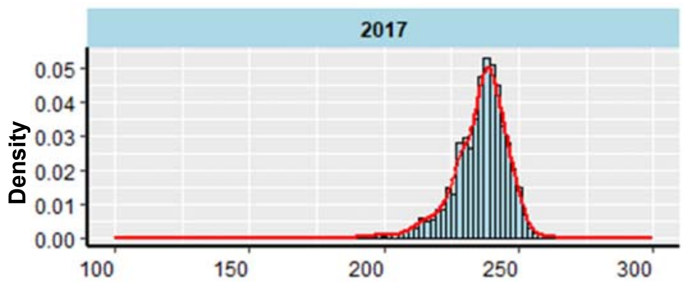

Figure 2: Posterior predictive distribution of size of total stock in 2017 (unit: 100 million m^2)

Compared with a single point, deterministic estimate of annual stock size, the BMA approach taken by this study produces a profile for annual stock size, i.e. the posterior predictive distribution (Figure 2). This probabilistic estimate of annual stock size captures both models' and the model-specific parameters' uncertainties. Having depicted all possible pathways of stock evolution, it provides a full distribution of existing stock size per year and therefore helps to improve the reliability and robustness of not only the estimate of existing stock, but also the forecasting of future total stock which is a function of the existing stock and the underlying survival models and parameters.

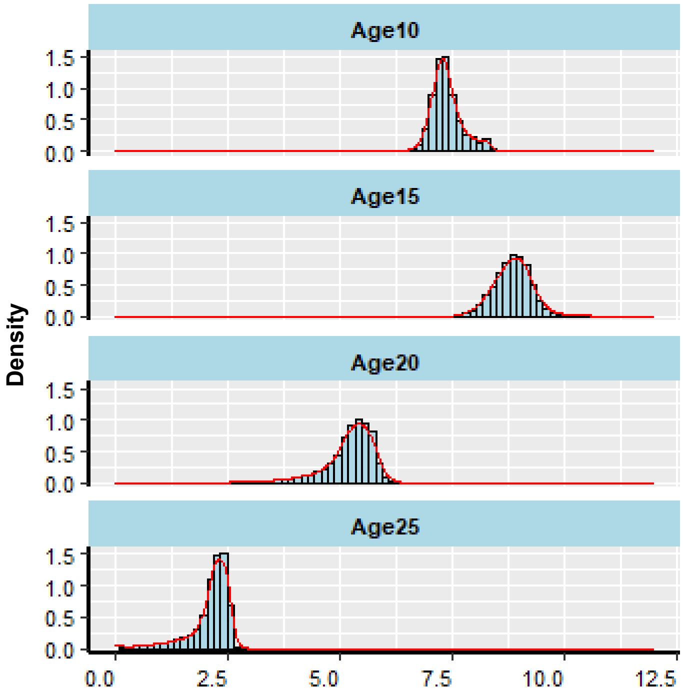

Figure 3: Posterior predictive distribution of sizes of substocks at various ages in 2017 (unit: 100 million m^2)

Furthermore, the dynamic building stock turnover model developed by this study offers additional insights into the composition of building stock through explicitly modelled building aging process. For each parameter vector in the parameter space of a candidate model in the mode space, the annual total stock is disaggregated into age-specific substocks, each of which goes through an aging process subject to age-specific demolition rate determined by the hazard function specified by this particular parameter vector of this particular candidate model. For each year, the substock of new buildings constructed in this year and the substocks of existing buildings at various ages that remain in use collectively create the age profile of the entire stock. For the year after, the stock's age profile is updated due to new construction, aging and demolition. These on-going dynamics, which result in the turnover of the overall stock and detailed representation of age-specific substocks, are fully captured in the dynamic model and, more importantly, are further characterised probabilistically by the BMA model ensemble through the posterior distributions of model-specific parameters and PMPs of candidate models. This allows to obtain the full distribution of each age-specific substock in any given year. Figure 3 shows the posterior predictive distributions of substocks aged 10, 15, 20 and 25 within the total stock in 2017.

The BMA model ensemble and its results have significant implications for the energy consumption and carbon emissions of buildings. Firstly, the possible lifetime distribution profile specified by a parameter vector in the parameter space of a candidate model enables explicit estimate of annual new construction and demolition, which are fundamental to quantifying the initial and demolition embodied energy and carbon per year. Secondly, model granularity at the level of age-specific sub-stocks offers a detailed representation of buildings' heterogeneity with respect to operational energy performance, which is expected to improve due to stringent design codes and technological advances. Thirdly, more importantly, the ability to model the temporal stock dynamics integrates embodied and operational dimensions of building energy and carbon. By simultaneously investigating both dimensions, it is possible to explore their importance relative to each other in the context of future building sectoral developments in green building materials, strengthening design codes for new buildings, and scaling up energy-related retrofits of existing buildings. This presents a fuller picture of stock-level lifecycle energy and carbon.

Across the three dimensions, the uncertainties associated with model-specific parameter vectors and candidate models, as fully captured by the BMA model ensemble, along with uncertainties of other parameters and input variables needed for modelling energy, can be propagated into the emergent stock-level outputs, such as annual total embodied energy and annual total operational energy of total stock. The full Bayesian approach and the resultant probabilistic distribution of stock-level outputs can mitigate the risk of potential over- or under-estimate that would otherwise be more likely to be produced by deterministic models. This creates a powerful modelling framework with enhanced robustness and reliability, thereby allowing for more effectively experimenting and analysing policies aiming to decarbonise buildings in the broader context of peaking China's economy-wide emissions by 2030.

4. CONCLUSIONS

This paper presents a statistical model to estimate total stock size of urban residential buildings in China, for which official data only exists up to 2006. It firstly develops a probabilistic dynamic model characterising the building aging and demolition process and overall stock turnover, then operationalises the model by separately using various candidate parametric survival models as an integral part of the overall model, and finally applies Bayesian Model Averaging (BMA) to create a model ensemble to combine predictions of the stock evolution pathway made by each candidate survival model based on their respective posterior model probabilities.

This study is a first-of-its-kind attempt to take a full Bayesian approach to investigate model and parameter uncertainties that were not taken account of by limited existing models targeting Chinese building stock, which is a strategically important but under-researched area. The modelling approach and the results can serve as a baseline for further studies on forecasting building stock development trajectory and analysing energy and carbon impacts, with particular regard to modelling and analysing policy scenarios to investigate the trade-offs across embodied-versus-operational energy and carbon emissions facing Chinese residential buildings.

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