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Improved Multi-ellipsoidal Uncertainty Set-based Robust Optimization for Microgrid with Correlated Wind Power

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ABSTRACT

Reducing the conservatism of robust optimization in an efficient way has always been a challenging problem for microgrid day-ahead dispatch with wind power integrated. In this paper, we address this problem by formulating a multi-ellipsoidal uncertainty set (MEUS) that is able to capture the strong temporal correlation of forecast error of wind power (WPFE) as well as the conditional correlation of WPFE with the forecast power, and combining it with a box uncertainty set. The dimension of each ellipsoid is optimized based on a comprehensive evaluation index to reduce the invalid region, so as to improve the conservatism of the model. A two-stage robust optimization model of microgrid is built based on the improved MEUS, which is cast into a mixed-integer second-order cone programming problem and solved by column and constraint generation algorithm. The effectiveness of the proposed method is verified by numerous simulations with actual data.

Keywords: microgrid, temporal and conditional correlation, improved multi-ellipsoidal uncertainty set, two-stage robust optimization

NONMENCLATURE

C_m	Parameter related to confidence
$\boldsymbol{C}_{lb}$	Lower bound of BUS
$\boldsymbol{C}_{ub}$	Upper bound of BUS
$\boldsymbol{c}$	Coefficient column vector of objective function
D	Total days of historical data
d	Day index of historical data
K	Weight coefficient
$\boldsymbol{L}_m$	Cholesky decomposition $\boldsymbol{R}_m = \boldsymbol{L}_m^{\mathrm{T}}\boldsymbol{L}_m$
M	Number of ellipsoids in MEUS
m	m-th ellipsoid in MEUS
$n_{\max,T,d}$	The maximum number of periods of actual WP included in T-dimensional MEUS in day d
N	Dispatch periods per day
O_T	Comprehensive index
$\boldsymbol{R}_m$	Covariance matrix of m-th ellipsoid
T	Dimension of single ellipsoid in MEUS
$\boldsymbol{U}_{MEUS}$	MEUS
$\boldsymbol{u}$	Uncertain variables related to WP
$\boldsymbol{u}_m$	Uncertain WP vector of m-th ellipsoid
$\boldsymbol{U}_{BUS}$	BUS
$\boldsymbol{U}$	Improved MEUS
$V_{\mathrm{BUS},T,d}$	Intersection volume of BUS and T-dimensional MEUS in day d
$V_{\mathrm{BUS},d}$	Volume of BUS in day d
$\boldsymbol{x}$	Vector of first-stage variables
$\boldsymbol{y}$	Vector of second-stage variables
$\boldsymbol{\mu}_m$	The center of m-th ellipsoid
ζ_T	Integrity index
η_T	Efficiency index
$\boldsymbol{\Omega}(\boldsymbol{x},\boldsymbol{u})$	Feasible region of $\boldsymbol{y}$ for fixed $\boldsymbol{x}$ and $\boldsymbol{u}$
Γ	Budget of uncertainty

1. INTRODUCTION

The way to deal with the uncertainties of renewable energy source (RES) for day-ahead dispatch of microgrid (MG) has significant influence on system economy and reliability [1]. Unlike the stochastic optimization and scenario-based methods that require accurate probability distribution of RES generation, robust optimization (RO) addresses the uncertainties via a given set and seeks the optimal solution for all possible realization of RES generation, thus is widely used in practical engineering.

The key of RO is to establish a proper uncertainty set that can capture the characteristic of uncertain variables with limited region as small as possible. Ellipsoidal uncertainty set (EUS) is regarded as an efficient solution for this objective due to its ability of depicting the correlation among variables. Ref [2] introduced different approaches to construct uncertainty sets based on historical data for robust Unit Commitment (UC) problem. Ref [3] adopted the EUS to fit the spatial-temporal correlated wind power and presented an affinely adjustable robust transmission constrained UC model, but only the correlation of adjacent periods is considered. Ref [4] transformed the EUS into a convex hull that contains the extreme scenarios to avoid a higher-order optimization problem when dealing with the economic dispatch of distribution networks that is nonlinear. However, the conservatism of the proposed method will be significantly increased when considering the temporal correlation of uncertain variables with higher dimensions, as the convex hull contains more region than EUS. Ref [5] utilized the minimum volume enclosing ellipsoid algorithm to construct the uncertainty set of correlated wind power. It formulated a mixed-integer second-order cone programming (MISOCP) problem for the stochastic and RO of energy and reserve dispatch problem. However, the multi-periods economic dispatch as well as temporal correlations of RES generation is not considered.

In this paper, we formulate the possible realization of wind power as an improved multi-ellipsoidal uncertainty set (IMEUS). The shape of IMEUS is up to the temporal correlation of forecast error of wind power (WPFE) as well as the conditional correlation of WPFE with the forecast power. Then, we use the IMEUS to establish a two-stage RO model to realize a multi-periods economic dispatch of MG.

2. FORMULATION OF IMEUS

The IMEUS proposed in this paper is data-driven based on historical wind power (WP) as well as the forecast value. We apply the conditional normal copula (CNC) function to establish the relationship of WPFE with the forecast power [6], so that the samples of the possible realization of WP can be updated based on the day-ahead forecast power.

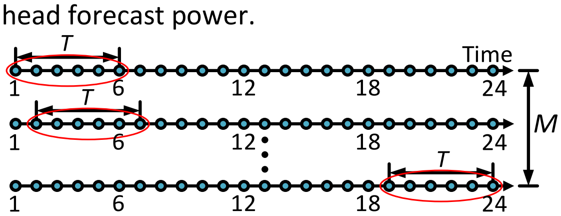


Fig 1 Rolling method of MEUS

The traditional method considers a 24-dimensional EUS to model the temporal correlations for the day-ahead dispatch problem. However, weak correlation among distant periods and high dimensionality make EUS conservative. In this paper, we firstly split the 24-dimensional ellipsoid into M T-dimensional ellipsoids, as demonstrated in Fig 1. Each ellipsoid contains T adjacent periods of WP, which has strong correlation. The MEUS is the intersection of these M ellipsoids, i.e.:

$$\boldsymbol{U}_{MEUS} = \left\{ \boldsymbol{u} \middle| \begin{array}{l} (\boldsymbol{u}_m - \boldsymbol{\mu}_m)^{\mathrm{T}} \boldsymbol{R}_m^{-1} (\boldsymbol{u}_m - \boldsymbol{\mu}_m) \le C_m, \\ m = 1, 2, \cdots, M \end{array} \right\} \tag{1}$$

Then, we formulate a bus uncertainty set (BUS) that also considers the conditional correlation of WPFE with the forecast power as:

$$\boldsymbol{U}_{BUS} = \left\{ \boldsymbol{u} \middle| \boldsymbol{C}_{lb} \le \boldsymbol{u} \le \boldsymbol{C}_{ub} \right\} \tag{2}$$

To overcome the shortcomings of BUS with larger invalid region and the conservatism caused by the extreme scenarios included in the tail of MEUS, the proposed IMEUS is formulated as the intersection of MEUS and BUS.

The dimension T is determined based on evaluation indexes to optimize the performance of MEUS. The integrity index ζ_T in (3) and efficiency index η_T in (4) are utilized to form a comprehensive index O_T in (5). The integrity index is used to ensure that more actual data is contained in MEUS, and the efficiency index is used to limit the volume of MEUS to reduce the conservatism.

$$\zeta_T = \frac{1}{D} \sum_{d=1}^{D} \frac{n_{\max,T,d}}{N} \tag{3}$$

$$\eta_T = 1 - \frac{\log_{10}\left(\frac{1}{D} \sum_{d=1}^{D} V_{\mathrm{BUS},T,d} \right)}{\log_{10}\left(\frac{1}{D} \sum_{d=1}^{D} V_{\mathrm{BUS},d} \right)} \tag{4}$$

$$O_T = K \cdot \zeta_T + (1 - K) \cdot \eta_T \tag{5}$$

3. TWO-STAGE RO BASED ON IMEUS

In this paper, a two-stage RO model based on IMEUS is formulated for a MG including WP. The detailed model of the two-stage RO can be found in Ref [7]. The compact form is expressed as (6).

$$\min_{\boldsymbol{x}} \left\{ \max_{\boldsymbol{u} \in \boldsymbol{U}} \min_{\boldsymbol{y} \in \boldsymbol{\Omega}(\boldsymbol{x},\boldsymbol{u})} \boldsymbol{c}^{\mathrm{T}} \boldsymbol{y} \right\} \tag{6}$$

$$\boldsymbol{U} = \left\{ \boldsymbol{u} \middle| \begin{array}{ll} \left\| C_m^{-1/2} \boldsymbol{L}_m (\boldsymbol{u}_m - \boldsymbol{\mu}_m) \right\|_2 \le 1, & \boldsymbol{C}_{lb} \le \boldsymbol{u} \le \boldsymbol{C}_{ub}, \\ u_t \ge B_t \cdot u_{F,t}, & \sum_{t=1}^{24} B_t \ge \Gamma, \\ t = 1, 2, \cdots, 24 & m = 1, 2, \cdots, M \end{array} \right\} \tag{7}$$

The inner max-min problem is cast into a MISOCP problem, and the binary expansion method is applied to make the model convex [5]. In addition, the budget of uncertainty Γ is introduced to adjust the conservatism degree of the optimal solution. Γ is an integer between 0 and 24, and defined as the minimum number of periods when $\boldsymbol{u}$ is taken as the forecast power of WP in the decision-making process. Therefore, the model is more conservative when Γ is smaller. Finally, $\boldsymbol{U}$ is transformed into an adjustable form as (7), where the MEUS of (1) is rewritten as second-order cone form by Cholesky decomposition.

4. CASE STUDY

4.1 Temporal correlation analysis

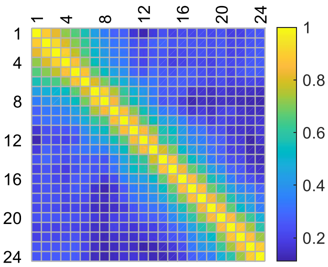


Fig 2 Temporal correlation of WPFE

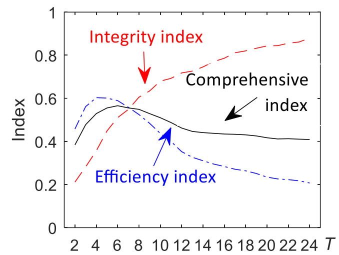


Fig 3 MEUS evaluation indexes at different T

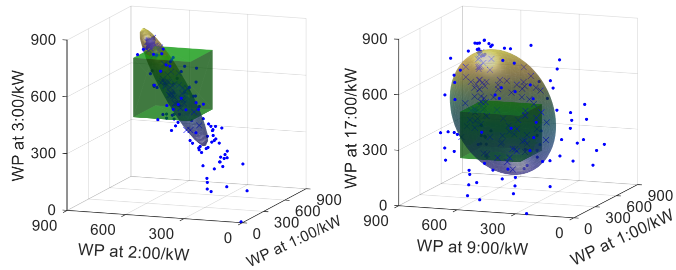
We use an actual WP data from a wind farm in China to verify the effectiveness of the proposed method. Fig 2 shows linear correlation coefficients of WPFE, representing temporal correlation among different periods. It is obvious that the strong correlation only exists among several adjacent periods.

The results of MEUS evaluation indexes at different T are shown in Fig 3, where K is set as 0.3. T=24 represents the EUS obtained by traditional method, whose ζ_T is higher but η_T is much lower. Both ζ_T and η_T increase along with the increase of T when $T \leq 5$, indicating the necessity to consider strong temporal correlation. It can been seen that the comprehensive index O_T is the best when T is selected as 6.

4.2 BUS and EUS at different time intervals

To further illustrate the influence of the temporal correlation on the performance of the formulated uncertain set, we provide the 3-dimensional BUS and EUS at different time intervals using the samples generated by CNC. Both BUS and EUS consider the conditional correlation of WPFE and the forecast power, while EUS considers the temporal correlation of WPFE at the same time. Fig 4 presents the scatter of the samples as well as the constructing EUS and BUS. The WP is standardized with the reference value of 100kW, and Table 1 shows the volume of BUS and EUS respectively.

As shown in Fig 4 and Table 1, the WP at 1:00, 2:00 and 3:00 are highly correlated, as a result, EUS has a smaller volume than BUS. However, the WP at 1:00, 9:00 and 17:00 are almost independent, and EUS shows even worse performance than BUS. Therefore, EUS can provide a better performance than BUS only if the strong correlation of periods is considered.



(a) Adjacent periods (b) Distant periods

Fig 4 Scatter of WP samples and 3-dimensional BUS and EUS at different time intervals

Table 1 Volumes of sets in Fig 4

	a)	b)
Volume of BUS	28.97	24.17
Volume of EUS	13.42	101.46

It can be also seen in Fig 4(a) that the EUS contains worse scenarios of WP than BUS. The WP at 1:00, 2:00 and 3:00 are all very small as the temporal correlation is considered, which may make the solution of RO more conservative and will be more serious when the dimensionality increases. It motivates us to combine the advantages of the EUS and BUS.

4.3 RO with different uncertainty sets

We define 5 optimal dispatch models of MG, i.e.:

Model 1: Deterministic optimization model.

Model 2: RO model based on BUS.

Model 3: RO model based on EUS.

Model 4: RO model based on MEUS, T=6.

Model 5: RO model based on IMEUS, T=6.

Table 2 Average cost and unbalanced power of each model

	Model 1	*Model 2*	*Model 3*	*Model 4*	*Model 5*
Day-ahead cost (RMB)	5380	7313	7651	7485	7152
Balancing cost (RMB)	2341	387	144	260	510
Total cost (RMB)	7721	7700	7795	7745	7662

The budget of uncertainty Γ is set as 14 for *Model 2* to *Model 5*. The parameters of MG are cited from Ref [7]. We use WP data of 8 months to train CNC model, and use WP data in the 9th month to test the results, as shown in Table 2.

In Table 2, the balancing cost is the difference between cost for purchasing additional electricity and the revenue from selling excess electricity. It is related to the unbalanced power between the day-ahead exchange power plan and the actual exchange power of MG with the distribution networks. It can be seen that the day-ahead cost of *Model 1* is the lowest as no uncertainty is considered, but with a large balancing cost. The balancing cost of *Model 2* to *Model 5* has significantly reduced, indicating that the cost for compensating the unbalanced power is decreased after considering the uncertainty of WP. *Model 3* and *Model 4* are too conservative so that the total cost is even higher than the deterministic optimization method. As *Model 4* based on MEUS only considers the strong temporal correlation among adjacent periods, the conservatism is improved compared with *Model 3*. However, the performance of MEUS is still limited owing to the reason discussed in *4.2*. *Model 5* combines the advantages of BUS and MEUS, and has realized a better trade-off between economy and conservatism with the lowest total cost.

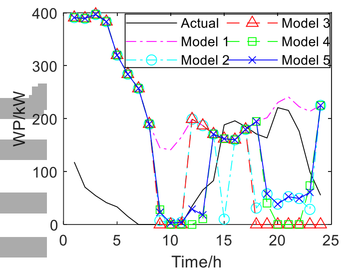


Fig 5 Realization of WP

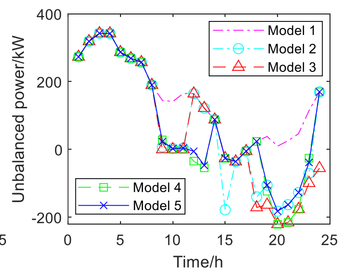


Fig. 6 Unbalanced power

Fig 5 and Fig 6 demonstrate the realization of WP in day-ahead dispatch and the unbalanced power for each model in a certain day. Compared *Model 2* with other EUS-based models, the main difference is that the WP at 15:00 is selected as an independent worse scenario in *Model 2*. While the EUS-based models prefer to select continuous periods as worse scenarios due to the temporal correlation. *Model 3* and *Model 4* contain many extreme scenarios during 18:00 to 24:00, making the solution too conservative. *Model 5* limits the conservatism through the boundary of BUS and considers the temporal correlation via MEUS, which makes the realization of WP in day-ahead dispatch more in line with the actual scenario.

In Fig 6, it is obvious that the solution determined by deterministic optimization method needs to purchase electricity from distribution networks at most of the day, corresponding to a higher balancing cost. *Model 3* and *Model 4* have to sell the excess electricity to distribution networks with a lower price during 18:00 to 24:00, causing loss of interest. *Model 5* shows the best performance with minimum unbalanced power, illustrating the advantage of the proposed IMEUS method.

5. CONCLUSIONS

This paper proposes an IMEUS modeling method based on CNC model, which considers the strong temporal correlation of WPFE among adjacent periods and the conditional correlation of WPFE and forecast power. The IMEUS combines the advantages of BUS and MEUS, and is applied to a two-stage RO model for day-ahead dispatch of a MG. Compared with traditional EUS and BUS, the RO model using IMEUS can improve the conservatism and thus increase the economic benefits in our simulation. Although our work considers the correlated wind power, it needs extensive historical data to formulate the IMEUS. In future work, we will consider more uncertain factors, e.g. electricity price, and try to simplify the model and improve the training efficiency for limited data.

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