

Dynamic Power Flow Analysis Considering the Primary Frequency Regulation Based on the Fast and Flexible Holomorphic Embedding Method

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ABSTRACT

A higher penetration of renewable energy poses challenges to the operation of the power system. However, compared with the traditional power flow analysis, dynamic power flow models are more realistic. On the other hand, though the holomorphic embedding (HE) method and its variants, such as the fast and flexible holomorphic embedding method (FFHE) have shown a great potential in the traditional power flow analysis, there is no research on the dynamic power flow based on the HE method. Therefore, this paper proposes a dynamic power flow model considering the primary frequency regulation based on the FFHE method, which is based on the HE method but shows a better performance than the original HE method. The case study shows that the proposed model can obtain a dynamic power flow solution with the same accuracy as the result obtained from the Newton Raphson iteration, and the calculation efficiency is higher.

Keywords: dynamic power flow, primary frequency regulation, fast and flexible holomorphic embedding, power system

1. INTRODUCTION

In power system operation, there are many disturbances, such as load disturbances, power generator shut-down and other equipment failures. Nowadays, with the aggravation of the global energy crisis and the growing environmental crisis, the development and utilization of renewable energy have brought about worldwide attention. An increasing number of countries in the world have made the utmost effort for the wind power generation and solar power generation. The inherent instability and randomness of

renewable energy can cause disturbances and active power imbalance in the power system, which will lead to changes of system frequencies.

Power flow analysis is crucial for the power system planning and operation. Prior literature about power flow methods are mostly based on numerical iterative techniques, such as the Gauss-Seidel method [1], Newton Raphson (NR) method [2], fast-decoupled (FD) Newton power flow method [3], and their variants. However, the numerical iterative methods mentioned above may fail to converge to operable solutions for the following reasons. First, there is no guarantee that the iteration will always converge, as this depends on the choice of the initial point; secondly, since the system has multiple solutions, it is impossible to control the solution to which it will converge [4]. To solve the above problems, a non-iterative power flow method called Holomorphic Embedding Load Flow method (HELM) was first proposed by Trias [4] in 2012. Then its modified version, the fast and flexible holomorphic embedding (FFHE) method proposed in [5] shows a better performance in solving power flow problems than traditional iterative techniques.

Considering the instability and randomness of renewable energy, the disturbances and active power imbalance that often occur in power system operation, it is necessary to construct the dynamic power flow model for practical power systems. However, to the best knowledge of the authors, there is no research about dynamic power flow models considering the primary frequency regulation based on the holomorphic embedding (HE) method.

Therefore, the main contributions of this paper are twofolds.

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1) Proposed a dynamic power flow model considering the primary frequency regulation based on the FFHE method, which introduces the real variable into the complex analysis.

2) Obtained the desired solutions based on the proposed model to analyze the dynamic power flow within the same tolerance as the NR method, and in less computation time.

2. DYNAMIC POWER FLOW MODEL CONSIDERING THE PRIMARY FREQUENCY REGULATION

In practical system operation, when disturbances occur, the active power balance between the generation and the demand will be broken, and the frequency of the system will deviate from the normal values due to the frequency regulation [6]. It can be learned that due to the static frequency characteristic of load and the frequency regulation characteristic of generators, the actual active power drawn by loads and active output power of generators will vary with the frequency when there are disturbances in the operation [7].

The initial power flow can be obtained from the NR method, the HE method and their variants. However, the power balance equations in the existing literature [4,5,8] ignore the frequency regulation of generator and load, which are not suitable for analyzing the power flow after disturbances. Supposing only the active power disturbances are considered, for an N -bus system with a single slack bus, the dynamic power flow problem considering the primary frequency regulation can be formulated as:

$$\begin{cases} P_{G0i} - K_{Gi}\Delta f - (P_{L0i} + \Delta P_i + K_{Li}\Delta f) - P_i = 0 \\ Q_{Gi} - Q_{Li} - Q_i = 0 \end{cases} \quad (1)$$

where P_{G0i} and P_{L0i} are the active power of the generator and the load at the bus i , for the initial stable state, respectively. Q_{Gi} and Q_{Li} are the reactive power of the generator and the load at the bus i , respectively. K_{Gi} and K_{Li} are the unit power regulation of the generator and the load at the bus i , respectively. ΔP_i is the initial active power disturbance of the bus i . Δf is the frequency deviation between the current and the initial stable state. P_i and Q_i are the active and reactive power injections of the bus i at the new equilibrium point, which is defined as follows.

$$P_i + jQ_i = U_i \sum_{j=1}^N Y_{ij}^* U_j^* \quad (2)$$

where U_i is the voltage of the bus i ; Y_{ij} is the (i,j) element of the bus admittance matrix. And the superscript $*$ is the conjugate symbol.

3. SOLUTION BASED ON FFHE

The FFHE method adopted in [5] is the technique of embedding the original equation into a large problem containing newly introduced complex variables. Then by comparing the series coefficient, the linear equations can be derived to form the matrices which is used to compute the numerical solution to the original problem. Note that the FFHE method adopts a recursive computational scheme which is different from the iterative techniques, such as the NR method. This will lead to its advantage in calculating.

This paper constructs a set of holomorphic embedded functions based on the FFHE method, which are derivable from the dynamic power flow equation (1). Therefore, the original dynamic power flow model considering the primary frequency regulation can be eventually solved. Detailed steps of the proposed method are listed below.

3.1 Construct the embedded equations

To model the frequency regulation in (1), based on the embedded model of the original power flow equations in [4,8], this paper proposes the dynamic power flow model considering the primary frequency regulation with a complex variable s as follows.

$$\begin{aligned} U_i^*(s) \sum_{j=1}^N Y_{ij} U_j(s) + (K_{Gi} + K_{Li}) \Delta f(s) &= c_i \sum_{j=1}^N Y_{ij} c_j \\ &+ (K_{Gi} + K_{Li}) \Delta F + s \left\{ S_i - c_i \sum_{j=1}^N Y_{ij} c_j - (K_{Gi} + K_{Li}) \Delta F \right\}, \quad i \in \mathbf{q} \\ U_i(s) U_i^*(s) &= c_i c_i^* + s \left\{ |U_i^{sp}|^2 - c_i c_i^* \right\}, \quad i \in \mathbf{p} \\ U_i^*(s) \sum_{j=1}^N Y_{ij} U_j(s) + U_i(s) \sum_{j=1}^N Y_{ij}^* U_j^*(s) + 2(K_{Gi} + K_{Li}) \Delta f(s) & \\ &= s \left\{ 2P_i - (c_i \sum_{j=1}^N Y_{ij} c_j + c_i^* \sum_{j=1}^N Y_{ij}^* c_j^* + 2(K_{Gi} + K_{Li}) \Delta F) \right\} \\ &+ s \left\{ c_i \sum_{j=1}^N Y_{ij} c_j + c_i^* \sum_{j=1}^N Y_{ij}^* c_j^* + 2(K_{Gi} + K_{Li}) \Delta F \right\}, \quad i \in \mathbf{p} \end{aligned} \quad (3)$$

For an N -bus system with a single slack bus, let \mathbf{p} be the set of the PV buses, and let \mathbf{q} be the set of the PQ buses. S_i and P_i are the complex power injection of the PQ buses and the active power injection of the PV buses, respectively. The constant U_i^{sp} is the specified bus voltage magnitude of the bus i for the PV buses. In addition, the constant c_i ($c_i \neq 0$) and ΔF are adjustable, representing the initial guess of the bus voltage and the frequency deviation. Variables $U_i(s)$ and $\Delta f(s)$ are the voltage of the bus i and the frequency deviation of the power system. Note that the unknown variable Δf is introduced to the corresponding equation in the form of $\Delta f(s)$, like $U_i(s)$ in the original FFHE method. Considering

the physical significance of Δf , $\Delta f(s)$ will only have real-valued series coefficients, which is different from $U_i(s)$.

According to the points for constructing the embedded equations mentioned in [4], the germ solution to (3) at the reference state $s=0$ can be developed, and then the task of solving (3) is to obtain $U_i(s)$ and $\Delta f(s)$ at the target state $s=1$. This is because at the target state, the original equations (1) and (2) can be recovered from the embedded equations (3).

The germ solution to (3) is given as follows.

$$\begin{cases} U_i(0) = c_i \\ \Delta f(0) = \Delta F \end{cases} \quad (4)$$

3.2 Calculate the coefficients

The unknown variables $U_i(s)$ and $\Delta f(s)$ in (3) should be written as the power series expansions as follows to keep the model holomorphic.

$$U_i(s) = \sum_{n=0}^{\infty} u_{i,n} s^n, \quad \Delta f(s) = \sum_{n=0}^{\infty} \Delta f_n s^n \quad (5)$$

where $u_{i,n}$ and Δf_n are the coefficients for the s^n terms in the power series expansions of the bus voltage and the frequency deviation. Note that $U_i^*(s^*)$ in (3) should be written as $U_i^*(s^*) = \sum u_{i,n}^* s^n$ to meet the requirements of holomorphicity.

To compute the coefficients $u_{i,n}$ and Δf_n in (5), the following process should be performed. Firstly, since the germ solution (4) at $s=0$ is known, the leading coefficients directly can be listed as follows:

$$u_{i,0} = c_i, \quad \Delta f_0 = \Delta F \quad (6)$$

Taking the equations of the PQ buses for example, substitute the power series expansions into (3), and the following equations can be obtained:

$$\begin{aligned} \sum_{n=0}^{\infty} u_{i,n} s^n \sum_{j=1}^N Y_{ij} \sum_{n=0}^{\infty} u_{j,n} s^n + (K_{Gi} + K_{Ui}) \Delta f_n s^n = c_i \sum_{j=1}^N Y_{ij} c_j \\ + (K_{Gi} + K_{Ui}) \Delta F + s \left\{ \tilde{S}_i - c_i \sum_{j=1}^N Y_{ij} c_j - (K_{Gi} + K_{Ui}) \Delta F \right\} \end{aligned} \quad (7)$$

By comparing the coefficients of s^n , the following relationship can be obtained:

(1) for $n=1$,

$$\sum_{n=0}^1 u_{i,n} \sum_{j=1}^N Y_{ij} u_{j,(1-n)} + (K_{Gi} + K_{Ui}) \Delta f_1 = \tilde{S}_i - c_i \sum_{j=1}^N Y_{ij} c_j - (K_{Gi} + K_{Ui}) \Delta F$$

(2) for $n \geq 2$,

$$\sum_{n=0}^n u_{i,n} \sum_{j=1}^N Y_{ij} u_{j,(n-n)} + (K_{Gi} + K_{Ui}) \Delta f_n = 0$$

Applying the same procedure and considering in the other equations in (3), the linear equations for the unknown coefficients can be derived, which are incorporated with the above relationship to construct the whole linear relationship of all unknown coefficients.

Let \mathbf{X}_n be the vector of the unknown coefficients. \mathbf{A}_n and

\mathbf{B}_n are the matrix and vector calculated by the known parameters and the coefficients for lower terms, such as $u_{i,n'}$, $n' < n$, respectively. The whole linear relationship can be written as:

$$\mathbf{A}_n \mathbf{X}_n = \mathbf{B}_n \quad (8)$$

Given that the germ solution is known for $u_{i,n}$ and Δf_n at the reference state, the unknown coefficients can be derived in the following procedure:

$$\begin{aligned} u_{i,0} = c_i \rightarrow u_{i,1} \rightarrow u_{i,2} \rightarrow u_{i,3} \cdots \rightarrow u_{i,n} \\ \Delta f_{i,0} = \Delta F \rightarrow \Delta f_{i,1} \rightarrow \Delta f_{i,2} \rightarrow \Delta f_{i,3} \cdots \rightarrow \Delta f_{i,n} \end{aligned} \quad (9)$$

It is worth noting that different from the Jacobian matrix in the NR method updated in each iteration, the coefficient matrix \mathbf{A}_n will keep invariant in solving the model.

3.3 Obtain the rational approximants

Padé approximants are rational approximants to power series and they are widely used in analytic continuation for better convergence [9]. After computing all coefficients for the power series of the variables, the Padé approximant for each variable in (3) can be obtained through (10) to represent the numerical solution.

$$X_{i,n} = \det(\Psi(\mathbf{L}_i(\Delta s))) / \det(\Psi(\mathbf{M}_i(\Delta s))) \quad (10)$$

Note that \mathbf{L}_i and \mathbf{M}_i are the matrices that can be derived from the known coefficients for each variable. The specific solution process is introduced in [5].

3.4 Termination criterion

In this paper, the convergence criteria is chosen to be the mismatch in the original power flow equations of each bus type, such as complex power injection for the PQ buses and active power injection for the PV buses. Let the numerical solution obtained from 3.3 replace the unknown variables in (3) and evaluate the max mismatch η_n of (3). If $\eta_n < \varepsilon$ (ε stands for the maximum allowable tolerance), the calculation stops; otherwise, let $n=n+1$, and then go back to 3.2.

4. CASE STUDY

To analyze the performance of the proposed dynamic power flow model based on the FFHE method, detailed results from the test on the IEEE 39-bus system are presented in this section. The termination criterion is set by reducing the max mismatch to 1.0E-6 p.u. (baseMVA=100). All the simulation is established on an Intel(R) Core(TM) i5-1035G7 CPU @ 1.20GHz processor equipped with 8 GB of RAM.

The detailed data of the IEEE 39-bus system are provided by MATPOWER, and the generator outage

occurs at bus 30. The primary frequency control characteristics of generators can be obtained from [10].

The initial values of the state variables after the disturbance are obtained based on the power flow result of the initial stable state, and the initial guess of the frequency deviation is set to be 0. The active power of each generator (per unit), the frequency deviation value (Hz) after the primary frequency regulation, and the computation time obtained from the FFHE method and the NR method are listed in Table 1, and the results verify the proposed model.

Table 1
The results of the frequency regulation based on the FFHE method and the NR method

Symbol	The initial stable state	The current stable state	
		FFHE	NR
G30	2.5000	/	/
G31	6.7787	7.1360	7.1360
G32	6.5000	6.7072	6.7072
G33	6.3200	6.5685	6.5685
G34	5.0800	5.2736	5.2736
G35	6.5000	6.6963	6.6963
G36	5.6000	5.8210	5.8210
G37	5.4000	5.5612	5.5612
G38	8.3000	8.5472	8.5472
G39	10.0000	10.4192	10.4192
$\Delta f(\text{Hz})$	/	-0.1143	-0.1143
T(s)	/	0.037	0.049

It is well known that other generators will increase the output when some generators set down in the system. In addition, only considering the primary frequency regulation, the frequency of the system will decrease as well. Therefore, the results are consistent with the qualitative analysis. As shown in Table 1, when the generator at bus 30 is out of service, the frequency deviation value is -0.1143 Hz and the active power of each generator (per unit) increases.

4.1 Computation accuracy

According to Table 1, the output of each generator (per unit) and the frequency deviation obtained are equal for both the FFHE method and the NR method. To further verify and analyze the computation accuracy of the proposed model, the differences between the state variables obtained from the FFHE method and the NR method are compared in Table 2.

Table 2
The differences between the results based on the FFHE method and the NR method

Symbol	Difference type	
	maximum	average
voltage magnitude (p.u.)	1.9789E-8	1.9459E-9
voltage angle (rad)	2.9036E-8	4.0076E-9
frequency deviation (Hz)	2.3475E-8	2.3475E-8

The maximum differences among the voltage magnitude, the voltage angle, and the frequency deviation are within 1.9789E-8 p.u., 2.9036E-8 rad, and 2.3475E-8 Hz, respectively. This shows that the proposed model based on the FFHE method can provide considerable calculation accuracy like the NR method.

4.2 Computation speed

The computation time of both the FFHE method and the NR method are given in Table 1. It shows that the FFHE method needs less time than the NR method to converge. The reason is that the coefficient matrix A_n of the linear equations stay constant when solving the proposed model by the FFHE method, while the Jacobin matrix in the NR method varies during each iteration and needs more time than the proposed model.

5. CONCLUSION

In this paper, a dynamic power flow model considering the primary frequency regulation based on the fast and flexible holomorphic embedding method is proposed. Considering the frequent disturbances in power systems, the proposed model is tested on the dynamic power flow analysis of the IEEE 39-bus system. Simulation results show that the proposed model based on the FFHE method is effective in analyzing the dynamic power flow when disturbances occur. The proposed model based on the FFHE method can obtain the dynamic power flow within the same tolerance as the Newton Raphson method, and in less computation time.

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