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# Power Flow Analysis of Cogeneration System Based on Improved Modelling Method of District Heating Network

Zhang Chen, Jun Liu*, Xinglei Liu

Shaanxi Key Laboratory of Smart Grid, School of Electrical Engineering, Xi'an Jiaotong University, Xi'an, Shaanxi 710049, China

## ABSTRACT

The researches on Integrated Energy System continue to thrive nowadays, since it can promote the use of renewable energy and increase energy consumption efficiency. In this paper, a novel modelling method of District Heating Network (DHN) considering the change of the thermodynamic states of the working fluid is proposed, based on the basic components in DHN, including the pipelines, pressure sources (PSs) and junctions. And a corresponding DHN model constructed of these components is established. By applying this method, DHNs with diverse topologies, even with different supply and return networks, can be analyzed. In order to solve the equations efficiently, Newton's method with its partial derivatives obtained by a numerical method is applied, and an appropriate set of initial values is also discussed. Finally, case studies of a cogeneration system are performed to demonstrate the validity of the proposed models as well as the analyzing methods.

**Keywords:** Cogeneration systems, District heating network models, thermodynamic properties of working fluid, modified Newton's method.

## NONMENCLATURE

| *Abbreviations* | |
|---|---|
| DHN | District Heating Network |
| MES | Multi-Energy Systems |
| PS | Pressure Source |
| PSN | Power Supply Network |

## 1. INTRODUCTION

Due to the reasons of fossil resource depletion, climate change, and policy incentives, sustainable energy including wind power and photovoltaic has been growing rapidly [1], while energy conservation and low-carbon emissions have attracted worldwide attention [2,3]. In order to achieve lower carbon emissions and higher efficiency of energy utilization, multi-energy systems (MESs) have been widely studied and applied in recent decades [4]. The integration of gas, electricity and heat carriers can offer great potential for better managing various energy resources, decreasing consumption and enabling a higher share of renewable energy [5].

Mathematical modelling of the Integrated MESs has been studied extensively so far, including both district level and multi-region level energy networks [6]. Energy hub concept was proposed in 2007 [7] to model the district level energy networks. It represents an interface between diverse energy infrastructures using a coupling matrix [8]. A combined analysis of electricity and heat networks had been conducted using a method similar to calculating a load flow problem in an AC power grid [9]. Such analogy between thermal network and electrical network is also used to model enthalpy transfer using electric circuit equivalents [10], so that one can use the software originally designed for power grid to solve the heat transfer problems. Besides, the dynamics of DHNs also start to get noticed by researchers. A uniform framework for modeling heat losses and transfer delays in the Laplace domain was proposed in [11]. Apart from combined heat and power network, a matrix modelling method of trigeneration systems, where cooling energy is included, was proposed in [12]. In the meantime, dynamic interactions of the three energy systems (gas, heat and power) are modelled with the help of the singular perturbation theory [13].

However, previous researches on DHN, focus merely on the heat networks whose supply pipe is in parallel with the return pipe, the components between them are modelled merely by the pressure drop $\Delta p$ and the heat power $Q$. In addition, the DHNs are mostly modelled from the perspective of network rather than specific components, which cannot take into account of the detailed losses of the heat transmission process. To overcome these deficiencies, a novel modelling method of DHN considering the change of the thermodynamic states of the working fluid is proposed.

## 2. MODELLING OF THE BASIC COMPONENTS IN DHN

### *2.1 Determining the thermal dynamic properties of working fluid according to real-time working state*

The working fluid in DHN is typically hot water, which owns all kinds of the physical properties of fluids, like specific internal energy $u$, density $\rho$, kinematic viscosity $\nu$, thermal conductivity $k$, specific heat capacity $c_p$. They are all functions of the working state of the fluid. Since phase transition does not occur during a normal operation process of DHN, the states can be well represented by the pressure and temperature ($p$, $T$) of the working fluid.

Since the normal working region of DHN is not extensive, with temperature at approximately 293.15~373.15K and pressure at approximately $p_1 = 0.1\text{MPa}$ ~ $p_2 = 10\text{MPa}$, the fitting method is chosen in this paper for simplicity. In this region, by fitting according to the water property tables, the following polynomial expressions of (1) to (8) proposed below can guarantee an accuracy of over 95%.

$$z = \frac{T - 323.2}{33.17} \tag{1}$$

$$u = 138.4z + 208.8 \tag{2}$$

$$\rho = -3.818z^2 - 14.19z + 990.2 \tag{3}$$

$$\nu = 39.5154z^4 - 93.7315z^3 + 121.2974z^2 - 275.181z + 554.8359 \tag{4}$$

$$k = -10.16z^2 + 39.39\text{z} + 645.4 \tag{5}$$

$$f_1(z) = 3.064z^4 - 5.893z^3 + 8.473z^2 + 11.83z + 4181 \tag{6}$$

$$f_2(z) = 2.4397z^4 - 3.1862z^3 + 4.9103z^2 + 14.349z + 4159.3 \tag{7}$$

$$c_p(p,T) = f_1(z) + \frac{f_2(z) - f_1(z)}{p_2 - p_1}(p - p_1) \tag{8}$$

where: $z$ is an intermediate variable which has the same unit as $T$. The units in these expressions are as follows. Temperature: K; specific internal energy: kJ/kg; density: kg/m$^3$; kinematic viscosity: 10$^{-9}$m$^2$/s; thermal conductivity: mW/(m·K); specific heat capacity: J/(kg·K).

After the basic properties are calculated using the expressions above, some other properties can also be derived.

$$\mu = \rho \nu \tag{9}$$

$$\text{Pr} = \frac{c_p \mu}{k} \tag{10}$$

$$\text{Re} = \frac{vD}{\nu} \tag{11}$$

where: $\mu$ is the dynamic viscosity; $\text{Pr}$ is the Prandtl number; $\text{Re}$ is the Reynolds number; $v$ is the velocity of fluid; $D$ is the hydraulic diameter of a pipe.

### *2.2 Detailed modelling of pipelines*

DHN is essentially a complicated pipe network. The pipeline not only undertakes the task of transporting working fluid, but also has the ability of transferring heat in its radial direction. The heat exchange system of heat source, heat station and heat transfer pipeline system are uniformly modelled as a heat exchange pipeline model in this paper, with the equivalent heat dissipation coefficient being modelled as thermal resistance.

This paper introduces a virtual internal node $I$ so that the pipe can be divided into 2 segments. Actually, the number of segments can be even larger, with higher accuracy but lower speed of calculation. When the fluid inside the pipeline is working under the steady state, the pressure drop relations are:

$$p_A - p_I = \Delta p_{AI} \tag{12}$$

$$p_B - p_I = \Delta p_{BI} \tag{13}$$

where: subscripts $A$, $B$ denote the two ports of the pipe. The pressure losses within different flow regions are different. The loss equations can be found in [14].

Since an internal node is introduced, the density $\rho$ and kinematic viscosity $\nu$ in the pressure loss equations will be the related parameters at the internal node:

$$\rho = \rho_I = \rho(T_I) \tag{14}$$

$$\nu = \nu_I = \nu(T_I) \tag{15}$$

The conversion of mass under the steady state is:

$$\dot{m}_A + \dot{m}_B = 0 \tag{16}$$

where: $\dot{m}$ is the mass flow rate into the port. Subscripts $A$, $B$ denote the two ports of the pipe. When fluid flows into a port, its mass flow rate is regarded as positive, according to the specified reference direction.

The enthalpy flow rate $\dot{H}$ into a port of pipeline is:

$$\dot{H} = \dot{m}\left(u + \frac{p}{\rho}\right) \quad (17)$$

The reference direction is the same for the enthalpy and mass flow rate. When fluid flows into the port, $u$, $p$ and $\rho$ are supposed to be the values at the port. When fluid outflows from the port, $p$ and $\rho$ are supposed to be values at the port, while $u$ will be the value at the internal node. And the energy conservation equation for the pipe will be:

$$\dot{H}_A + \dot{H}_B + Q_H = 0 \quad (18)$$

According to the heat transfer process of a pipe [15,16], the heat transfer $Q_H$ can be divided into the convection $Q_{conv}$ and conduction $Q_{cond}$:

$$Q_H = Q_{conv} + Q_{cond} \quad (19)$$

$$Q_{conv} = -\dot{m}c_p\left(T_A - T_H\right)\left(1 - e^{-NTU}\right) \quad (20)$$

$$Q_{cond} = \frac{kS_{surf}}{D}\left(T_H - T_I\right) \quad (21)$$

where: $S_{surf}$, $D$, $T_H$ are the surface area ,hydraulic diameter, surface temperature of the pipe. To get the convection heat transfer, the convective heat transfer coefficient is needed (included in $NTU$: the number of transfer unit), which is related with the Nusselt number $\mathrm{Nu}$ and Reynolds number $\mathrm{Pr}$. As mentioned previously, they can be derived from the basic states, which may be calculated by:

$$M_{avg} = \frac{M\left(p_I, T_A\right) + M\left(p_I, T_I\right)}{2} \quad (22)$$

where: $M$ can be replaced by $k$, $c_p$ and $\mu$.

The heat transfer process outside the pipe wall can be modelled by thermal resistance, as shown in Fig 1. Fig 1(a) is more often used, which consists of a temperature source and a thermal resistance.

$$T_{HS} - T_H = R_H Q_H \quad (23)$$

where: $T_{HS}$ is the temperature of the outer surface of the pipe, $R_H$ is the equivalent radial thermal resistance of the pipe.

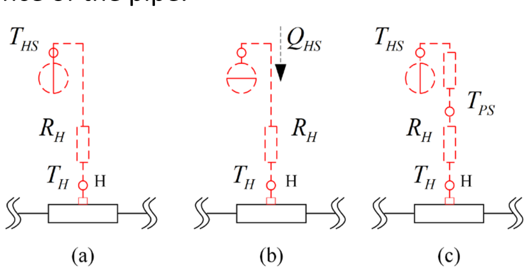



Fig 1 Schematic diagram of heat transfer process outside the pipe wall

### 2.3 Modelling of the pressure source

The function of a PS is to boost the pressure across its two ports connected, with pressure difference $p_{cmd}$:

$$p_B - p_A = p_{cmd} \quad (24)$$

Its mass conservation equation is the same as (16), while the energy conservation equation is:

$$\dot{H}_A + \dot{H}_B + \dot{W} = 0 \quad (25)$$

Due to no requirement to calculate the pressure loss and heat transfer in a PS, ($p_I$, $T_I$) can be simplified to only one variable $u_I$. And then the enthalpy flow equation is the same as (17).

The mechanical power $\dot{W}$ is calculated by:

$$\dot{W} = \dot{m}_A \frac{p_{cmd}}{\left(\frac{\rho\left(T_A\right) + \rho\left(T_B\right)}{2}\right)} \quad (26)$$

For a motor-driven PS, the conversion efficiency is:

$$\eta_e = \dot{W} / \dot{E} \quad (27)$$

where: $\dot{E}$ is the input electric power of the PS.

### 2.4 Modelling of junction

When a stream of fluid needs to diverge or several streams of fluids need to converge, a junction is used.

For a junction with $n$ ports, the mass and energy conservation equations are:

$$\sum_{n=A,B,C\ldots} \dot{m}_n = 0 \quad (28)$$

$$\sum_{n=A,B,C\ldots} \dot{H}_n = 0 \quad (29)$$

The calculation method of enthalpy flow of junction is the same as the PS. The pressure values at all ports are the same since the pressure loss is neglected inside the junction:

$$p_A = p_B = p_C = \ldots = p_N \quad (30)$$

## 3. POWER FLOW OF COGENERATION SYSTEM

A cogeneration system is typically composed of a DHN and a PSN [17]. They can be coupled by components like CHP units, motor-driven PSs, heat pumps, etc. Analyzing a DHN is actually solving a set of algebraic equations. Thus, the first thing to do is to find a set of unknown variables and an equal number of constraints that can depict the whole system under actual operating conditions. The variable vector $\mathbf{x}$ is defined as:

$$\mathbf{x} = [\mathbf{x}_p^N, \mathbf{x}_T^N, \mathbf{x}_m^B, \mathbf{x}_{p_I}^P, \mathbf{x}_{T_I}^P, \mathbf{x}_{T_H}^P, \mathbf{x}_{Q_H}^P, \mathbf{x}_{u_I}^C, \mathbf{x}_{u_I}^J]^T \quad (31)$$

In which the variables are: node pressures, node temperatures, mass flow rate of branches, pressure and temperature inside pipes, temperature at the wall of pipes, heat transfer power of pipes, specific internal energy of PSs and junctions respectively. The dimension of this vector is:

$$\dim(\mathbf{x}) = 2|\mathbb{N}| + |\mathbb{B}| + 4|\mathbb{P}| + |\mathbb{C}| + |\mathbb{J}| - 1 \tag{32}$$

in which the symbols are: number of nodes, number of branches, number of pipes, number of PSs and number of junctions, respectively.

And the mismatch vector can be written as:

$$\Delta\mathbf{F} = [\left(\Delta\mathbf{F}^{N}\right)^{T}, \left(\Delta\mathbf{F}^{P}\right)^{T}, \left(\Delta\mathbf{F}^{C}\right)^{T}, \left(\Delta\mathbf{F}^{J}\right)^{T}]^{T} \tag{33}$$

where: the vectors are the mismatches about the nodes, pipes, PSs and junctions respectively.

Since the partial derivatives in the Jacobian matrix are rather complicated according to the proposed models, and they also change with the method of determining thermodynamic properties, a numerical method of obtaining each partial derivative is proposed, as shown in Fig 2.

**Algorithm**: Calculate the numerical derivatives in Jacobian Matrix.

```
for( j = 1; j ≤ dim(x); j++ )
  {
  step = 1.0
  pd_0 = [ΔF([x(1:(i−1)),x(i)+step,x((i+1):l)]) − ΔF([x(1:(i−1)),x(i)−step,x((i+1):l)])] / 2step
  step/ = 2
  pd_1 = [ΔF([x(1:(i−1)),x(i)+step,x((i+1):l)]) − ΔF([x(1:(i−1)),x(i)−step,x((i+1):l)])] / 2step
  d_0 = max(abs(pd_1 − pd_0)) // max(abs()) returns the maximum absolute component of a vector.
  step/ = 2
  pd_2 = [ΔF([x(1:(i−1)),x(i)+step,x((i+1):l)]) − ΔF([x(1:(i−1)),x(i)−step,x((i+1):l)])] / 2step
  d_1 = max(abs(pd_2 − pd_1))
  while (d_1 < d_0 || |d_0| > 1e − 3)
    {
    if (|d_0| < 1e − 6)
      break
    pd_1 = pd_2
    d_0 = d_1
    step/ = 2
    pd_2 = [ΔF([x(1:(i−1)),x(i)+step,x((i+1):l)]) − ΔF([x(1:(i−1)),x(i)−step,x((i+1):l)])] / 2step
    d_1 = max(abs(pd_2 − pd_1))
    }
  J(:, i) = pd_1 // assign a column of J at one time
  }
```

Fig 2 Algorithm of calculating numerical derivatives

The following method is proposed to assign the initial values for vectors in (31). (i) For pressures and temperatures, they can use the average values of the whole network. (ii) For specific internal energy, it can be derived based on average temperature. (iii) For heat transfer power at consumers and sources, their designed values can be utilized. For transmission losses, 0 is a reasonable initial value. (iv) For mass flow rate at a consumer, it can be estimated using the formula of specific heat capacity. The mass flow rates at other branches can be obtained according to the mass conservation law.

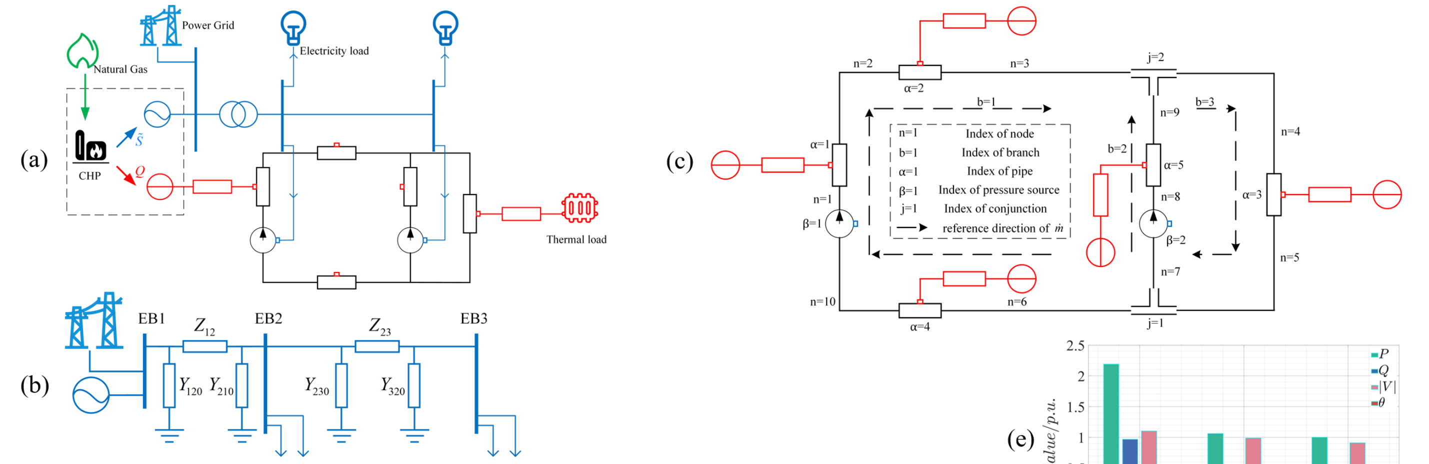
After solving all the unknown variables in DHN, the mechanical power input in PSs will be obtained; and (27) can be used to get the corresponding electrical power, which is classified into the load of PSN. By applying load flow analysis in PSN, the cogeneration system can be solved ultimately.

## 4. CASE STUDY

In order to verify the improved DHN models proposed in chapter 2 and the calculation method proposed in chapter 3, a demonstration network of cogeneration system shown in Fig 3(a) is tested. The heat source is from the output of a CHP unit, while the electricity is supplied by both the CHP and the power grid. Fig 3(b) shows the model of PSN part composed of 3 buses connected by 1 transformer and 1 transmission line. The model of DHN is shown in Fig 3(c), with the numbers of objects clearly marked on it. There are 5 pipes, 2 PSs and 2 junctions in the DHN. Parameters of both the PSN and the DHN are specified in Fig 3(d) (not shaded part). The impedance, admittance, voltage and power parameters of the PSN are per unit values. The analysis process begins from the DHN, and then goes to the PSN. The latter, being a radial network, can be solved easily by the forward-back substitution method which is widely adopted in power systems.

According to the calculation methods in Section 3, it can be seen from Fig 4 that after 5 iterations, the error of the mismatch vector drops below $10^{-8}$. The convergence process behaves in quadratic characteristic with the Y-axis, which validates the high efficiency of the proposed method. The results of the DHN are shown in the shaded part of Fig 3(d), and the PSN in Fig 3(e). From the results, we can learn that the pressure mainly drops on transmission pipes, while the heat transfer of pipes depends highly on thermal resistance. There is little temperature variation on branch b=2 since the boundary condition of this pipe is configured to adiabatic. A comparative calculation based on the traditional method [18] is also carried out, and the main results are similar, which validates the component models of this paper. The differences mainly occur in the long-distance transmission pipeline (Fig 3(f)), with nearly 30% error in $\Delta p$ and 50% in $\Delta T$, indicating the necessity of using a more precise model for multi-region

DHNs. The calculation is performed with Intel Core i7-10700K CPU @ 3.80GHz, and a 16GB ram, coded in the Microsoft Visual Studio Community 2019. The running time of this program is about 0.078s.




(d)

| | L/m | D/m | $R_H$ (K/W) | $T_{HS}$ (K) | $Q_H$ (10kW) | | | | |
|---|---|---|---|---|---|---|---|---|---|
| pipe1 | 50 | 0.1128 | 1.88E-06 | 373.15 | 228.1444 | Sbase | 500kVA | Y230 | 0.0+j5.0700e-3 |
| pipe2 | 5000 | 0.1128 | 2.82E-03 | 293.15 | -2.5355 | Z12 | 0.0096+0.1920j | Y320 | 0.0+j5.0700e-4 |
| pipe3 | 50 | 0.1128 | 1.88E-06 | 323.15 | -227.0303 | Z23 | 0.0668 + 0.0701j | V1 | 1.1+0.0j |
| pipe4 | 5000 | 0.1128 | 2.82E-03 | 293.15 | -1.31570645 | Y120 | -0.0092+0.2160j | S2 | 1.0+0.0j |
| pipe5 | 50 | 0.1128 | ∞ | 293.15 | 0 | Y210 | 0.0102 - 0.226j | S3 | 1.0+0.0j |

| | p/kPa | T/K | | p/kPa | T/K | | | |
|---|---|---|---|---|---|---|---|---|
| node1 | 1801.325 | 330.4552 | node6 | 951.26938 | 330.448361 | PS1 | pcmd=1.7MPa | ηe=0.9 |
| node2 | 1793.086 | 365.0528 | node7 | 951.26938 | 330.448361 | PS2 | pcmd=0.02MPa | ηe=0.9 |
| node3 | 969.103 | 364.8727 | node8 | 971.26938 | 330.448361 | mdot1 | 15.8009kg/s | |
| node4 | 969.103 | 353.7108 | node9 | 969.10303 | 330.448887 | mdot2 | 7.5821kg/s | |
| node5 | 951.2694 | 330.4484 | node10 | 1101.325 | 330.455199 | mdot3 | 23.3829kg/s | |

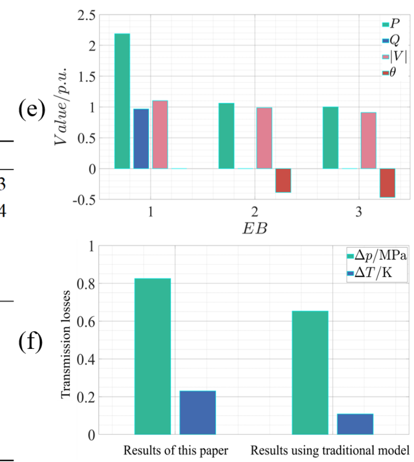



Fig 3 Diagrams, data and calculation results of the cogeneration system in casy study

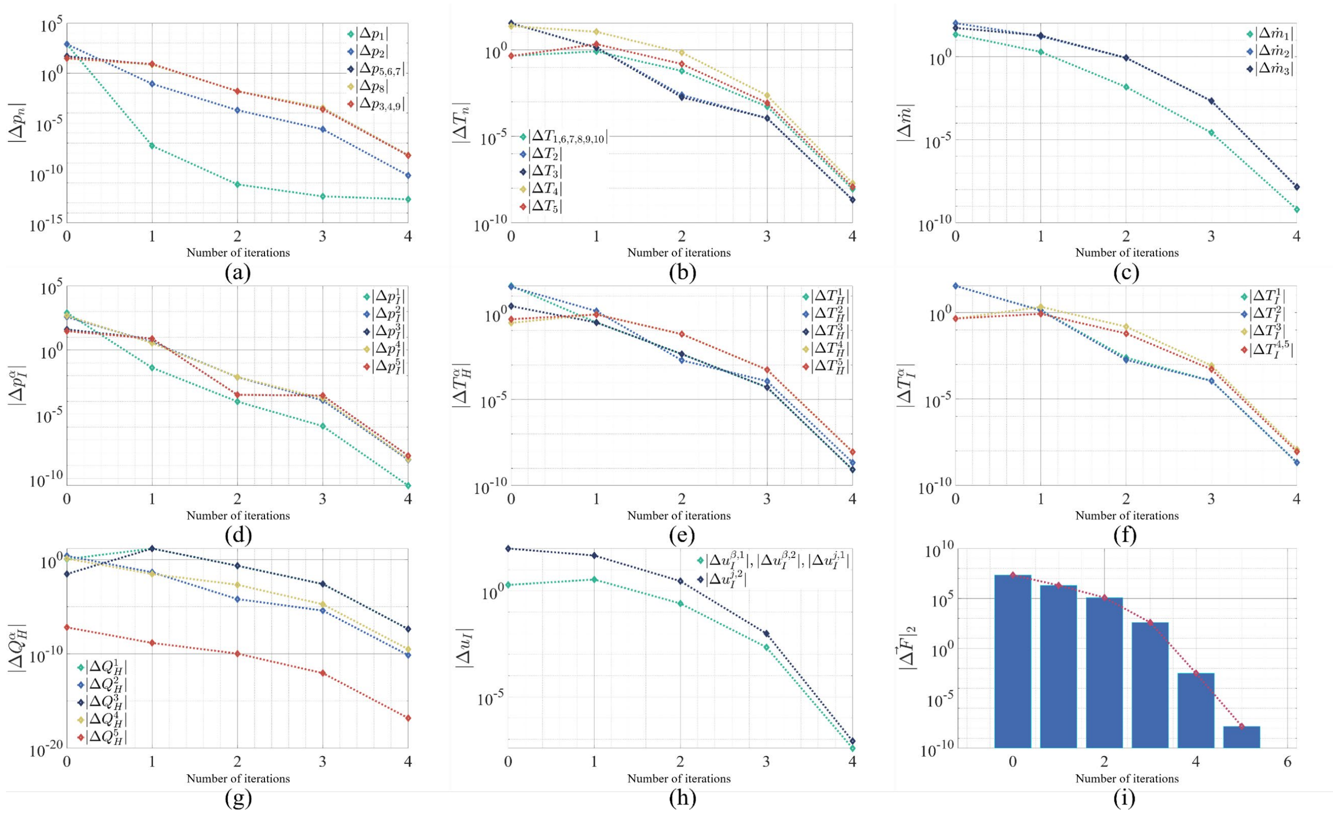



Fig 4 Convergence characteristics of the proposed method

## 5. CONCLUSION AND PROSPECT

An improved modelling method of DHN is proposed in this paper. The main contributions are: (1) It can be used to analyze DHNs of any topologies, not restrict to parallel supply and return fluid networks. (2) Changes of thermodynamic properties are taken into consideration. (3) Detailed heat transfer, pressure loss processes are analyzed in detail in formulae (19-22). (4) An efficient calculating method is applied and a general way of assigning initial values is discussed. (5) A case study demonstrates the models and calculating methods proposed previously and a comparison with traditional method is presented to illustrate the necessity of adopting new method in multi-region DHN analysis.

The limitations of this paper are: (1) Modelling of other components of DHN, like the heat pumps, are not included. (2) Calculating a more detailed model means additional computational time, and a faster and better calculating method is needed.

Future researches may include: (1) Topology of DHN which can optimize the heat delivery process. (2) Inventing new components in DHN which may flexibly control the heat power flow based on the physical processes mentioned in this paper. (3) Faster numerical methods calculating the power flow in DHN.

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## REFERENCE

[1] David E.H.J. Gernaat, Harmen-Sytze de Boer, Louise C. Dammeier et al., The role of residential rooftop photovoltaic in long-term energy and climate scenarios. Applied Energy 2020; 279:115705.

[2] M. Zhang, Q. Wang, D. Zhou et al., Evaluating uncertain investment decisions in low-carbon transition toward renewable energy. Applied Energy 2020; 240:1049-1060.

[3] T. Liu, Y. Wang, Q. Song et al., Low-carbon governance in China – Case study of low carbon industry park pilot. Journal of Cleaner Production 2018; 174:837-846.

[4] Pierluigi Mancarella. MES (multi-energy systems): An overview of concepts and evaluation models. Energy 2014; 65:1-17.

[5] Elisa Guelpa, Aldo Bischi, Vittorio Verda et al., Towards future infrastructures for sustainable multi-energy systems: A review. Energy 2019; 184:2-21.

[6] W. Huang, N. Zhang, Y. Cheng et al., Multienergy Networks Analytics: Standardized Modeling, Optimization, and Low Carbon Analysis. Proceedings of the IEEE 2020; 108-1411-1436.

[7] M. Geidl, G. Koeppel, P. Favre-Perrod et al., Energy hubs for the future. IEEE Power and Energy Magazine 2007; 5:24-30.

[8] Soheil Derafshi Beigvand, Hamdi Abdi, Massimo La Scala. A general model for energy hub economic dispatch. Applied Energy 2017; 190:1090-1111.

[9] Xuezhi Liu, Jianzhong Wu, Nick Jenkins et al., Combined analysis of electricity and heat networks. Applied Energy 2016; 162:1238-1250.

[10] T. Lan and K. Strunz. Modeling of the Enthalpy Transfer Using Electric Circuit Equivalents: Theory and Application to Transients of Multi-Carrier Energy Systems. IEEE Transactions on Energy Conversion 2019; 34:1720-1730.

[11] J. Yang, N. Zhang, A. Botterud et al., On An Equivalent Representation of the Dynamics in District Heating Networks for Combined Electricity-Heat Operation. IEEE Transactions on Power Systems 2020; 35:560-570.

[12] Gianfranco Chicco, Pierluigi Mancarella. Matrix modelling of small-scale trigeneration systems and application to operational optimization. Energy 2009; 34:261-273.

[13] F. Shen, P. Ju, M. Shahidehpour et al., Singular Perturbation for the Dynamic Modeling of Integrated Energy Systems. IEEE Transactions on Power Systems 2020; 35:1718-1728.

[14] Frank M.White. Fluid Mechanics. 8th ed. WCB McGraw-Hill, 2015.

[15] John H.Lienhard IV/John H.Lienhard V. A heat transfer textbook; 3rd ed. Phlogistion press; 2005.

[16] Frand P.Incropera et al. Fundamentals of Heat and Mass Transfer; 6th ed. John Wiley & Sons; 2005.

[17] J. Liu, Y. Chen, C. Duan et al., Distributionally Robust Chance-Constraint Optimal Power Flow with Wasserstein-Moment Metric in Active Distribution Networks, 10th International Conference on Applied Energy (ICAE2018), HongKong, Aug. 22-25, 1-5.

[18] X. Liu, J. Liu, X. Zhao. Energy Flow Calculation Method of Combined Cooling, Heating and Power System with Terminal Cooling Network Model. 2020 IEEE Power and Energy Society General Meeting (IEEE PES GM2020). Montreal, CA, Aug. 2-6, 1-5.

 