

# Contingency Management in Integrated Electricity and Gas Systems Considering Gas Flow Dynamics

Xueyong Tang<sup>1</sup>, Sheng Wang<sup>2</sup>, Bin Sun<sup>1</sup>, Qingsheng Li<sup>1</sup>, Yi Ding<sup>2\*</sup>

1 Power Grid Planning & Research Center, Guizhou Power Grid Co. , Ltd.

2 College of Electrical Engineering, Zhejiang University

## ABSTRACT

The growing capacity of gas-fired generating units has intensified the interaction between the electricity and gas systems. Though a more flexible operation can be achieved by integration, it also raises a serious issue on managing contingencies when some of the components in the integrated electricity and gas systems (IEGS) fail during the operation. This paper proposes a contingency management scheme in the IEGS considering the influence of the gas flow dynamics. Firstly, the partial derivative equations which describe the continuity and motion of gas flow are discretized using the finite-difference scheme. A second-order cone reformulation of the discretized equations is then proposed. Moreover, the optimal load shedding problem during the contingency state of the IEGS is formulated and then solved. Finally, 24-bus IEEE Reliability Test System and Belgium natural gas transmission system are used to validate the proposed technique.

**Keywords:** integrated electricity and gas systems, contingency management, optimal load shedding, gas flow dynamics

## 1. INTRODUCTION

The worldwide transition towards a low-carbon and efficient energy production has promoted the utilization of cleaner fuel resources to generate electric power, such as natural gas [1]. In UK, the electricity generation from natural gas was 273.4 TWh in 2018, taking 37.98% among all kinds of fuels [2]. This leads to the large-scale installation of gas-fired units (GFU), which has intensified the interaction between the electricity and natural gas systems. The concept of integrated electricity and gas

systems (IEGS) is therefore proposed. However, the severe blackout in Taiwan, China in 2017 has witnessed that without proper handling, such interdependency may lead to failure propagations, causing more severe consequences compared with those in two formerly isolated systems [3].

Contingency management is designed for the situation in which the random failure of energy supply facilities occurs, e.g., the gas wells or the electric power generators. This subject has been thoroughly discussed in the electricity systems in the past few decades, while it is still at an early stage when the gas system is incorporated [4]. In most of the previous studies, the main goal of contingency management was to minimize the load shedding in both electricity and gas systems, and meanwhile reduce the operation cost [5]. The operating conditions of IEGS components were determined using the steady-state based integrated optimal power flow (SIOPF) technique in [6]. Multi-linear probabilistic energy flow of IEGS was proposed in [7] to handle uncertain situations such as wind power fluctuations. Decomposition [8] and linearization techniques [9] can be supplementary for relaxing the physical constraints of the electricity or gas power flow and achieving a better computation efficiency, convergence, and robustness.

However, steady-state based contingency management is not always the optimum option in the operational phase. Unlike the electric power flow, the transient process of the gas flow could last from minutes to hours after the change of system state, especially in the transmission-level. Some researchers studied the time-varying flow rate in the gas system considering its dynamics. Monte Carlo simulation technique is used in [10] to study the system state transition, as well as the

change of gas flow. The gas flow dynamics are utilized for developing an equivalent model of gas networks in [11]. The effects of gas flow dynamics on the unit commitment was further studied in [12].

Despite that the post effects of the component failures on the gas flow are simulated in these researches, the contingency management decisions are still made based on the steady-state analysis. It is evidenced that the gas flow dynamics can be utilized to mitigate the load shedding if with proper strategy. For example, the gas stored in the pipeline, also known as the linepack, can cover the gas shortage for a short period during the operation [13]. With this idea in mind, an optimal control model of transient gas flow was proposed in [14]. It was further extended into the coordinated scheduling for the electricity and gas systems in [15], and its advantage over the SIOPF was compared. However, the formulation of the optimal control problem is a bit complicated for contingency management. It inherits a large set of nonlinear constraints from the partial derivative equations (PDE) of gas flow dynamics, which makes it difficult to guarantee a fast convergence and robustness of the solution.

To address these research gaps, this paper proposes a fast calculation method for contingency management in the IEGS. First, the basic structure of the IEGS is introduced. Then, the PDEs which describe the continuity and motion of gas flow dynamics are discretized using the finite-difference scheme. A second-order cone (SOC) reformulation technique of the discretized equations is proposed. By doing so, the nonconvexities are eliminated so that the further optimization problems can be addressed by off-the-shelf solvers. Based on that, the contingency management of the IEGS is formulated in the form of an optimal control problem. Finally, we use the integrated 24-bus IEEE Reliability Test System and Belgium natural gas transmission system to validate the proposed technique. The numerical results are also compared with the steady-state based optimal load shedding to demonstrate the superiority of the proposed method.

## 2. STRUCTURE OF THE IEGS

As illustrated in Fig 1, the IEGS transports the electricity generated by GFUs and traditional non-gas fossil generating units (TFU), as well as the natural gas from gas wells and storages, to satisfy the demands of electricity and gas from various locations. Among those units and facilities, the GFU is most critical. It links the

two energy systems by consuming gas to generate electricity.

During the IEGS operation, random failures of gas sources or generators could occur, and transfer the IEGS into a contingency state. The generators and gas sources have to be re-dispatched, or the electricity and gas loads may be curtailed to maintain a balanced operation. As aforementioned, the gas flow dynamics are beneficial for mitigating the load shedding. Therefore, it is essential to incorporate the gas flow dynamics into the contingency management.

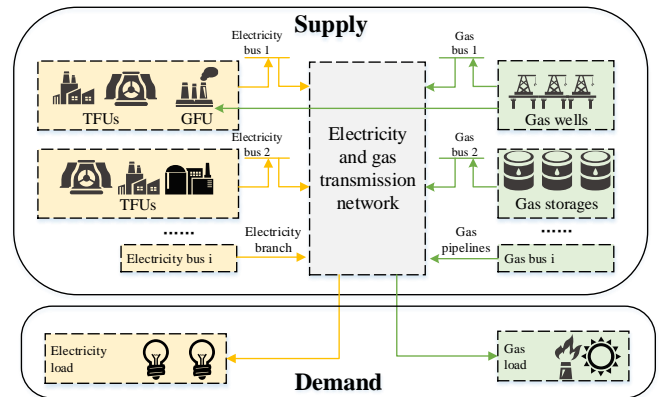


Fig 1 Structure of the studied IEGS.

## 3. REFORMULATION OF DYNAMIC GAS FLOW EQUATIONS

The gas flow dynamics in a pipeline are governed by two PDEs, namely continuity and motion equations. In a horizontally placed gas pipeline, the dissipative and isothermal gas flow is described by [16]:

$$f_{co}(x, t) = \frac{B^2}{\rho_0 A} \frac{\partial q}{\partial x} + \frac{\partial p}{\partial t} = 0 \quad (1)$$

$$f_{mo}(x, t) = \frac{\partial p}{\partial x} + \frac{\rho_0}{A} \frac{\partial q}{\partial t} + \frac{2\rho_0^2 B^2}{F^2 D A^2} \frac{q|q|}{p} = 0 \quad (2)$$

where  $B$  is the isothermal wave speed of gas.  $\rho_0$  is the gas density at the standard temperature and pressure.  $A$  is the cross-sectional area of the pipeline.  $D$  is the diameter of the pipeline.  $F$  is the Fanning transmission factor.  $q$  and  $p$  are the quantities of gas flow and gas pressure, respectively.

The derivative regarding the time domain has little influence on the accuracy of (2), especially in the transmission pipelines with a relatively steady flow rate and large capacity [17]. Discretizing the above PDEs for the pipeline from bus  $i$  to  $j$  (the notation  $ij$  is omitted) using Wendroff formula [18]. This yields:

$$\frac{1}{B^2}(p_{m+1,k+1} + p_{m,k+1} - p_{m+1,k} - p_{m,k}) + \quad (3)$$

$$\frac{\Delta t \rho_0}{\Delta x A} (q_{m+1,k+1} - q_{m,k+1} + q_{m+1,k} - q_{m,k}) = 0$$

$$4((p_{m+1,k+1} + p_{m+1,k})^2 - (p_{m,k+1} + p_{m,k})^2) + \quad (4)$$

$$\frac{\Gamma \rho_0 B^2 \Delta x}{F^2 DA^2} (q_{m+1,k+1} + q_{m+1,k} + q_{m,k+1} + q_{m,k})^2 = 0$$

where  $\Delta x$  and  $\Delta t$  are the step sizes in space and time domains.  $m$  is the index of pipeline sections.  $k$  is the index of time sequence.  $\Gamma \in \{-1, 1\}$  represents the direction of the gas flow in the steady-state.

Assuming the gas flow doesn't change direction after the contingency [11]. Then, (4) can be further relaxed into SOC constraints:

$$\left\| \begin{array}{c} p_{m,k+1} + p_{m,k}, \\ \sqrt{\frac{\rho_0^2 B^2}{F^2 DA^2}} \Delta x \\ (q_{m+1,k+1} + q_{m+1,k} + q_{m,k+1} + q_{m,k}) \end{array} \right\|_2 \leq p_{m+1,k+1} + p_{m+1,k}, \Gamma = 1 \quad (5)$$

$$\left\| \begin{array}{c} p_{m+1,k+1} + p_{m+1,k}, \\ \sqrt{\frac{\rho_0^2 B^2}{F^2 DA^2}} \Delta x \\ (q_{m+1,k+1} + q_{m+1,k} + q_{m,k+1} + q_{m,k}) \end{array} \right\|_2 \leq p_{m,k+1} + p_{m,k}, \Gamma = -1 \quad (6)$$

The SOC relaxation is exact for  $q$ , but not exact for  $p$ . Hence, the quantity of load shedding will not be affected if we further use the relaxed equations in the contingency management problem.

## 4. CONTINGENCY MANAGEMENT SCHEME OF THE IEGS

### 4.1 Initial and boundary conditions

For the gas flow dynamics are essentially described by PDEs, it is necessary to specify the initial and boundary conditions. The initial conditions are given by the operating condition of the IEGS in the normal state. The gas flow is steady which can be calculated using the ISOPF. The operation of the IEGS aims to minimize the operating cost  $C_{IEGS}$  by controlling the generation of TFUs and GFUs, and the gas production of gas sources:

$$\text{Min}_{sg_i, g_{i,j}, g_{i,j}^{gf_u}} C_{IEGS} = \sum_{i \in GB} gp_i sg_i + \sum_{i \in EB} \sum_{j \in NG_i} cst_{i,j}(g_{i,j}) \quad (7)$$

Subject to:

$$sg_i^- \leq sg_i \leq sg_i^+ \quad (8)$$

$$g_{i,j}^- \leq g_{i,j} \leq g_{i,j}^+ \quad (9)$$

$$g_{i,j}^{gf_u,-} \leq g_{i,j}^{gf_u} \leq g_{i,j}^{gf_u,+} \quad (10)$$

$$sg_i - gd_i - \sum_{j \in NG_i^{gf_u}} g_{i,j}^{gf_u} / \zeta_{i,j} - \sum_{j \in \Omega_i^g} q_{ij} = 0 \quad (11)$$

$$\sum_{j \in NG_i^{gf_u}} g_{i,j}^{gf_u} + \sum_{j \in NG_i} g_{i,j} - ed_i - \sum_{j \in \Omega_i^e} f_{ij} = 0 \quad (12)$$

$$q_{ij} = C_{i,j} \text{sgn}(p_i - p_j) \sqrt{|p_i^2 - p_j^2|} \quad (13)$$

$$ef_{i,j} = (\theta_i - \theta_j) / X_{i,j} \quad (14)$$

$$|f_{ij}| \leq f_{ij}^+ \quad (15)$$

$$|q_{ij}| \leq q_{ij}^+ \quad (16)$$

where  $sg_i$  is the gas production at bus  $i$ .  $gp_i$  is the gas purchasing price.  $g_{i,j}$  is the electricity generation of TFU  $j$  at bus  $i$ .  $cst_{i,j}$  is the generation cost function for TFU.  $EB$  and  $GB$  are the sets of electricity and gas buses.  $NG_i$  and  $NG_i^{gf_u}$  are the sets of TFU and GFU at bus  $i$ , respectively.  $gd_i$  and  $ed_i$  are the gas and electricity loads at bus  $i$ .  $\zeta_{i,j}$  is the efficiency of the GFU.  $\Omega_i^e$  and  $\Omega_i^g$  are the sets of electricity branches and gas pipelines connected to bus  $i$ .  $f_{ij}$  and  $q_{ij}$  are the electricity and gas flows from bus  $i$  to  $j$ .  $C_{ij}$  is a characteristic parameter of the pipeline, depending on the length, absolute rugosity, and some other properties.  $\text{sgn}(x)$  is the signum function, where  $\text{sgn}(x) = 1$  if  $x \geq 0$ , and  $\text{sgn}(x) = -1$  if  $x < 0$ .

We denote the solution of the optimal operation problem as  $sg_i^*$ ,  $p_i^*$ , and  $q_{ij}^*$ . During the contingency management, the gas pressures in the pipeline segment  $m$  in time sequence  $k$ ,  $p_{ij,m,k}$ , should be controlled within the secure limits:

$$p_{ij}^- \leq p_{ij,m,k} \leq p_{ij}^+ \quad (17)$$

After formulating (3), (5), and (6) for all pipelines, the initial condition for the discretized PDEs is specified as:

$$p_{ij,m,0} = \sqrt{p_i^{*2} - \text{sgn}(p_i^* - p_j^*) q_{ij}^{*2} (C_{ij}^2 L_{ij})^{-1} m \Delta x} \quad (18)$$

$$q_{ij,m,0} = q_{ij}^* \quad (19)$$

where  $L_{ij}$  is the length of the pipeline  $ij$ .

For a generalized connected pipeline system, the boundary conditions are specified as:

$$p_{ij,0,k} = p_{j_1,0,k} (\forall j_1 \in \Omega_i^g, \forall k) \quad (20)$$

$$p_{ij,0,k} = p_{j_2,0,k} (\forall j_2 \in \Omega_i^g, \forall k) \quad (21)$$

$$sg_i^* - gd_i + \sum_{j \in \Omega_i^g} q_{ji,M_{ij},k} - \sum_{j \in \Omega_i^g} q_{ij,0,k} = 0, \forall k \quad (22)$$

where  $M_{ij}$  is the number of pipeline segments in pipeline  $ij$ .

### 4.2 Formulation of the optimal load shedding problem

The objective of the optimal load shedding problem for the contingency management is to find the best control strategy of IEGS components, to minimize the electricity and gas loads shedding, as well as the operation cost over the contingency period. The control variables include: (1) gas production of the gas sources  $sg_{i,k}$ ; (2) electricity generation of TFUs and GFUs,  $g_{i,j,k}$  and  $g_{i,j,k}^{gf}$ ; (3) the quantities of electricity and load shedding,  $ec_{i,k}$  and  $gc_{i,k}$ .

$$\text{Min}_{sg_{i,k}, g_{i,j,k}, ec_{i,k}, gc_{i,k}} J = \sum_{k=1}^{NK} \left( \sum_{i \in EB} \left( CDF^e ec_{i,k} + \sum_{j \in GEN_i} cst_{i,j}(g_{i,j,k}) \right) + \sum_{i \in GB} \left( sg_{i,k} P_i^g + CDF^g \cdot gc_{i,k} \right) \right) \quad (23)$$

Subject to:

a) Initial condition constraints: (18), (19), and (24).

$$sg_{i,0} = sg_i^*, g_{i,j,0} = g_{i,j,0}^*, g_{i,j,0}^{gf} = g_{i,j,0}^{gf*} \quad (24)$$

b) Gas system dynamic constraints (6) for all the pipelines.

c) Gas boundary condition constraints (20)-(22) for  $k=1,2,\dots, NK$ .

d) Electricity power flow constraints (12), (14), and (15) for  $k=1,2,\dots, NK$ .

e) Upper and lower boundaries (8)-(10), (16), and (17) for  $k=1,2,\dots, NK$ :

where  $CDF^e$  and  $CDF^g$  are the customer damage function of electricity and gas loads, respectively [19]. The formulated contingency management scheme is a SOC programming problem, which is solved using the Gurobi solver [20].

## 5. SOLUTION PROCEDURES

The contingency management scheme proposed in this paper aims to determine the new operating state of the IEGS after contingency happens. The solution procedures are summarized as follows:

- 1) Determine the normal operating schedule of IEGS in the day-ahead according to the forecast electricity and gas loads. It is solved using SIOPF according to (7)-(16).
- 2) Receive the contingency information, such as the failure of generating units. Set the parameters  $\Delta t$  and  $\Delta x_{ij}$  for discretizing the PDEs of gas flow dynamics into (4)-(6).
- 3) Calculate the initial condition of the contingency state in the gas system according to (18), (19), and (24).

- 4) Formulate the contingency management problem according to (4)-(6), (17), and (20)-(23). Solve the SOC programming problem using the Gurobi solver.

## 6. CASE STUDY

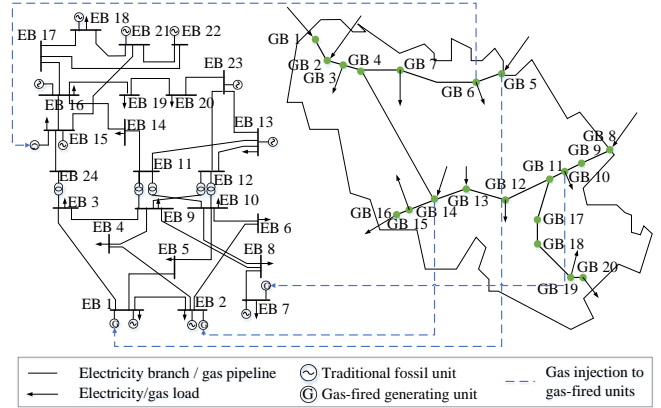


Fig 2 Integrated IEEE RTS and Belgium natural gas transmission system.

In this section, the proposed contingency management scheme is applied to a test IEGS, consisting of the IEEE 24-bus Reliability Test System [21] and Belgium natural gas transmission system [22]. The oil/steam and oil/combustion turbine generating units at the electricity bus (EB) 1, 2, 7, and 15 are replaced with the GFUs of the same capacity, connecting to gas bus (GB) 5, 14, 11, and 6, respectively. The efficiencies of GFUs are set according to [23]. The prices of the gas producers/storages at GB 1, 2, 5 and 8, 13, 14 are 0.085 and 0.062  $\$/m^3$ , respectively. Simulations are performed on a Lenovo laptop with an Intel® Core™ i7-8565U 1.80GHz and a 16GB memory.

In the first case, we assume that a severe failure occurs. The capacity of the gas source at GB 1 has reduced by 9.28  $Mm^3/day$ , and the failure lasts for 7 hours. Three different contingency management schema are compared:

- Strategy A: the load shedding is determined by SIOPF [6].
- Strategy B: the operating condition of IEGS is determined by SIOPF, and the load shedding is determined by the simulation of gas flow dynamics [10].
- Strategy C: the proposed contingency management scheme in this paper.

The result of gas and electricity load shedding at representative buses are presented and compared in Fig. 3. As can be observed, strategy A has the largest gas load shedding at all the GBs. In strategy B, the load shedding

appears lower at the beginning, increasing gradually from zero to the steady-state values in the first 40 minutes. This transient process of gas flow can take longer in a longer transmission pipeline with a larger capacity. This can be a compromised strategy in the IEGS where the electricity and gas systems can be coordinated in a relatively longer timeframe, e.g. hourly based schedule, but cannot be coordinated in real-time, e.g. ancillary service. In strategy C, the electricity and gas systems can be fully coordinated in real-time, which demonstrates the most superior performance on the load shedding.

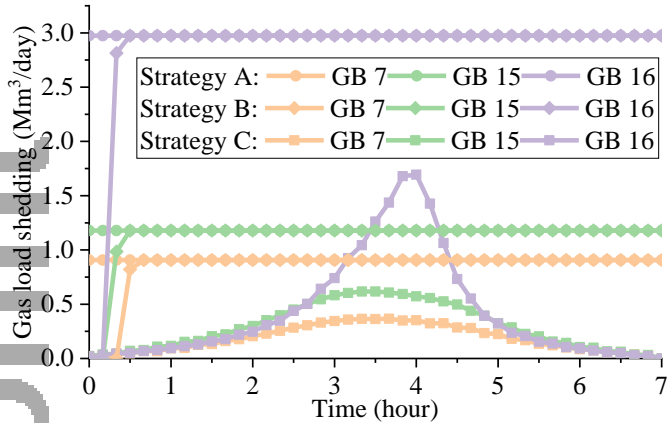


Fig 3 Comparison of gas load shedding in different strategies

The computation times of different strategies are compared in Table 1. Strategy A is fastest for it only entails solving a nonlinear programming problem at a single time point. Strategy C is a more complicated optimization problem involving large-scale variables at different time points. Owing to the proposed SOC relaxation technique, it achieves the minimum load shedding within a quite reasonable time.

Table 1 Computation times of different strategies

Strategy	Solver	Computation Time (s)
A	Interior point method in Matpower	0.061
B	Interior point method in Matpower + fsolve in Matlab	35.59
C	Interior point method in Matpower + Gurobi	2.61

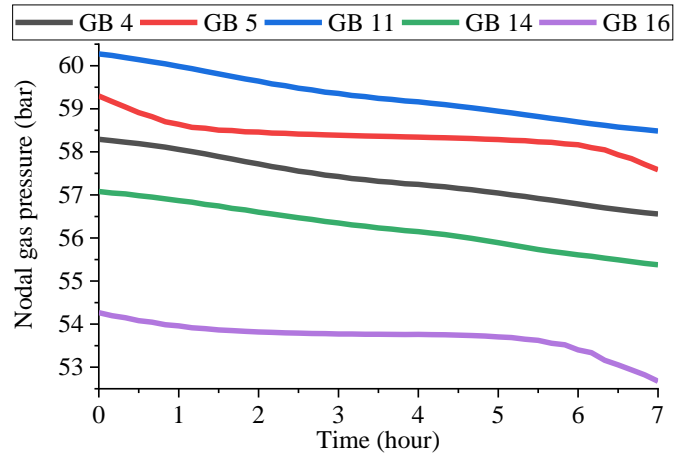


Fig 4 Nodal gas pressure during the contingency state

The gas pressures at critical GBs in strategy C are further presented in Fig. 4. For security reasons, the lower boundaries of gas pressure are set to 0.97 times of their pressures in the normal operating state. As presented in Fig. 4, all the GBs reach their lower boundaries at the end of the studied period. It can be concluded that though strategy C leads to minimal load shedding, it is delivered by using the linepacks. The overuse of linepacks will lower the pressure in the gas network, which makes it vulnerable against the gas load fluctuations and possible failures in the future. Hence, there exists a trade-off between the load shedding and gas pressure, which is, in another word, the trade-off between the capability of solving the crises at the moment, and the capability of withstanding the risk in the future.

To further investigate this matter, we compare the total gas load shedding of IEGS with different durations of gas source failure and different lower boundaries of gas pressures in strategy C. The simulation results are presented in Fig. 5. The gas load shedding is higher when the duration of failure increases and lower gas pressure limits are higher. The long duration of failure indicates a large quantity of gas shortage. Besides, due to the tight pressure limits, there is also less available linepack. It is worth noting that from a certain point, the gas source failure can be fully handled with linepacks without any load shedding.

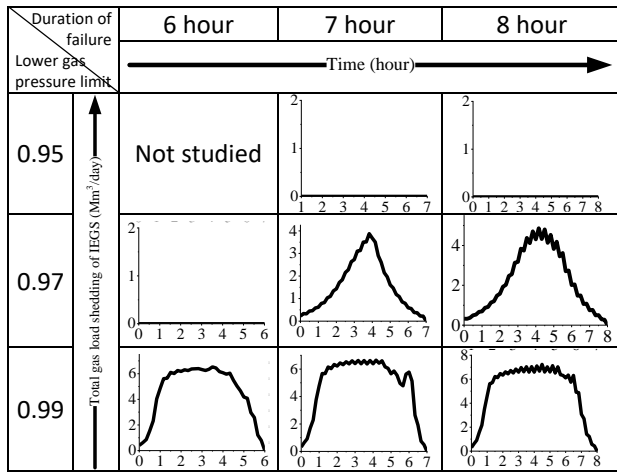


Fig 5 Comparison of total gas load shedding with different durations of failure and different lower gas pressure boundaries

## 7. CONCLUSION

This paper proposes a contingency management scheme in the IEGS considering the gas flow dynamics. The numerical simulations demonstrate the superiority of the proposed strategy. The gas load shedding is minimized when the gas flow dynamics are fully utilized. However, the overuse of linepack will lower the gas pressures, leading to potential risks in the future. Therefore, the load shedding in the contingency state and gas pressures should be carefully balanced. The conclusions strongly indicate the necessity of incorporating gas flow dynamics into contingency management in the IEGS.

## ACKNOWLEDGEMENT

The research is supported by the Key Scientific and Technological Project of China Southern Power Grid 067600KK52190010: Research on Key Technologies of Regional Multi-Energy System Planning and Operation Based on Digital Twins.

## REFERENCE

- [1] Wang S, Ding Y. Optimal sizing and asset utilization efficiency analysis of a distributed multi-energy system considering energy substitution and load uncertainty. *Journal of Global Energy Interconnection*. 2019;2(5):426-32.
- [2] Department for Business EIS. Digest of UK Energy Statistics (DUKES). In: Department for Business EIS, editor. 25 July 2019.
- [3] Hui H, Ding Y, Luan K, Xu D. Analysis of "8•15" Blackout in Taiwan and the Improvement Method of Contingency Reserve Capacity Through Direct Load Control. 2018 IEEE

Power & Energy Society General Meeting (PESGM)2018. p. 1-5.

- [4] Li G, Bie Z, Kou Y, Jiang J, Bettinelli M. Reliability evaluation of integrated energy systems based on smart agent communication. *Applied Energy*. 2016 Apr;167:397-406.
- [5] Wang P, Ding Y, Goel L. Reliability assessment of restructured power systems using optimal load shedding technique. *IET Generation, Transmission & Distribution*. 2009;3:628-40.
- [6] Sheng W, Yi D, Chengjin Y, Can W, Yuchang M. Reliability evaluation of integrated electricity-gas system utilizing network equivalent and integrated optimal power flow techniques. *Journal of Modern Power Systems and Clean Energy*. 2019.
- [7] Chen S, Wei Z, Sun G, Cheung KW, Sun Y. Multi-Linear Probabilistic Energy Flow Analysis of Integrated Electrical and Natural-Gas Systems. *IEEE Transactions on Power Systems*. 2017;32:1970-79.
- [8] Lei YK, Hou K, Wang Y, Jia HJ, Zhang P, Mu YF, et al. A new reliability assessment approach for integrated energy systems: Using hierarchical decoupling optimization framework and impact-increment based state enumeration method. *Applied Energy*. 2018;210:1237-50.
- [9] Juanwei C, Tao Y, Yue X, Xiaohua C, Bo Y, Baomin Z. Fast analytical method for reliability evaluation of electricity-gas integrated energy system considering dispatch strategies. *Applied Energy*. 2019;242:260-72.
- [10] Yu W, Song S, Li Y, Min Y, Huang W, Wen K, et al. Gas supply reliability assessment of natural gas transmission pipeline systems. *Energy*. 2018;162:853-70.
- [11] Zhou Y, Gu C, Wu H, Song Y. An Equivalent Model of Gas Networks for Dynamic Analysis of Gas-Electricity Systems. *IEEE Transactions on Power Systems*. 2017;32:4255-64.
- [12] Yang J, Zhang N, Kang C, Xia Q. Effect of Natural Gas Flow Dynamics in Robust Generation Scheduling Under Wind Uncertainty. *IEEE Transactions on Power Systems*. 2018;33:2087-97.
- [13] Bao Z, Chen D, Wu L, Guo X. Optimal inter- and intra-hour scheduling of islanded integrated-energy system considering linepack of gas pipelines. *Energy*. 2019;171:326-40.
- [14] Zlotnik A, Chertkov M, Backhaus S. Optimal control of transient flow in natural gas networks. 2015 54th IEEE Conference on Decision and Control (CDC)2015. p. 4563-70.
- [15] Zlotnik A, Roald L, Backhaus S, Chertkov M, Andersson G. Coordinated Scheduling for Interdependent Electric Power and Natural Gas Infrastructures. *IEEE Transactions on Power Systems*. 2017;32:600-10.
- [16] Cameron I. Using an Excel-Based Model For Steady-State And Transient Simulation. PSIG Annual Meeting. St. Louis, Missouri: Pipeline Simulation Interest Group; 1999. p. 39.
- [17] Herrán-González A, De La Cruz JM, De Andrés-Toro B, Risco-Martín JL. Modeling and simulation of a gas distribution pipeline network. *Applied Mathematical Modelling*. 2009;33:1584-600.

- EnerarXiv-preprint
- [18] Clegg S, Mancarella P. Integrated Modeling and Assessment of the Operational Impact of Power-to-Gas (P2G) on Electrical and Gas Transmission Networks. IEEE Transactions on Sustainable Energy. 2015;6(4):1234-44.
  - [19] Wang S, Shao C, Ding Y, Yan J. Operational reliability of multi-energy customers considering service-based self-scheduling. Applied Energy. 2019;254:113531.
  - [20] Bixby BJ--hn, newcastle, edu. au/ipdu/talks/IPDU Bixby.pdf. CAREY M, CRAWFORD I. Scheduling Trains on a Network of Busy Complex Stations. The Gurobi Optimizer. Transportation Re-search Part B, 2007, 41 (2): 159-178; 2011.
  - [21] Grigg C, Wong P, Albrecht P, Allan R, Bhavaraju M, Billinton R, et al. The IEEE Reliability Test System-1996. A report prepared by the Reliability Test System Task Force of the Application of Probability Methods Subcommittee. IEEE Transactions on Power Systems. 1999;14(3):1010-20.
  - [22] De Wolf D, Smeers Y. The gas transmission problem solved by an extension of the simplex algorithm. Management Science. 2000;46(11):1454-65.
  - [23] Unsihuay C, Lima JWM, Souza ACZd. Modeling the Integrated Natural Gas and Electricity Optimal Power Flow. 2007 IEEE Power Engineering Society General Meeting. Tampa, FL, USA 2007. p. 1-7.