

Coordinated Pricing of Urban Electrified Transportation Networks: A Stackelberg Game Theoretic Perspective

Yujie Sheng¹, Qinglai Guo^{1*}, Tianyu Yang¹, Zhe Zhou²

1 Department of Electrical Engineering, Tsinghua University, Beijing, China

2 Tsinghua-Berkeley Shenzhen Institute, Shenzhen, China

ABSTRACT

This paper investigates the coordinated pricing of urban electrified transportation networks enabled by the dynamic wireless charging technology of electric vehicles (EVs) in the future. The in-motion charging demand of EVs will create stronger interdependency between the operation of power distribution networks (PDN) and traffic networks (TN). The PDN locational marginal pricing and TN congestion pricing will affect the route choices of multi-class vehicles (i.e. vehicles charging or not charging). The aggregated traffic flow and charging load of vehicles will in return affect the operations and pricing of the PDN and TN. To investigate the benefit of coordinated pricing, the Stackelberg game is employed to model the above interaction between the utilities and strategies of PDN, TN, and vehicles under coordinated and uncoordinated pricing scenarios respectively. Case studies demonstrate the benefits of the coordinated pricing of the coupled networks.

Keywords: electric vehicle, wireless charging, distribution network, transportation network, pricing, Stackelberg game

NOMENCLATURE

Abbreviations

EV	Electric vehicle
PDN	Power distribution network
TN	Transportation network
CT	Congestion toll
LMP	Locational marginal pricing
CEV	Charging EV
NCV	Non-charging vehicles

O-D	Origin-destination
TAP	Traffic assignment problem
UE	User equilibrium
LMP	Locational marginal pricing
<i>Symbols</i>	
T_N	TN node set
T_A	TN link set
(r, s)	O-D pairs
q^{rs}	O-D flow
K^{rs}	Feasible path set
f_k^{rs}	Path traffic flow
x_a	Link traffic flow
c_k^{rs}	Path travel cost
ω	Monetary value of time
τ_a	Congestion toll
λ_a	Link charging price/LMP
E_a	Link charged energy
t_a	Link travel time
M	TN-PDN bus incidence matrix
E_N	PDN bus set
E_L	PDN branch set
P_t	Bus load
P_c	Bus charging load
P_d	Bus fixed load
p_i^g / q_i^g	Bus active/reactive power generation
p_{ij}^l / Q_{ij}^l	Branch active/reactive power flow
$r_{ij}^l / x_{ij}^l / z_{ij}^l$	Branch resistance/reactance/impedance
P_{0j}^l	Power purchased from main grid
a_i / b_i	Local generation price coefficients
ρ	Main grid electricity price

1. INTRODUCTION

The problem of air pollution and the greenhouse effect is receiving growing concern from all over the world, leading to the trend of electrification in transportation. Electric vehicles (EVs) are regarded as a promising alternative to traditional gasoline vehicles for their higher energy efficiency and less air pollutant emission. Driven by various promotion policies, the penetration rate of EVs in urban traffic network has been increasing rapidly. It's reported that the global electric car fleet has exceeded 7.2 million in 2019 [1].

A major obstacle to EV popularization is their limited driving range and the resulting range anxiety of drivers. The rapid development of charging technology is a solution to this problem, which greatly changes the feature of EV charging load and the interaction between power distribution networks (PDN) and transportation networks (TN). With conventional plug-in slow charging, EVs are usually treated as time-flexible at fixed locations. TN and PDN are only coupled by the drivers' traveling demand. With plug-in fast charging stations, EV drivers have more flexible choices of charging locations during their trip. The fast charging loads show spatial transferable features and strengthen the interdependency between TN and PDN. In the near future, wireless power transfer technologies will electrify the roads in TN as charging infrastructure and enable EVs to be charged in motion, eliminating range anxiety, reducing the battery size, and mitigating the long waiting time for charging. It is expected that the wireless charging power can move a car at a speed of 75 mph in the near future. In this paper, we envision an electrified transportation network with dynamic wireless charging. Under this scenario, the traffic routing and the charging strategies of EVs show more close interdependency, and the operation and regulation (pricing) of TN and PDN are strongly coupled by the charging EVs. The TN operators aim at mitigating traffic congestion via minimum congestion tolls (CTs) on roads. The PDN operators aim at supplying the loads with minimum cost and mitigating line congestion through locational marginal pricing (LMPs). The CTs will influence all the drivers' route choices, including the EVs charging (CEVs) or the vehicles not charging (NCVs). The LMPs will influence the routing and charging choice of CEVs charged on the road. On the other hand, the aggregated traffic flow and charging load of those vehicles will affect the operations of TN and PDN and potentially impact CTs and LMPs. As the utilities of TN, PDN, and CEVs, NEVs are mutually influenced by their

strategies, game-theoretical approaches are more appropriate.

In recent years, a few studies discussed the TN-PDN coupling under the wireless charging scenario. Reference [2] studied the optimal prices of electricity and roads to maximize social welfare, where two pricing models are proposed under different assumptions of authoritarian. However, the flexibility of congestion pricing to achieve a certain traffic flow pattern was not considered. The impact of wireless charging roads on the electricity market was investigated in [3]. The EV mobility was modeled as a queuing network based on the statistics from traffic information systems, while the model of TN was not considered. The short-term operation of PDN and TN coupled with LMPs and EVs was studied in [4] and decentralized optimization was employed to address the coordination. However, the regulation measures of TN (e.g. CTs) were not considered. Reference [5] proposed a static optimal traffic-power flow model to determine the best generation schedule and CTs. The model was furthered improved to a multi-period one to consider time-varying electricity and traffic demands in [6], while the impact of the charging load on electricity prices was ignored. Overall, to our best knowledge, no existing studies have investigated the pricing of electrified transportation networks from a game-theoretical perspective.

To fill this gap, we employ game theoretical models to investigate the TN-vehicle-PDN interaction in the pricing of electrified transportation networks and study the benefit of coordinated pricing. The main contributions of this work can be summarized as follows:

- 1) As a benchmark, a three-level Stackelberg game model is established to describe the uncoordinated pricing (i.e. LMPs and CTs) of PDN and TN. The pricing equilibrium of the game is analyzed and solved.
- 2) A two-level Stackelberg game model is established to describe the coordinated pricing in the electrified transportation networks, where NCVs and CEVs are charged with different tolls. The benefits from this cooperation are demonstrated.

The rest of this paper is organized as follows: Section 2 presents the modeling of the game players (i.e. TN-vehicle-PDN). In Section 3, the Stackelberg game framework is described under the two scenarios. Then, case studies are presented in Section 4. Finally, Section 5 concludes this paper.

2. PLAYER MODELING

2.1 Vehicles: Routing & Charging

The traffic flow and wireless charging load distribution in the TN is the aggregated effect of the routing and charging decisions of a large population of individual drivers and modeled as a traffic assignment problem (TAP) in this paper.

The TN topology can be depicted as a graph $G_T = [T_N, T_A]$, where T_N and T_A are node set and link set respectively. The drivers' traveling demand is modeled as several clustered origin-destination (O-D) pairs (r, s) and the O-D flow q^{rs} (veh/h) from the origin r to the destination s . There might be several available paths between one O-D pair (r, s) , which constitutes a feasible path set K^{rs} . The traffic flow on one path $k \in K^{rs}$ is denoted as f_k^{rs} (veh/h) and the traffic flow on one link $a \in T_A$ is denoted as x_a (veh/h). The topology relation between paths and links is depicted by an indicator variable: $\delta_{ak}^{rs} = 1$ if link a is on path k , otherwise $\delta_{ak}^{rs} = 0$.

The TAP calculation is to assign the traveling demand of each O-D pair to its available paths (f_k^{rs}), and determine the link flow x_a (veh/h) on each link a , which is similar to power flow in PDN. Each individual driver seeks to minimize their own travel costs c_k^{rs} via optimal routing. The travel cost on one path k is the summation of the link travel cost on the path.

$$\min c_k^{rs} = \sum_{a \in T_A} c_a^{rs} \delta_{ak}^{rs} \quad (1)$$

For drivers without charging demand (denoted by n), the link travel cost includes the time cost and congestion tolls.

$$c_{an}^{rs} = \omega t_a + \tau_a \quad (2)$$

where t_a is the link travel time (min), ω is the monetary value of time (\$/min), τ_a is the congestion toll charged on the link (\$). For EV drivers charging in motion (denoted by c), the charging cost associated with the electricity price on the link is additionally taken into account.

$$c_{ac}^{rs} = \omega t_a + \tau_a + \lambda_a E_a \quad (3)$$

where λ_a is the link charging price (\$/kWh). E_a is the charged energy of the link (kWh), which is assumed to be a constant proportional to the link length.

Due to the congestion effect of urban TN, the link travel time t_a increase with the link flow x_a , which is described by the Bureau of Public Roads (BPR) function [21]:

$$t_a(x_a) = t_a^0 [1 + 0.15(x_a / c_a)^4] \quad (4)$$

where the link flow consists of the two kinds of vehicles with or without charging demand:

$$x_a = x_{ac} + x_{an} \quad (5)$$

As the routing and charging choices (strategy) of numerous individual drivers will influence the travel cost (utility) of each other, the problem can be described as a non-cooperative and non-atomic game, where no individual player has a significant impact but the aggregate behavior (i.e. link flow) of them can change the payoffs (i.e. travel cost). When no drivers can reduce their trip costs by unilaterally changing to another path, the game reaches an equilibrium called user equilibrium (UE) in traffic engineering [7]

$$\text{If } f_k^{rs} > 0, \text{ then } c_k^{rs} = u^{rs}, \forall k \in K^{rs}, \forall (r, s) \quad (6)$$

$$\text{If } f_k^{rs} = 0, \text{ then } c_k^{rs} \geq u^{rs}, \forall k \in K^{rs}, \forall (r, s)$$

where the travel costs on all utilized routes are identical and minimal as u^{rs} . The game can be regarded as a potential game, and the user equilibrium can be solved by an equivalent optimization model:

$$\min F_{veh}^{UE} = \sum_{a \in T_A} \int_0^{x_a} \omega t_a(v) dv + \sum_{a \in T_A} \tau_a x_{an} + \sum_{a \in T_A} (\tau_a + \lambda_a E_a) x_{ac} \quad (7)$$

s.t. (4)–(5)

$$x_a = \sum_{(r,s)} \sum_{k \in K^{rs}} f_k^{rs} \delta_{ak}^{rs}, \sum_{k \in K^{rs}} f_k^{rs} = q^{rs}, f_k^{rs} \geq 0 \quad (8)$$

where the additional constraints in (8) are flow conservation condition and non-negativity constraint of path flow.

2.2 TN Operator: Congestion Pricing

The above UE model describes the best response of numerous vehicle drivers (i.e. the route choice and the resulting traffic flow and charging load) to given congestion tolls and charging price. From the perspective of the TN operator, the congestion tolls are implemented to adjust the drivers' route choice and avoid traffic congestion. As a non-profit operator, the TN operator aims at charging the minimum congestion tolls while controlling the traffic congestion under a certain level. The pricing of congestion tolls can be modeled as the Stackelberg game, where TN operator and vehicles are regarded as leader and followers respectively. The best response of the TN operator in the game is

$$\min F_{TN}^{CT} = \sum \tau_a x_a \quad (9)$$

$$\text{s.t. } \sum t_a(x_a) x_a \leq T_p \quad (10)$$

$$0 \leq \tau_a \leq \tau_a^{\max}, \forall a \quad (11)$$

where $x_a \in \arg \min (7)$

where the objective (9) is the total congestion tolls charged. (10) limits the total travel time (congestion) under a certain level T_p . (11) is the pricing range of congestion tolls. The problem above is bilinear programming with bilinear objective, which is non-convex and brings computational burden. Here we assume the value of T_p as its minimum possible value

under the traffic flow constraints, which regards the mitigation of traffic congestion as the primary goal.

$$\begin{aligned} \min T_p &= \sum t_a(x_a)x_a \\ \text{s.t. (8)} \end{aligned} \quad (12)$$

As the model is a convex one, the optimum traffic flow pattern x_a^* can be solved uniquely, which is also called a social optimum TAP pattern. Then the problem (9) can be converted into a linear one and the best toll setting can be calculated easily. Under the toll setting, the traffic flow under the UE state will reach x_a^* automatically. The solution method of the complete model with different T_p will be comprehensively discussed in our future works.

2.3 PDN Operator: Locational Marginal Pricing

The electrified roads are served by a PDN, which is usually a radial network represented by a graph $G_E = [E_N, E_L]$, where E_N and E_L denote the bus set and branch set respectively. Loads of each bus consist of the regular fixed loads and the moving charging loads.

$$P_t = P_d + P_c \quad (13)$$

The charging load is assumed to be proportional to the CEV flow x_{ac} on the electrified road.

$$P_c = \eta M x_{ac} \quad (14)$$

where η is a parameter associated with charging power, M is the incidence matrix between TN roads and PDN buses.

With given loads, the best response of the PDN operator is to make economic dispatch and serve the loads with minimum cost. It is assumed that locational marginal pricing (LMP) is used for the charging prices. A widely used convexified alternating current optimal power flow model in PDN is employed as follows.

$$\min F_{PDN}^{GEN} = \sum_{i \in E_N} \left[a_i (p_i^g)^2 + b_i p_i^g \right] + \rho \sum_{j \in \pi(0)} P_{0j}^l \quad (15)$$

$$P_{ij}^l + p_j^g - r_{ij}^l i_{ij}^l = \sum_{k \in \pi(j)} P_{jk}^l + p_j^d, \forall i \quad (16)$$

$$Q_{ij}^l + q_j^g - x_{ij}^l i_{ij}^l = \sum_{k \in \pi(j)} Q_{jk}^l + q_j^d, \forall i \quad (17)$$

$$U_j = U_i - 2(r_{ij}^l P_{ij}^l + x_{ij}^l Q_{ij}^l) + (z_{ij}^l)^2 i_{ij}^l, \forall i \quad (18)$$

$$i_{ij}^l U_i \geq (P_{ij}^l)^2 + (Q_{ij}^l)^2, \forall i \quad (19)$$

$$i_{ij}^l \leq i_i^l, P_{ij}^l \geq 0, Q_{ij}^l \geq 0, \forall i \quad (20)$$

$$p_i^f \leq p_i^g \leq p_i^r, q_i^f \leq q_i^g \leq q_i^r, U_i^f \leq U_i \leq U_i^r, \forall i \quad (21)$$

where the objective (15) is the production cost including the cost of local generation and purchasing electricity from the main grid. (16)-(19) are basic power flow constraints, where (19) is relaxed to convert the

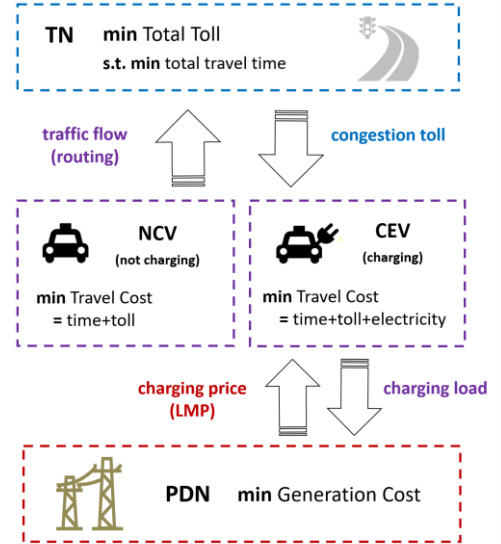


Fig 1 Uncoordinated Pricing Framework

program into a second-order cone one. (20)-(21) are operation constraints. Importantly, note that the charging prices (LMP) are the dual variables to the constraint (16).

3. GAME FRAMEWORK & SOLUTION

In order to assess the benefit of power-traffic cooperation in pricing, the game frameworks of uncoordinated pricing and coordinated pricing are introduced respectively in this section.

3.1 Uncoordinated Pricing

The game framework of uncoordinated pricing is shown in Fig 1, with the best response of each player (i.e. TN operator, PDN operator, and vehicles) introduced in detail in the last section. The game appears as a three-level Stackelberg game. Based on our assumptions, the pricing game equilibrium can be solved via the process as follow:

- 1) Considering the traffic flow constraints, the congestion control goal of TN operator i.e., the social optimum traffic flow pattern is determined by (12).
- 2) With the charging load determined by (14), the best response of PDN operator, i.e. optimal power flow and the corresponding charging price (LMPs) is calculated by (15).
- 3) The corresponding congestion toll to be set is solved by the best response of the TN operator (9), which takes the best response of vehicles (user equilibrium) into consideration.

3.2 Coordinated Pricing

The coordinated pricing game framework is shown in Fig 2. The PDN and TN operator jointly set the tolls with the goal of minimizing the social cost including generation cost and total travel time cost. Among them, the charging price and congestion toll are no longer distinguished but replaced by CEV toll and NCV toll, in which a part of the CEV toll is used to afford the charging cost. The framework appears as a two-level Stackelberg game and can be solved by the two steps as follow:

3.2.1 Optimal power-traffic flow

As the goal of an coordinated toll setting, the optimal power traffic flow model is established to minimize the social cost composed of generation cost and total travel time cost.

$$\min F_{ETN}^{OPTF} = F_{TN}^{CT} + F_{PDN}^{GEN} \quad (22)$$

$$= \sum_{i \in E_N} [a_i (p_i^g)^2 + b_i p_i^g] + \rho \sum_{j \in \pi(0)} P_{0j}^l + \omega \sum_a t_a(x_a) x_a$$

$$\text{s.t. Cons-TN}=\{(4)-(5),(8)\}, \text{Cons-PDN}=\{(15)-(21)\} \quad (23)$$

$$\text{Cons-Coupling}=\{(13)-(14)\}$$

where the constraints include TN constraints, PDN constraints, and network coupling constraints.

3.2.2 Optimal Power-Traffic Pricing

Based on the optimal power traffic flow, the optimal toll is solved to minimize the total toll charged:

$$\min F_{ETN}^{TOLL} = \sum_{a \in T_A} \tau_a^c \cdot x_{ac}^{OPTF} + \sum_{a \in T_A} \tau_a^n \cdot x_{an}^{OPTF} \quad (24)$$

$$\text{s.t.} \sum_{a \in T_A} (t_a(x_{ac/n}^{OPTF}) + \tau_a^{c/n}) \cdot x_{ac/n}^{OPTF} = \sum_{rs} q^{rs} \cdot \lambda^{rs} \quad (25)$$

$$\sum_{a \in T_A} (t_a(x_{ac/n}^{OPTF}) + \tau_a^{c/n}) \cdot \delta_{akc/n}^{rs} \geq \lambda^{rs}, \forall k, r, s \quad (26)$$

$$\tau_a^c, \tau_a^n \geq 0, \forall a \quad (27)$$

$$\sum_{a \in T_A} \tau_a^c \cdot x_{ac}^{OPTF} \geq F_{PDN}^{char}, \forall k, r, s \quad (28)$$

where τ_a^c and τ_a^n are the tolls of CEV and NCV respectively. The constraint condition (25)-(26) is the

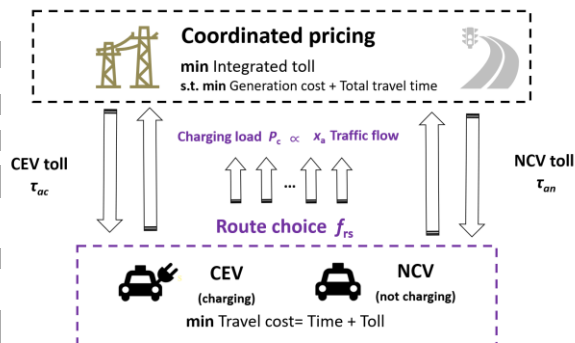


Fig 2 Coordinated Pricing Framework

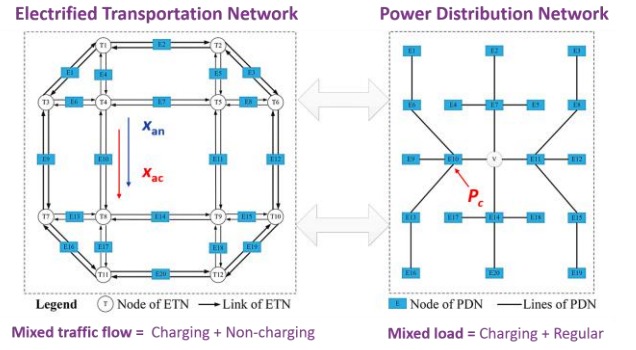


Fig 3 Electrified Transportation Network

feasible set of toll derived from the UE (8) - (10). (28) is the additional constraints of EV toll to make sure the EV toll meets the power generation cost from charging loads. The cost is assumed as proportional to the load.

4. CASE STUDIES

The case study is modified from the electrified transportation network case from [5], where the modification of data is provided online [8]. The penetration of CEVs is set to be 50% of the total vehicles.

4.1 Uncoordinated Pricing

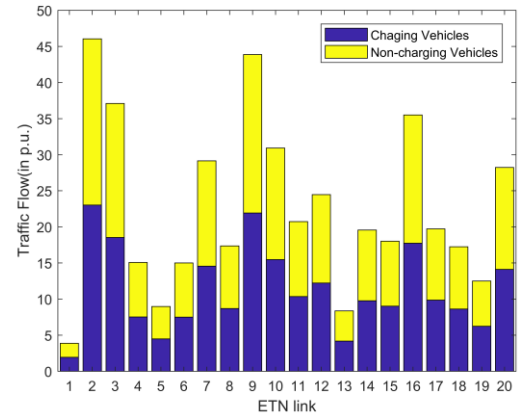


Fig 4 Traffic flow pattern under uncoordinated pricing

As a benchmark, the power-traffic flow under uncoordinated pricing is first calculated. Due to space limitation, the resulting traffic flow pattern is shown in Fig 4, and the charging load is proportional to the CEV flow. The total time cost of TN is 2236.6. The generation cost of PDN is 4053.5. And the total toll charged in TN is 1618.4.

4.2 Coordinated Pricing

To show the benefit of coordinated pricing, the power-traffic flow under coordinated pricing is then calculated. The resulting traffic flow pattern is shown in Fig 5. Compared with Fig 4, the total traffic flow of each road in TN is basically the same, but the ratio of CEVs (blue) to NCVs (yellow) are different, which leads to different charging load distribution network in PDN. Such a difference shows the flexibility of the electrified transportation network. According to the PDN topology, the charging load is more uniform and closer to the main grid and local generation bus, which reduces the power loss and the generation cost.

The operation costs under the two scenarios are compared. The total time cost of TN is 2236.9, which is only 0.01 % higher than the uncoordinated scenario. The generation cost of PDN is 4007, which is 2% lower than the uncoordinated scenario. And the net toll charged in TN (i.e. the difference between total toll 2983.7 and charging cost 2952.6) is 31.2, which is greatly lower than the uncoordinated pricing scenario. There are two reasons for the toll reduction. One is that a part of the toll functions as the charging price in our assumption. The other is the natural congestion effect of TN: when more NCVs are attracted to one road, the CEVs are naturally moved to other roads under the user equilibrium mechanism.

5. CONCLUSION

In this paper, the uncoordinated and coordinated pricing of the electrified transportation network is analyzed and compared. The results show that the pricing cooperation between TN and PDN can save the comprehensive operation cost and greatly reduce the net toll charged on travelers, which shows the flexibility of the electrified transportation network. Future works will further develop the solution method of pricing equilibrium and the fairness of pricing. The dynamics of

TN and the retail electricity pricing are also worth taking into consideration.

ACKNOWLEDGEMENT

This work was sponsored by Tsinghua-Toyota Joint Research Institute Cross-discipline Program.

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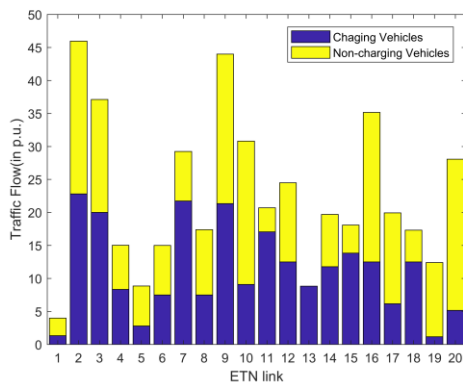


Fig 5 Traffic flow pattern under coordinated pricing