# Investigation on oscillations characteristics of self-excited thermoacoustic system forced by acoustic wave

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## ABSTRACT

Self-excited thermoacoustic oscillations usually occur in many practical systems such as rocket motors, gas turbines and cryogenic distribution systems. We study the dynamics of a self-excited cryogenic thermoacoustic system subjected to acoustic forcing. The effect of the driving phase on the amplitude of forced oscillation is analyzed. The results show that the variation of driving phase has not affected the maximum amplitude of pressure oscillation. The coupling oscillation characteristics are clarified through the phase portraits. By analyzing the data with Poincare map, we found a range of nonlinear dynamics, including (i) a shifting of the 1-period oscillation towards k-period oscillation as the forcing phase increases; (ii) an accompanying transition from single-frequency model to two-frequency model. The results suggest that such oscillators can be used to represent thermoacoustic selfexcited systems subjected to similar forcing.

**Keywords:** Thermoacoustic oscillation, Instability, Heatto-sound conversion, Active control

## 1. INTRODUCTION

Stable cryogenic environment plays critical role in determining the performance of many applications, such Single-Pressure Refractive-Index as the Gas Thermometry system [1], Taiwan Phone Source [2], Canadian Light Source [3] and Superconducting Linac [4]. However, these systems are susceptible to self-excited thermoacoustic instabilities, especially under a large temperature gradient. In the large-scale helium refrigerator, cryogenic transfer lines are the key parts to transfer the coolant to the superconducting magnet and accelerator. When a gas column in a long cylindrical tube is subjected to strong temperature gradients, it may spontaneously oscillate with large amplitudes. The energy transferred from unsteady heat source to acoustic wave is the main cause of self-excited thermoacoustic oscillation.

To mitigate such thermoacoustic oscillation, approaches [5, 6] involving adding a restriction or baffles, installing thermal intercepts, providing a dynamic damper via a throttle value, orifice, or porous plug are proposed. However, these are passive control approaches whose drawbacks are that modifying system will be time consuming and costly. Active approaches are applied to control the instabilities of thermoacoustic systems. Zhao et al. [7]. carried out experimental evaluation of anti-sound approach in damping selfsustained thermoacoustic oscillations. A model-based active adaptive controller for thermoacoustic instability was proposed [8]. Experimental study on forced thermoacoustic oscillation driven by loudspeaker demonstrated that forced oscillation had a higher selectivity for driving frequency [9]. However, the drive amplitude and frequency will not be affected by the start time, only the drive phase will produce different results due to the start time. The trigger point lies in the unknown nature of the self-oscillation, so only for the driving phase, the influence of the start time on the system oscillation characteristics is studied. Compared with the driving amplitude and driving frequency, the control of driving phase is much more complicated. However, how the driving phase affect the self-excited oscillation is unclear. Therefore, we take an active approach to study the interaction between self-excited oscillation and forced oscillations. Our aim is to see how the driving phase affects the characteristics of coupling oscillations.

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In the following, to reveal the impacts of driving phase on the pressure amplitude of the cryogenic oscillation, this paper adopts a moving piston to drive the cryogenic helium in the tube to achieve active control of self-excited oscillations. To select effective driving phase in practical application, the coupling oscillations characteristics under different driving phase are analyzed. To better control self-excited oscillations, the investigation on the control method, especially the start modes of external disturbance are carried out in this paper.

#### 2. MODEL AND METHODOLOGY

#### 2.1 Model

Fig.1(a) depicts the two-dimensional computational domain of the cryogenic helium tube model coupling with a driven piston, including a typical guarterwavelength helium tube (the diameter is 2.85 mm) and a driven piston. The cryogenic helium tube is open (acoustic outlet) at one end (x=0), and the other end (x=L, L is the length of the helium tube channel) connects with the externally driven zone, i.e., the movement zone of the piston (the length is 5 mm). Hence, the surface of the piston is a movable wall of the model. The open end of the model connects with a cryogenic helium container whose temperature is  $T_c$ =4.5 K and the other end is in a normal temperature environment (T<sub>h</sub>=300 K). Then a nonlinear temperature distribution in horizontal axis direction along the tube wall is formed, as indicated in Fig.1(b), which is prescribed by imposing the userdefined function (UDF) on the solid wall. A detailed introduction about the nonlinear temperature function has been reported in our previous study [10], therefore we will not give a further description. The mesh configurations in helium tube channel (I) and external driven zone (II) are shown in Fig.1(c). To meet the requirements of the dynamic mesh zone and computational cost, the grids in the zone (II) are denser than those in the zone (I). In particular, the model should adopt a rectangular grid in the zone (II).





wall. (c) Grids in helium tube channel (I) and external driven zone (II)

## 2.2 Solution of governing equations

In the computational fluid dynamics (CFD) simulation, the cryogenic helium tube system is simplified into a two-dimensional form. The CFD technology is adopted to study the physical processes that cause thermoacoustic oscillation of the system. The equations of conservation of mass, momentum, and energy are given as follows.

Equation of conservation of mass:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

Equation of conservation of momentum:

$$\frac{\partial(\rho\vec{u})}{\partial t} + \nabla \cdot (\rho\vec{u}\vec{u}) + \nabla p = \nabla \cdot \overline{\overline{\tau}}$$

Equation of conservation of energy:

$$\frac{\partial}{\partial t}(\rho E) + \nabla \cdot (\vec{u}(\rho E + p)) = \nabla \cdot [\kappa \nabla T + (\overline{\tau} \cdot \vec{u})]$$

where  $\vec{u}=u\vec{i}+v\vec{j}$  is the velocity vector, i and j indicate the coordinates, t denotes the time;  $\kappa$  is the thermal conductivity,  $\overline{\tau}$  is the stress tensor.

The internal energy:

$$E = h - p \left/ \rho + \left| \vec{u} \right|^2 \right/ 2$$

where E and h are internal energy and enthalpy. Helium is assumed to be an ideal gas whose temperature T, pressure p, and density  $\rho$  can be related by the equation of state  $p = \rho R_g T$ , where the gas constant  $R_g = 2078.5 J K^{-1} k g^{-1}$  for helium.

The temperature gradient is defined by the UDF described by the following equation:

 $T(x) = 147.45 * \tanh[100 * (x - x_0)] - 147.45 * \tanh[100 * (x + x_0)]$ +147.5 \* tanh[100 \* (2 - x - x\_0)] - 147.5 \* tanh[100 \* (2 - x + x\_0)] + 300

where: x -horizontal coordinate of the helium tube [m],  $x_0$  -position of the flection point [m].

The external forced thermoacoustic oscillation driven by a moving piston is achieved by introducing a UDF to Fluent.

where  $v=v_p * \sin(2\pi f_F * t + \phi_0)$ 

 $v_{p}$  - maximum speed of the piston movement [m/s],

v - speed of the piston movement [m/s],

 $f_F$  -forced frequency of the piston [Hz],

 $\phi_0$  -driving phase of the piston.

#### 3. RESULTS AND DISCUSS

3.1 Self-excited thermoacoustic oscillation

Without external forcing, pressure fluctuations of self-excited oscillation in the stable stage is monitored as illustrated in figure 2. It can be seen from figure 2 that the system still reaches a pressure oscillation of about 0.55 kPa, when the system is in the stable stage. This indicates the necessity of applying a control approach in minimizing self-excited oscillations in the cryogenic system. Therefore, we propose an active control approach in which a cryogenic tube is driven by a piston. In the subsequent section, the effect of the driving phase of the piston on the oscillation characteristic of the cryogenic helium tube will be investigated.



Fig 2 Pressure fluctuations of self-excited oscillation in the stable stage

## 3.2 Effect of driving phase

Figure 3 shows the relationship between the driving phase of the piston and the oscillation pressure amplitude without self-excited oscillations. The results show that the variation of driving phase has not affected the maximum amplitude of pressure oscillation. When the driving speed is 7 m/s and the frequency is 32 Hz, the pressure oscillation amplitude is 0.66 kPa. As the driving phase increases, the phase of the pressure oscillation continues to advance. When the driving phase is 180°, the phase of the pressure oscillation is ahead of the case with 0° phase by 180°.



Fig 3 Effect of driving phase  $\phi_0$  on pressure amplitude of the system without self-excited oscillations

Since the driving phase is sensitive to the time, to achieve precise control of oscillations, we fit the pressure oscillation function, as shown in table 1. The result shows that the phase of the pressure oscillation is different from the drive phase.

Table 1 Fitting functions of oscillating pressure		
Cases	$\phi_0$	Fitting functions
1	Self-excited	$A_1 = 0.55 * \sin(64\pi * t + 11.35)$
2	0°	$A_2 = 0.66 * \sin(64\pi * t + 0.99)$
3	30°	$A_3 = 0.66 * \sin(64\pi * t + 1.53)$
4	60°	$A_4 = 0.66 * \sin(64\pi * t + 2)$
5	90°	$A_5 = 0.66 * \sin(64\pi * t + 3.34)$
6	150°	$A_1 = 0.66 * \sin(64\pi * t + 2.65)$
7	180°	$A_1 = 0.66 * \sin(64\pi * t - 2.21)$

According to anti-sound technique, unwanted oscillation can be cancels out by introducing another sound source. The waves act in such a way that the self-excited oscillation and external sound source are out of phase by 180°. To obtain a suitable driving phase, the effect of driving phase  $\phi_0$  on the phase difference  $\phi / \pi$  between forced oscillation and self-excited oscillation is shown in figure 4. As the drive phase increases, the phase difference  $\phi / \pi$  gradually decreases. The dotted line position indicates that the phase difference is 180°. From the result we can see that the phase difference is 180°, when the driving phase is near 60°.



Fig 4 Effect of driving phase  $\phi_0$  on the phase difference between forced oscillation and self-excited oscillation

## 3.3 Coupling oscillation characteristics

Figure 5 shows the effect of external sound on the fast Fourier transform (FFT) spectrum. In figure 5 (a), the frequency of self-excited mode is 32 Hz. While for a forced mode, the system presents a two-frequency model.



Fig 5 Effect of external sound on the FFT spectrum (a) self-excited oscillation (b) coupling oscillation.

Figure 6 shows the effect of external sound on dynamic characteristics. For self-excited model, the phase portrait is a limit circle as demonstrated in figure 6 (a). When the system is subjected to the periodic external force, the phase portrait [10] changes from a closed curve to multiple circles (b), and the Poincare map changes from two points (c) to multiple points (d), i.e., 1-period to k-period.



Fig 6 Effect of external sound on the phase portraits (a, b) and Poincare maps (c, d).

### 4. CONCLUSIONS

In this study, we study the dynamics of a self-excited cryogenic thermoacoustic system subjected to acoustic forcing. The effect of the driving phase on the amplitude of forced oscillation shows that the variation of driving phase has not affected the maximum amplitude of pressure oscillation. To provide a guide for the selection of driving signal, the dynamics of a self-excited cryogenic thermoacoustic system subjected to acoustic forcing are investigated. By analyzing the data with Poincare map, we found a range of nonlinear dynamics, including (i) a shifting of the 1-period oscillation towards k-period oscillation as the forcing phase increases; (ii) an accompanying transition from single-frequency model to two-frequency model.

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