

# A Well Test Model Study of Multi Fracture-vug Combination for Fractured Vuggy Carbonate Reservoirs

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## ABSTRACT

Fractured vuggy carbonate reservoirs usually develop several large-scale caves which are the main reservoir space of oil-gas. The volume and position of caves are important parameters for production guidance, but there are no effective well test methods of calculating the cave volume of multi-cave reservoirs at present. This paper establishes a multi fracture-vug composite model for fractured vuggy carbonate reservoirs, divides reservoirs into various fracture-vug seepage zones according to the cave distribution, establishes a mathematical model through the coupling of seepage zones, and then obtains analytical solutions of the mathematical model using dimensionless and Laplace transformation. By plotting the log-log type curves of well test, the paper analyzes the flow characteristics of the model. It is found that the pressure derivative of inter-porosity flow segment from caves to fractures is presented as a V-shaped concave, and the pressure and pressure derivative curves at the fracture flow segment take on the parallel lines with the slope of 0.5. A parameter sensitivity analysis indicates: the bigger the cave is, the steeper the V-shaped concave of the pressure derivative curve goes; the farther the distance between cave and wellbore is, the more hysteretic the V-shaped concave becomes. Finally, the paper demonstrates the field example analysis and application by fitting measured pressure data to calculate the volume and distance of caves.

**Keywords:** Fractured vuggy reservoirs; Well test method; Transient pressure analysis

## NONMENCLATURE

### Symbols

$p_f$	fracture pressure, MPa
$p_v$	cave pressure, MPa
$p_w$	wellbore pressure, MPa

$p_i$	initial pressure, MPa
$\phi_f$	Fracture porosity, dimensionless
$x$	Position coordinates, m
$K_f$	Fracture permeability, D
$t$	Time, h
$\mu$	oil viscosity, mPa·S
$C_{ft}$	Fracture compression factor, MPa <sup>-1</sup>
$C_{vt}$	Cave compression factor, MPa <sup>-1</sup>
$q_o$	total production of the well, m <sup>3</sup> /d
$B$	oil formation volume factor, m <sup>3</sup> /m <sup>3</sup>
$A_f$	Fracture cross-sectional area, m <sup>2</sup>
$C$	storage constant, MPa <sup>-1</sup>
$S$	skin factor, dimensionless
Superscript	
$j$	The $j^{\text{th}}$ seepage zone
Subscript	
$j$	The $j^{\text{th}}$ cave
$D$	Dimensionless

## 1. INTRODUCTION

Fractured vuggy carbonate reservoirs develop many large-scale caves and fractures. Caves are the primary reservoir space, and fractures are great seepage channels connecting caves. For river reservoirs and fault controlling dissolution reservoirs, it is often the case that the reservoirs grow several caves and their positions have certain directionality [1]. Identifying the volume and distance of caves is of great significance for production guidance in the later production period.

Since the 1960s, many scholars have proposed and developed the well test theory of multi-continuum medium for carbonate reservoirs [2-4]. They think that reservoirs can be regarded as the superposition of several continuous medium systems in space and inter-porosity flow exists in these systems. However, the

multi-medium model is based on the continuum assumption, which is not applicable to large-scale fracture-vug reservoirs and fails to explain the cave volume. From the beginning of the 20th century, some scholars take discrete fracture-vug distribution into account and establish well test models that successfully explain the fracture-vug parameters [5-7]. Gao holds that the cave flow is Barree-Conway high-speed non-Darcy flow and establishes the numerical well test model for well drilling in a cave [5]. Lu adopts the wave equation to represent the flow in a large-scale cave with an analytical well test model built for well drilling in the cave [6]. And Lin Ran gives a well test analysis for the dual fracture-vug serial-parallel composite model [7]. However, most of the well testing models above focus on one or two caves, which are inapplicable to the multi fracture-vug reservoirs.

This paper expands the dual fracture-vug composite model to the multi fracture-vug combined model. We divide reservoirs into multiple seepage zones and establish a mathematical model for analytical solutions through the coupling of seepage zones, which resolves the well test problem of multi fracture-vug reservoirs.

## 2. PHYSICAL MODEL AND ASSUMPTIONS

The physical model is shown in Fig 1. The fractured vuggy reservoir with a couple of caves can be Simplified to the physical model.

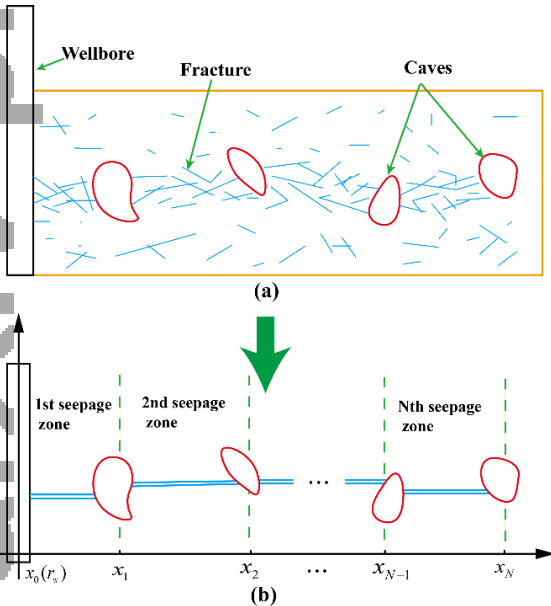


Fig 1 Schematic diagram of the physical model: (a) A fractured vuggy reservoir with a couple of caves. (b) Simplified well test physical model.

We can see that several large caves are distributed in the formation along a certain direction, connected in

series by large-scale fractures. Among them, Fracture 1 is in connection with the wellbore. The formation is divided into N fracture-vug seepage zones according to the cave distribution. Physical model assumptions are as follows:

(1)the caves are filled with single-phase oil; (2)The vertical production well is produced at a constant rate; (3)The large scale caves are regarded as equipotential body; In the fracture of every seepage zone, the flow of fluid is isothermal Darcy flow; (4)total compressibility (rock and fluid) is low and a constant; (5)considering the impact of wellbore storage and skin factor, ignoring the impact of gravity and capillary forces; (6) At time  $t = 0$ , the pressure is uniformly distributed in a reservoir, equal to initial pressure.

## 3. MATHEMATICAL MODEL AND SOLUTION

### 3.1 Mathematical Model

the governing differential equations for the fracture of the  $j^{\text{th}}$  zone:

$$\frac{\partial^2 p_f^j}{\partial x^2} = \frac{\phi_f^j \mu_f C_{ft}}{3.6K_f^j} \cdot \frac{\partial p_f^j}{\partial t} \quad (1)$$

$$(x_{j-1} \leq x < x_j) \quad (j=1,2,3,\dots,N)$$

Infinite condition:

$$p_f^j|_{t=0} = p_v^j|_{t=0} = p_i \quad (j=1,2,3,\dots,N) \quad (2)$$

Inner boundary conditions:

$$-\frac{86.4K_f^1}{\mu} \cdot A_1 \cdot \frac{\partial p_f^1}{\partial x} \Big|_{x=x_0} = q_o B \quad (3)$$

External boundary conditions:

$$-\frac{86.4K_N}{\mu_N} \cdot A_N \cdot \frac{\partial p_f^N}{\partial x} \Big|_{x=x_N} = V_N C_{vt} \frac{dp_v^N}{dt} \quad (4)$$

Pressure continuity equation:

$$p_f^j \Big|_{x=x_j} = p_f^{j+1} \Big|_{x=x_j} = p_v^j \quad (5)$$

Flow continuity equation(Coupling condition):

$$\frac{86.4K_f^j}{\mu} \cdot A_f^j \cdot \frac{\partial p_f^j}{\partial x} \Big|_{x=x_j} = \frac{86.4K_f^{j+1}}{\mu} \cdot A_f^{j+1} \cdot \frac{\partial p_f^{j+1}}{\partial x} \Big|_{x=x_j} \quad (6)$$

$$+V_j C_{vt} \frac{dp_v^j}{dt} \quad (j=1,2,3,\dots,N)$$

### 3.2 Laplace transform of mathematical models

Define dimensionless quantities:

$$x_D = \frac{x}{r_w}, \quad \omega_{(f,v)} = \frac{\phi_{(f,v)} C_{(f,v)t}}{\phi_f C_{ft} + \phi_v C_{vt}}, \quad A_D^j = \frac{A_j}{r_w^2}$$

$$P_{(w,f,v)D} = \frac{86.4 \times K_f^1 r_w}{q_o \mu B} (p_i - p_{(w,f,v)}), \quad V_D^j = \frac{V_j}{r_w^3}$$

$$t_D = \frac{3.6K_f^1 t}{\mu(\phi_f^1 C_{ft} + \phi_v^1 C_{vt}) r_w^2}, M_1^j = K_f^j / K_f^1.$$

This paper omits the dimensionless model, and the Laplace transform of the dimensionless mathematics model as equation (7)-(11):

$$\frac{d^2 \overline{p_{FD}^j}}{dx_D^2} = \omega_f^j s d \overline{p_{FD}^j} \quad (7)$$

$$(x_D^{j-1} \leq x_D < x_D^j) (j=1,2,3...N)$$

$$\overline{p_{FD}^j} \Big|_{x_D=x_{jD}} = \overline{p_{FD}^{j+1}} \Big|_{x_D=x_{jD}} = \overline{p_{VD}^j} \quad (j=1,2,3...N-1) \quad (8)$$

$$\frac{d \overline{p_{FD}^j}}{dx_D} \Big|_{x_D=x_{jD}} = -\frac{1}{A_{FD}^1 \cdot s} \quad (9)$$

$$\frac{d \overline{p_{FD}^N}}{dx_D} \Big|_{x_D=x_{ND}} = -\frac{\omega_v^N V_D^N}{A_D^N} s \overline{p_{VD}^N} \quad (10)$$

$$M_1^j \frac{d \overline{p_{FD}^j}}{dx_D} \Big|_{x_D=x_D^j} = M_1^{j+1} \frac{d \overline{p_{FD}^{j+1}}}{dx_D} \Big|_{x_D=x_D^j} + \frac{\omega_v^j V_D^j}{A_D^j} s \overline{p_{VD}^j} \quad (j=1,2,3...N-1) \quad (11)$$

The general solution of equation (8) is:

$$\overline{p_{FD}^j} = B_j e^{\sqrt{\omega_f^j \cdot s} \cdot x_{jD}} + C_j e^{-\sqrt{\omega_f^j \cdot s} \cdot x_{jD}} \quad (12)$$

$$(x_D^{j-1} \leq x_D < x_D^j) (j=1,2,3...N)$$

The  $B_j$  and  $C_j$  are undetermined constants, Substituting into the solution equation (8)(9)(10)(11), there are  $2N$  unknown constants and  $2N$  equations, so they can be solved by simultaneous solution. When  $x_D^j = x_D^0$ , The wellbore pressure can be expressed as:

$$\overline{p_{SD}} = B_1 e^{\sqrt{\omega_f^1 \cdot s} \cdot x_{wD}} + C_1 e^{-\sqrt{\omega_f^1 \cdot s} \cdot x_{wD}} \quad (13)$$

When considering the effects of wellbore storage and skin factor, we can use the following equation derived from Duhamel theory to calculate:

$$\overline{p_{wD}} = \frac{s \overline{p_{SD}} + S}{s + C_D s^2 (s \overline{p_{SD}} + S)} \quad (14)$$

#### 4. TYPICAL WELL TEST CURVES

By the Stehfest numerical inversion method, taking the triple fracture-vug model as an example, we can get the log-log well test type curves as Fig 2 shows, and the model can be divided into 8 flowing stages:

Stage I: Wellbore storage effect stage, the pressure and derivative curves are straight lines with slope of 1;

Stage II: Fracture linear flowing stage, the pressure and derivative curves are parallel lines with slope of 0.5. This stage usually cannot be observed easily due to the skin effect;

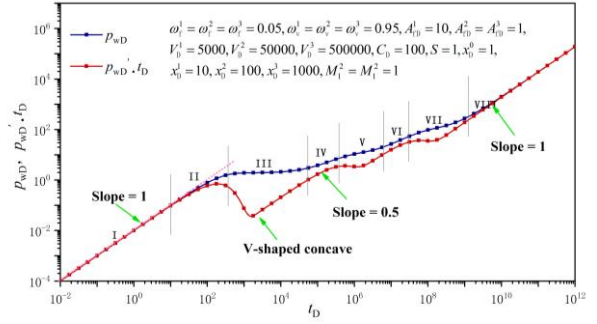


Fig 2 The type curves of triple fracture-vug model

Stage III: Inter-porosity flow stage of the cave in Fracture-vug I Zone, the first cave starts to feed fluid toward fractures causing the rapid decrease of derivative curves and the formation of V-shaped concave;

Stage IV: Fracture linear flowing stage in Fracture-vug II Zone where the pressure and derivative curves are parallel lines with the slope of 0.5;

Stage V: Inter-porosity flow stage of the cave in Fracture-vug II Zone and the curve shape is the same as that of Stage III;

Stage VI and Stage VII: Fracture linear flowing stage and the inter-porosity flow stage of the large cave in Fracture-vug III Zone and the curve shape is the same as that of Stage IV and Stage III.

Stage VIII: Pseudo-steady flowing stage of the overall system where the pressure and pressure derivative curves coincide with each other as a straight line with the slope of 1.

#### 5. PARAMETER SENSITIVITY ANALYSIS

We take the dual fracture-vug composite model as the example to analyze the influence of cave volume and distance on the typical curves.

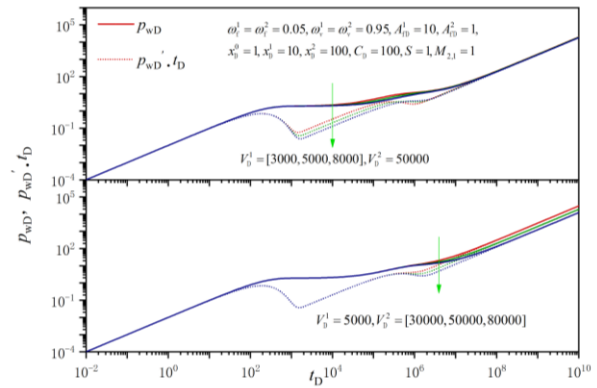


Fig 3 Effect of the volume of Caves

As Fig 3 shows, the volume of Cave 1 increases, the first concave of the pressure derivative curve goes deeper, the inter-porosity flow of Cave 1 increases. Similarly, as the volume of Cave 2 increases, the second

concave goes deeper. This conclusion applies to the volume of any cave in the multi fracture-vug models.

As Fig 4 shows, the distance between Cave 1 and wellbore increases, the first concave moves right which indicates the response lag of inter-porosity flow of Cave 1. In the case of the same cave volume, the farther the distance of cave 1 is, the smaller the concave degree of the concave response segment is. Similarly, as the distance of Cave 2 increases, the first concave of the log-log curve moves right. This conclusion applies to the distance of any cave in the multi fracture-vug models.

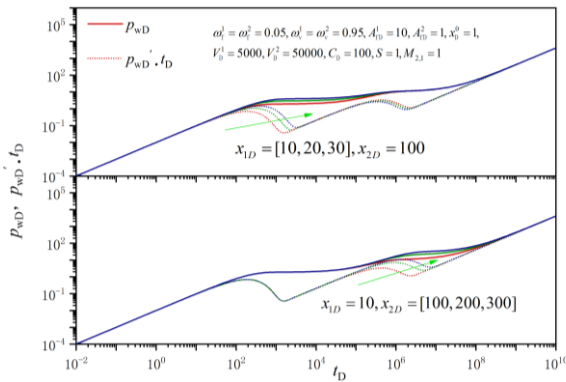


Fig 4 Effect of the distance of Caves

## 6. FIELD EXAMPLE

Well T40 is a production well in Tahe oilfield, China. According to well logging and seismic data, the fracture-vug system near the wellbore is well developed, and the far-wellbore may have reservoirs of a certain scale. We adopt the triple fracture-vug model to fit the buildup pressure data, and the Curve fitting map is shown in Fig 5. We calculate that the respective volumes of Cave 1 and Cave 2 are 7489m<sup>3</sup> and 56324m<sup>3</sup> and the respective cave distances are 25.1m and 189.2m.

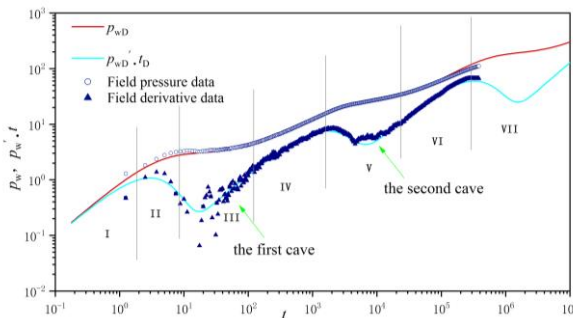


Fig 5 Type curve matching of the T40 well

## 7. CONCLUSIONS

(1) A multi fracture-vug composite model for fractured vuggy carbonate reservoirs was established. We divide reservoirs into various fracture-vug seepage

zones, establishes a mathematical model through the coupling of zones and obtains analytical solutions;

(2) By plotting the log-log type curves, we analyzed the flow characteristics of the model. The pressure derivative of inter-porosity flow segment from caves to fractures is presented as V-shaped concave, the pressure and pressure derivative curves at the fracture flow segment take on the parallel lines with the slope of 0.5.

(3) A parameter sensitivity analysis indicates: the bigger the cave is, the steeper the V-shaped concave of the pressure derivative curve goes; the farther the distance of cave is, the more hysteretic the V-shaped concave becomes.

(4) The triple fracture-vug model is applied to fit buildup the data of well T40. We calculate that the respective volumes of Cave 1 and Cave 2 are 7489 m<sup>3</sup> and 56324 m<sup>3</sup> and the cave distances are 25.1m and 189.2m.

## 8. ACKNOWLEDGEMENT

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