

# MPC-based lateral control algorithm for bus path following control

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## ABSTRACT

The route of buses is relatively fixed compared with private vehicles. This characteristic provides a suitable scenario for path following control. The purpose of path following control is to control the steering system of the vehicle to track the desired trajectory curve while satisfying performance such as safety and energy consumption. Model predictive control (MPC) method is used in this paper for vehicle lateral control. Preview is used to reduce the track error. The vehicle kinematic model is established as predictive model. The path following problem is described as a quadratic programming (QP) problem that can be solved in real-time situations. Simulations on Matlab/Simulink & TruckMaker are conducted, and real vehicle tests are completed. Results show that the proposed lateral control strategy can achieve the tracking error within 15 cm in different velocities and scenarios in the simulation.

**Keywords:** Model predictive control, vehicle lateral control, path following, quadratic programming, electric bus

## 1. INTRODUCTION

Intelligence, safety, and energy saving are the current research hotspots in the automotive industry. For autonomous electric bus, the reliability and robustness of path following is the basis of vehicle safety. Under this circumstance, key technical issues such as active safety, energy saving, and path following of autonomous vehicles need to be solved urgently.

The purpose of path following control is to control the steering system to enable the vehicle to track the desired trajectory curve while satisfying performance such as safety and energy consumption<sup>[1]</sup>. Model predictive control includes three parts: model prediction, rolling optimization and feedback correction. It is good in

adaptability and robustness<sup>[2]</sup>. In path following problems, the usage of MPC in current studies is still inadequate, the existing problems include: balancing the prediction accuracy and real-time performance of the algorithm, the impact of model accuracy on the control results is difficult to eliminate, and the test verification environment is immature<sup>[3]</sup>.

Researchers have focused on rationally simplifying predictive models and improving computational efficiency to obtain real-time performance. These two points are mutually restrictive. Preview is a suitable method to reduce the tracking error, despite that the predictive model is not accurate enough. And the simplified predictive model helps improve real-time performances.

In summary, a MPC-based vehicle lateral control strategy is proposed to realize the path following target. Preview is used to reduce the tracking error. The kinematic model of vehicle is linearized and discretized for real-time usage. The path following problem is described as a quadratic programming problem, and is solved by qpOASES in real time. Simulations on Matlab/Simulink&TruckMaker are conducted, and real vehicle tests are completed. Results show that the proposed lateral control strategy can achieve the tracking error within 15 cm in different velocities and scenarios.

## 2. MPC-BASED LATERAL CONTROL STRATEGY

### 2.1 Overall structure of control strategy

The overall structure of MPC-based vehicle lateral control strategy is shown in Figure 1. The inputs of control strategy are the vehicle state variables, steering angle and objective path. The output is front wheel angle, which is the control variable. The steering wheel angle is calculated based on front wheel angle. The

objectives of control strategy are to follow the target path with the smallest control variable value.

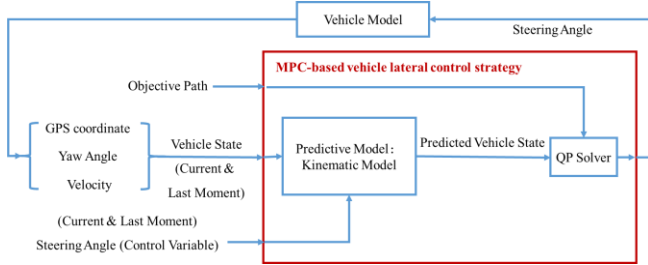


Fig 1 Overall structure of control strategy

The state variables and control variable of MPC-based strategy is listed in Table 1. The state variables are acquired from GPS, the target path is fixed, and the control variable is recorded by control strategy.

Table 1 Variable definitions

Control variable		
Front wheel angle	$\delta_f$	$u$
State variables		
X-axis position / m	$x$	$x_1$
Y-axis position / m	$y$	$x_2$
Yaw angle / rad	$\theta$	$x_3$
Output variables		
Tracking error of X-axis position / m	$x_{ref} - x$	$y_1$
Tracking error of Y-axis position / m	$y_{ref} - y$	$y_2$
Tracking error of Yaw angle / rad	$\theta_{ref} - \theta$	$y_3$

## 2.2 Predictive model

We use the kinematic model of vehicle as the predictive model, which is expressed as Formula 1.

$$\begin{aligned}\dot{x} &= v \cos(\theta + \beta) \\ \dot{y} &= v \sin(\theta + \beta) \\ \dot{\theta} &= \omega\end{aligned}\quad (1)$$

According to the definition of variables in Table 1, the expression of predictive model is Formula 2.

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u) = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} f_1(\mathbf{x}, u) \\ f_2(\mathbf{x}, u) \\ f_3(\mathbf{x}, u) \end{pmatrix} = \begin{pmatrix} v \cos(x_3 + \beta) \\ v \sin(x_3 + \beta) \\ \frac{v \tan u \cos \beta}{L} \end{pmatrix}\quad (2)$$

The Formula 2 is linearized near  $\mathbf{x}_t$  using Taylor series expansion algorithm, and we get Formula 3. ( $d\mathbf{x} = \mathbf{x} - \mathbf{x}_t$ )

$$d\dot{\mathbf{x}} = \mathbf{A}_0(t)d\mathbf{x} + \mathbf{B}_0(t)du\quad (3)$$

where

$$\mathbf{A}_0(t) = \begin{pmatrix} 0 & 0 & -v \sin(x_{3,t} + \beta_t) \\ 0 & 0 & v \cos(x_{3,t} + \beta_t) \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{B}_0(t) = \begin{pmatrix} 0 \\ 0 \\ \frac{v \cos(\beta_t)}{L \cos^2(u_t)} \end{pmatrix}$$

The Formula 3 is discretized using Euler's Method, and we get Formula 4.

$$\delta\mathbf{x}(k+1) = \mathbf{A}(T, k)\delta\mathbf{x}(k) + \mathbf{B}(T, k)\delta u(k)\quad (4)$$

where

$$\begin{aligned}\mathbf{A}(T, k) &= \mathbf{I} + \mathbf{A}_0(k)T \\ \mathbf{B}(T, k) &= \mathbf{B}_0(k)T\end{aligned}$$

We assume that in the period of prediction (in  $N_p$  steps) at each moment, the predictive model keeps the same, until the next moment. Thus, we have Formula 5.

$$\begin{aligned}\mathbf{A}(T, k) &= \mathbf{A}(T) \\ \mathbf{B}(T, k) &= \mathbf{B}(T)\end{aligned}\quad (5)$$

Thus, we use the linearized and discretized kinematic vehicle model to predict the vehicle states in MPC.

## 2.3 Objective function

We define the objective function as the combination of both tracking error and the control variable to achieve the control target by the smallest action, as is expressed in Formula 6.

$$J = \mathbf{Y}^T \mathbf{Q} \mathbf{Y} + \mathbf{U}^T \mathbf{R} \mathbf{U}\quad (6)$$

where

$$\mathbf{Y} = \mathbf{X} - \mathbf{X}_{ref} = \begin{pmatrix} \mathbf{x}(k+1) - \mathbf{x}_{ref}(k+1) \\ \mathbf{x}(k+2) - \mathbf{x}_{ref}(k+2) \\ \mathbf{x}(k+3) - \mathbf{x}_{ref}(k+3) \\ \dots \\ \mathbf{x}(k+N_p) - \mathbf{x}_{ref}(k+N_p) \end{pmatrix}$$

$$\mathbf{U}(k) = \begin{pmatrix} \mathbf{u}(k) \\ \mathbf{u}(k+1) \\ \mathbf{u}(k+2) \\ \dots \\ \mathbf{u}(k+N_p-1) \end{pmatrix}$$

The constraint is the change rate of control variable that is limited due to the mechanical characteristics.

## 2.4 Preview process

In predictive model, a curve preview method is used. According to the real-time vehicle position information, based on the front center point (location of GPS hardware), preview the position coordinates and yaw angle of this point at the time  $T_{pre}$  in the future, and use the coordinates and yaw angle of the preview point

as the reference point of the prediction algorithm. The preview time is a parameter that can be calibrated, and a set of suitable values needs to be calibrated according to the changes in state information such as vehicle speed and predicted step length. The schematic diagram of preview method is shown in Figure 2.

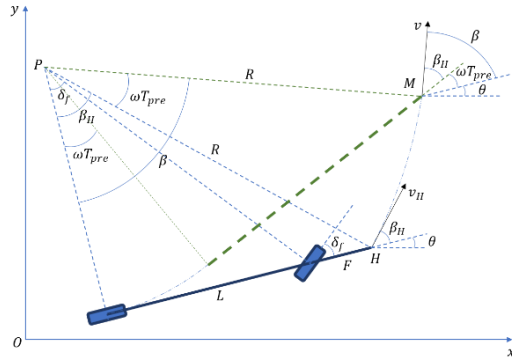


Fig 2 Preview method

If the vehicle is currently traveling in a straight line, that is, the front wheel angle is zero, the preview point is directly in front of the vehicle, which is a straight line preview. Otherwise, it is curve preview. Once the preview point is determined, it is regarded as a point fixed on the rigid body of the vehicle, that is, the yaw angle of the preview point is the yaw angle of the vehicle. The speed at preview point M and the speed at point H are numerically equal. The angular velocity at point M is equal to that of the vehicle. The relation of point H and point M is expressed by Formula 8-10.

$$v = v_H \quad (8)$$

$$\tan(\beta_H) = \frac{(L + F) \tan \delta_f}{L} \quad (9)$$

$$\beta = \beta_H + \omega T_{pre} \quad (10)$$

$$\omega = \frac{v \tan(\delta_f) \cos(\beta_H)}{L} \quad (11)$$

When the front wheel angle is zero, the position of point M is calculated by straight preview method, which can be expressed as Formula 12 and 13.

$$x_M = x_H + v T_{pre} \cos(\theta) \quad (12)$$

$$y_M = y_H + v T_{pre} \sin(\theta) \quad (13)$$

When the front wheel angle is not zero, the position of point M is calculated through curve preview method, which can be expressed as Formula 14-16.

$$R = \frac{v}{\omega} \quad (14)$$

$$x_M = x_H + R \cos(\theta) \sin(\omega T_{pre}) - R \sin(\theta) (1 - \cos(\omega T_{pre})) \quad (15)$$

$$y_M = y_H + R \sin(\theta) \sin(\omega T_{pre}) + R \cos(\theta) (1 - \cos(\omega T_{pre})) \quad (16)$$

## 2.5 QP solver

qpOASES is an online activity set to solve the following form of quadratic programming problems, as can be expressed in Formula 7.

$$\begin{aligned} \min_x \quad & \frac{1}{2} x^T \mathbf{H} x + x^T g(\omega_0) \\ \text{s.t.} \quad & \text{lb} \mathbf{A}(\omega_0) \leq \mathbf{A} x \leq \text{ub} \mathbf{A}(\omega_0), \\ & \text{lb}(\omega_0) \leq x \leq \text{ub}(\omega_0) \end{aligned} \quad (7)$$

where the Hessian matrix  $\mathbf{H}$  is a (semi-) positive definite symmetric matrix, and the gradient vector, boundary conditions, and constraints are all affine functions (functions consisting of first-order polynomials) with the parameter  $\omega_0$ .

## 3. SIMULATIONS AND VEHICLE TESTS

Simulations based on Matlab/Simulink&TruckMaker platform is conducted. The lateral control strategy is wrapped in S-function. The vehicle model in TruckMaker is close to the real vehicle, thus the simulation results are used to study the effects of the change of parameters. The structure of simulation platform is show in Figure 3.

Vehicle tests in a close loop road is conducted to verify the effectiveness and real-time performance of the control strategy.

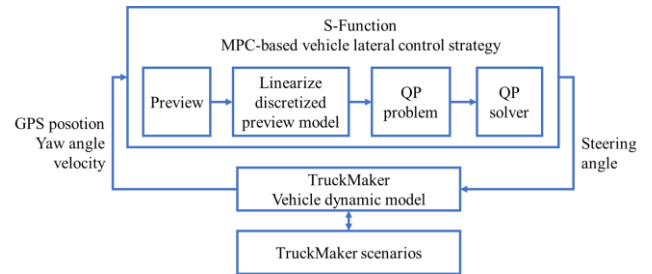


Fig 3 Simulation platform structure

Two scenarios including straight line and right-angle turn are designed. We assume that the friction coefficient is 0.8. Different velocities are simulated in the simulation platform and vehicle tests.

## 4. RESULTS AND DISCUSSION

The tracking errors and conditions are listed in Table 2. The tracking error is lower than 10 cm in all conditions. The parameters of each condition are calibrated, and the prediction step needs to be reduced with the increase of vehicle velocity. Take the 20m-radius right-angle turn with the velocity of 10km/h as an example, the tracking result is shown in Figure 4. The tracking error of different friction coefficient is shown in Figure 5.

Table 2 Tracking errors of simulation

Road type	Radius (m)	Velocity (km/h)	Maximum tracking error (cm)
straight	$+\infty$	10	1.00
		30	1.00
		50	1.00
		70	1.00
Right-angle	20	10	9.61
		20	9.40
	30	10	8.03
		20	7.06
	30	30	5.64
		30	5.64
	50	10	8.37
		20	4.19
		30	6.90
		35	4.34

The tracking result of right-angle turn with velocity of 15 km/h is shown in Figure 7. The target path is not smooth due to the signal noise. However, the vehicle can follow the path smoothly despite of the noises.

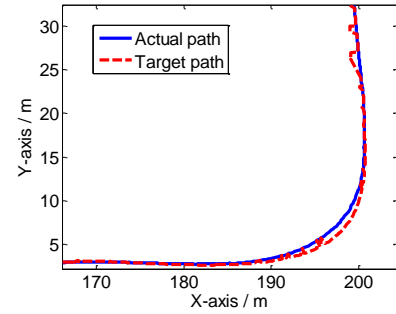


Fig 7 Tracking result of real vehicle

The results show that the proposed MPC-based vehicle lateral control strategy can realize a reliable and stable tracking action. Preview method can help reduce the tracking error effectively. In real vehicle tests, due to the noises in target path, the tracking error will reach a higher value than that in simulations. The real-time performance can be verified according to the real vehicle tests.

## 5. CONCLUSION

An MPC-based vehicle lateral control strategy is proposed in this study. Preview method is used to reduce the tracking error. Simulation and real vehicle test results show that the prediction step of MPC needs to be reduced with the growth of vehicle velocity, and the reliability and real-time performance is verified. The tracking error is within 15 cm in simulation platforms.

## ACKNOWLEDGEMENT

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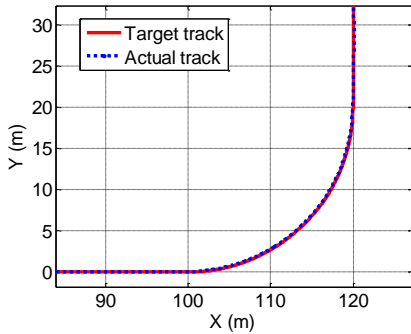


Fig 4 Tracking result

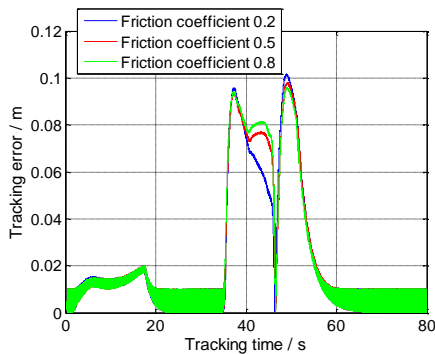


Fig 5 Tracking error

For real vehicle tests, in straight road conditions, the tracking error grows with the increase of velocity, as is shown in Figure 6.

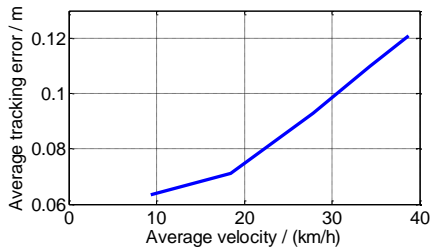


Fig 6 Trend of tracking error with velocity

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