# FuelNet: A precise fuel consumption prediction model using long short-term memory deep network for eco-driving

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#### ABSTRACT

It has been well recognized that driving behaviors significantly impact fuel consumption of vehicles. In this paper, we propose a FuelNet model based on Long Shortterm Memory Neural Network (LSTM NN), which can predict vehicle fuel consumption in a very accurate manner. First, we take the kinetic vehicle parameters and the corresponding fuel consumption parameters to build the FuelNet model, and analyze the correlations between the prediction accuracy and different combinations of input parameters. In addition, our model exhibits the superior capability for fuel consumption prediction (FCP) at different speed, and the comparison with different deep learning models as well as other physics model and data-driven methods suggests that FuelNet can achieve the best prediction performance in terms of both accuracy and stability. Finally, the application of FCP in distinct driving trajectories and abnormal fuel consumption detection performs well, which demonstrates the FuelNet also can provide guidance for eco-driving strategies.

**Keywords:** Fuel consumption prediction (FCP), Long short-term memory (LSTM), Deep network, FuelNet, Ecodriving strategies

#### INTRODUCTION

According to the World Health Organization, transportation emission is a significant and growing contributor to particulate air pollution, which makes up 30% of particulate matter emissions (PM) in European cities and 50% of PM emissions in OECD countries [1]. As a result, it is critical to estimate the cumulative fuel consumption of any generated trajectory in the time and space domains with fuel consumption prediction (FCP), drivers can make the optimal driving strategy based on energy efficiency to reduce vehicle emissions.

As we all know, fuel consumption can be measured directly in real-time by instruments. However, if we can predict the fuel consumption for any future trajectories of a vehicle, the optimal trajectory can be sent to the driver or the ECU, which helps the vehicle run in an energy-saving way. FCP is also used to solve a series of intelligent transportation-based issues, such as optimization of intersection traffic, smart lane-changing decisions for an autonomous vehicle, and prediction of remaining mileage.

Recently, many data driven models have been proposed to deal with FCP, including physics-based methods, statistical and regression methods, and artificial intelligence technology. The classic models based on physics are the vehicle specific power (VSP) model [2], Comprehensive Modal Emission Model [3], and VT-Micro model [4]. Jiménez et al. [2] proposed the concept of vehicle power ratio and applied it to fuel consumption estimation and vehicle emissions. They established a VSP-based emission model. The Comprehensive Modal Emission Model [3], developed by An F et al. can predict exhaust emissions and fuel consumption in real-time through clock-by-second driving patterns and real-time engine data. Rakha et al. [4] established a VT-Micro model for light-duty vehicles under thermal stability, which was determined by the regression coefficient and product combination of different accelerations and speeds.

Although these models can obtain reasonable results, they require second-by-second speed-fuel data, which is often unavailable with current connectedvehicle technology, and the calibration of coefficients is

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tedious. Artificial neural network models have been applied to establish the fuel consumption model of the tractor [5], vehicle [6] and hauling trucks in surface mines [7]. Xu et al. [8] developed a generalized regression neural network (GRNN) model to establish implicitly the relationship between truck fuel consumption and the truck driver's driving behavior obtained from the Internet of Vehicles. However, traditional neural network has higher requirements for input features and requires a long training time, but has low prediction accuracy and generalization performance.

With the rapid development of artificial intelligence technology, deep-learning [9] approaches make possible of strong learning ability, wide coverage, strong adaptability, and good portability. They can also handle high-dimensional and nonlinear relationships, such as convolutional neural networks [10], recurrent neural networks (RNNs) [11], long short-time memory (LSTM) neural networks [12], and gated recurrent units (GRU), which have achieved good results in the fields of computer vision, speech recognition, and natural language processing. Among these common algorithms, the RNN model [11], which is popular in time-series prediction, has a gradient disappearance problem in processing long-term dependency information, and the accuracy in predicting is not ideal. Compared with conventional RNNs, LSTM has the key component memory cell, which can capture the features of time series within longer time spans and overcome the gradient disappearance problem to solve many timeseries data-related problems, and it is considered particularly efficient for long-term time-series prediction, such as travel time prediction [13], pedestrian trajectory prediction [14], and traffic flow prediction [15]. The vehicle fuel consumption data that this paper plan to predict are time-series data, which have complex nonlinear relationships and fluctuate with time. Therefore, we chose the LSTM model for FCP.

The contributions of this study are three-fold. First, by taking advantage of the recent development of deep learning techniques, we design an LSTM-based vehicle FCP model and determine the optimal configuration of it, which is able to predict fuel consumption and provide a general guideline for drivers on choosing energy-efficient driving behavior. Second, our model can predict fuel consumption accurately in wide speed range of 10 to 80 km/h as well as for distinct driving trajectories (highspeed, optimal-speed, and stop-and-go) and applied well in abnormal fuel consumption detection. Third, the robustness and superiority of the proposed model on FCP are validated by comparing it with five well-recognized existing models.

The rest of this paper is organized as follows. Section 2 states the vehicle FCP problem to be studied in a general manner and introduces the structure of the proposed LSTM-based FuelNet model. Section 3 describes the selection process of the optimal configuration of the proposed model. Section 4 evaluates the proposed model by comparing it with five recognized models and discusses the application of FCP on eco-driving at a signalized intersection and abnormal fuel consumption detection. Section 5 concludes this paper and provides directions for future research.

#### 2. PROBLEM STATEMENT



In this study, we apply LSTM to predict the fuel consumption for any vehicle trajectory, which will then be used to guide the driving behavior and save energy. Fig. 1 shows the time-space trajectories of a vehicle passing a road segment before a signalized intersection. In this figure,  $Y_1$  represents the trajectory of the vehicle that drives at a high speed to go through the intersection before the green phase ends;  $Y_2$  represents the trajectory of a vehicle that decelerates to stop at the stop-line, and goes through the intersection in the green phase of the next signal cycle, and  $Y_3$  represents the trajectory of a vehicle that coasts slowly to go through the intersection just in the green phase of the next signal cycle. These three trajectories depart from the same time and space origin, which stands for three typical driving behaviors for a driver deciding to choose in the dilemma zone of an intersection.

Before selecting the optimal trajectory, it is necessary to predict accurately the fuel consumption of the candidate trajectories, which is time-series data. LSTM is a dedicated deep-learning network for timeseries training and identification, which can well solve the gradient disappearance problem in processing longterm dependency information that other deep learning models face. In this study, we constructed an LSTM- based FCP model with the most suitable input features and the optimal configuration of the training sequences.

#### 2.1 LSTM neural network model

Long short-term memory neural network (LSTM NN) is a variant of RNN, which was introduced by Hochreiter [12]. RNNs can only have short-term memory owing to the gradient disappearance problem. LSTM NN combines short-term memory with long-term memory through gate control, which can pass information selectively, and solves the gradient disappearance problem to some extent. LSTM NN has three gates to protect and control the cell states, and the most important gate is the forget gate, followed by the input gate, and finally the output gate (as shown in Fig 2). The forget gate makes LSTM can save the information from a long time ago, the input gate makes LSTM can prevent insignificant content from entering the memory, and the output gate determines the output value based on the cell state. The hidden layer of the original RNN has only one state, namely, h, which is very sensitive to short-term inputs but not to longterm inputs. Thus, the LSTM NN uses a cell state, namely, c, which is used to save the long-term state, and these two states flow with time [12].

# 2.2 Structure of the LSTM-based FuelNet model

To save storage, computing resources and computing time while ensuring the prediction accuracy of vehicle fuel consumption, this study uses the traditional 3-layer LSTM NN model, which contains an input layer, an output layer, and a hidden layer. The first layer is the input layer, with historical vehicle speed, GPS (longitude, latitude), acceleration, and fuel consumption data as input; the second layer is the hidden layer, which is used to store the number of nodes in the past state. The third layer is the output layer, which exports the predicted fuel consumption (as shown in Fig 3). Because both input and output are sequences, the LSTM structure used in this study can be called a sequence-to-sequence model. The detailed structure of the LSTM-based FuelNet model is shown in Fig 3. In this model, there is a direct connection between input and output; no intermediate conversion is required, and the output can be obtained directly from the input. In Fig 3, c represents the cell state, and [, ] means joining matrices  $\mathbf{h}_{t-1}$  and  $\mathbf{x}_t$  into one matrix.

The proposed LSTM-based FuelNet model can be formulated by Eq. (1):

$$\hat{v}(t+1) = f_{FCP}(X_{FCP}(t); \Phi) \tag{1}$$

The output variables of the FCP model are  $\hat{y}(t+1)$ , which denotes the predicted fuel consumption at the

next time step t + 1.  $\Phi = \{\phi_1, \phi_2, ..., \phi_n\}$  represents the set of parameters of the LSTM.  $X_{FCP}(t)$  is the input feature matrix at time t, which is generalized as follows:  $X_{FCP}(t) = \{x_1(t), x_2(t), ..., x_m(t)\},$  (2) where m denotes the number of input features. From

where *m* denotes the number of input features. From Table 1,  $X_{FCP}(t)$  can be written as follows:



 v
 a
 Py
 Pz
 y

 Fig 3 Structure of the LSTM-based FuelNet Model

t+n

[,]

t + 1

# B. OPTIMAL CONFIGURATION OF LSTM-BASED

# 3. OPTIMAL CONFIGURATION OF LSTM-BASED FUELNET MODEL

# 3.1 Screening of the inputs

**[**, ]

**[**, ]

*t* - 1

There are five variables that are closely related to fuel consumption in the data collected in this study: GPS (longitude, latitude), speed, and acceleration. However, using all these variables as input features may not necessarily achieve the best prediction performance and can decrease prediction efficiency. In order to choose the suitable feature variables as the input set to obtain optimal results, we examined the prediction performance of the model with five different combinations of the input parameters and performed eight groups of FCP experiments with speed inputs of 10-80 km/h.

The prediction results with five different variable combinations as input features and the error of each combination are shown in Fig 4 (in each sub-figure, the left is the prediction result, and the right is the prediction error). It can be seen in Fig 4 that the FuelNet model has the best performance in predicting fuel consumption when its input is a combination of speed and acceleration (error is between -0.73-0.59), and has the worst performance when the input is a combination of speed, acceleration, and GPS (error is between -0.93-1.95). This

is because speed and acceleration are the main manifestations of driving behavior; both have a significant relationship to fuel consumption.

То analyze the above prediction results quantitatively and check the applicability of the proposed model, the widely used evaluation indexes, root mean squared error (RMSE), relative error (RE), and coefficient of determination (R<sup>2</sup>) [16], are used in this study. RMSE represents the sample standard deviation of the differences between actual data and predicted values; the smaller RMSE and RE represent the better prediction performance. The closer the coefficient of determination value is to 1, indicating a better data fit, the higher the degree of interpretation of the dependent variable by the independent variable.

The results of quantitative analysis of the LSTMbased FuelNet model with different sets of inputs are listed in Table 1. The results with the best performance are marked in bold. It can be seen in Table 1 that the worst prediction performance is obtained when the input features are GPS and acceleration; when the input feature is speed or speed and acceleration, the prediction results are both better than other combinations. But a better prediction result can be observed compared with the one with only speed input in most instances.

### 3.2 Configuration tuning

The hyper-parameters in the LSTM network such as the hidden size, number of iterations, batch size and learning rate need to be carefully tuned, which directly affect the accuracy and efficiency of prediction.

According to a series of experiments, we choose 200, 50, 100 as the batch size, the number of iteration and the hidden size, respectively. And we found the bestperforming training set size being 20000. In addition, the optimization function and loss function are Adam (Adaptive Moment Estimation, which can adaptively adjust the learning rate) and mean absolute error, respectively. In this study, a personal computer with a CPU of 3.7 GHz is used to conduct the computational experiments with a deep-learning framework: Tensorflow1.9.0-CPU + keras + Spyder3.0.

# EXPERIMENT AND APPLICATION

# 4.1 Comparison with other fuel consumption estimation methods

Our proposed FuelNet was compared with a few well-regarded models such as VSP, VT-Micro, GRNN, RNN



Fig 4 Prediction results of FuelNet with five different sets of inputs: (a) Inputs: speed, GPS and acceleration;(b) Input: GPS and acceleration; (c) Input: GPS and speed; (d) Input: speed; (e) Input: acceleration and speed

Table 1 Prediction performances of LSTM-based FuelNet model with five different sets of inputs under different speed conditions

Input	10km/h			20km/h			30km/h			40km/h		
mput	RMSE	R <sup>2</sup>	RE	RMSE	R <sup>2</sup>	RE	RMSE	R <sup>2</sup>	RE	RMSE	R <sup>2</sup>	RE
Speed + GPS + acceleration	0.242	0.643	0.157	0.170	0.614	0.102	0.219	0.865	0.049	0.377	0.826	0.089
GPS + acceleration	0.173	0.666	0.110	0.161	0.950	0.019	0.230	0.851	0.066	0.370	0.832	0.073
Speed + GPS	0.131	0.808	0.070	0.111	0.835	0.063	0.247	0.828	0.084	0.376	0.827	0.059
Speed + acceleration	0.142	0.775	0.080	0.059	0.954	0.015	0.217	0.897	0.035	0.360	0.829	0.058
Speed	0.133	0.803	0.072	0.057	0.957	0.009	0.218	0.866	0.035	0.362	0.827	0.056
lagut	50km/h			60km/h		70km/h			80km/h			
input	RMSE	R <sup>2</sup>	RE	RMSE	R <sup>2</sup>	RE	RMSE	R <sup>2</sup>	RE	RMSE	R <sup>2</sup>	RE
Speed + GPS + acceleration	0.328	0.891	0.082	0.369	0.880	0.057	0.410	0.877	0.064	0.361	0.978	0.101
GPS + acceleration	0.321	0.901	0.051	0.368	0.891	0.084	0.399	0.879	0.066	0.335	0.981	0.042
Speed + GPS	0.330	0.896	0.067	0.368	0.891	0.072	0.401	0.875	0.071	0.345	0.980	0.087
Speed + acceleration	0.323	0.900	0.043	0.357	0.897	0.050	0.387	0.878	0.060	0.317	0.983	0.029
Speed	0.326	0.899	0.046	0.360	0.896	0.053	0.389	0.876	0.058	0.320	0.983	0.031

and GRU models under three speed conditions 10-30 km/h, 30–60 km/h, and 60–90 km/h. The experimental results are shown in Fig 5. From the prediction results, it can be seen that all six models performed well in predicting fuel consumption, but the fit degree of the proposed FuelNet is the best. The box plots are used to show the results of the relative errors. The maximum relative error value of the proposed FuelNet model is below 37%, and the maximum relative error value of the other five models is above 50%. From the absolute error histogram, it can be seen that the absolute errors of FuelNet mainly concentrate in the range of 0-0.1 at the three speeds, while other methods have a long-rail distribution on absolute errors.

Therefore, from these three sub-figures, it can be concluded that the proposed FuelNet model has the best prediction performance. In addition, the RE, RMSE, and R<sup>2</sup> of the six models are listed in Table 2. As can be seen in Table 2, deep learning is better than physical-principlebased methods and data-driven methods. There is no major difference in the prediction results between these deep-learning models. Compared with other algorithms, our proposed FuelNet model performs well and has the best performance in terms of RE, RMSE, and R<sup>2</sup>.



Fig 5 Prediction results of different models under three speed conditions: (a) 10–30 km/h; (b) 30–60 km/h; (c) 60–90 km/h

4.2 Applications of FuelNet

4.2.1 Case 1: Eco-driving at a signalized intersection



Fig 6 Driving trajectories for three driving conditions

Next, we use the proposed FuelNet to predict the fuel consumption of vehicles under different driving conditions to further demonstrate its universal applicability and practical application value. To show that the proposed FuelNet is suitable for vehicle FCP under different driving conditions, we chose to perform FCP experiments in the optimal-speed driving state, highspeed driving state, and stop-and-go driving state, and each group of experiments was repeated three times. Only one car at each time was used for the experiments.

The average speed of optimal-speed, high-speed, and stop-and-go driving conditions were 31.8 km/h, 68.3 km/h, and 33.9 km/h, respectively. The driving distance of each condition was 300 m. The driving trajectory of each condition above is shown in Fig 6. The experimental results are shown in Fig 7. It can be seen that under different driving conditions, the proposed FuelNet can better predict the fuel consumption of each trajectory. To determine the above prediction results more accurately, a quantitative analysis was performed. The results of the RMSE, RE, and R<sup>2</sup> calculations are listed in Table 3.

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It can be seen from Table 3 that the FCP results of the three types of trajectories are relatively good. The average value of the RMSE is 0.368; the average value of  $R^2$  is 0.901, and the average value of RE is 0.049. It can be concluded that the proposed FuelNet can predict the fuel consumption of optimal-speed trajectories, high-speed trajectories, and stop-and-go trajectories, which provides a reference for selecting driving behaviors that can save energy and reduce emissions.

### 4.2.2 Case 2: Detection of abnormal fuel consumption

FuelNet can also be used to detect abnormal vehicle fuel consumption. By comparing and analyzing prediction results of FuelNet with actual value, the driver can find abnormal fuel consumption faults as early as possible and can be assisted in screening the reasons of faults, so as to avoid excessive fuel waste in time.

The fuel consumption data used in this experiment is from Shaanxi Motor Truck, and the prediction result is shown in Fig 8. It can be seen that before the 402<sup>th</sup> second, the overlap of the predicted and real value is good with the small absolute error. But after then, the absolute error significantly increased, the maximum of which is up to 7.093. This situation was finally diagnosed by the technician, it was caused by the oil leakage from pipeline connected to the engine.

#### 5. CONCLUSION

In this paper, a vehicle FCP method based on LSTM NN, namely FuelNet, is proposed to provide a reference for eco-driving. It can model the long-term dependency characteristics of time-series data by selecting suitable LSTM NN parameters. Subsequently, to improve the prediction accuracy, we studied the influence of the

Model	10-30km/h			30-60km/h			60-90km/h		
Model	RMSE	R <sup>2</sup>	RE	RMSE	R <sup>2</sup>	RE	RMSE	R <sup>2</sup>	RE
FuelNet	0.135	0.936	0.021	0.394	0.978	0.055	0.376	0.975	0.038
VSP	0.907	0.526	0.479	1.351	0.687	0.573	1.300	0.693	0.230
VT-Micro	1.105	0.610	0.409	1.273	0.713	0.623	1.345	0.680	0.256
GRNN	0.353	0.803	0.247	0.832	0.784	0.132	0.711	0.800	0.213
RNN	0.202	0.909	0.051	0.530	0.961	0.120	0.612	0.934	0.110
GRU	0.558	0.617	0.337	0.821	0.906	0.178	0.982	0.830	0.154

Table 2 Prediction performances of different models under three speed conditions



Fig 7 Prediction results of FuelNet models with three driving conditions: optimal speed (left), high speed (middle), stop-and-go (right)

Table 3 Prediction	performances	of FuelNet	models with	three drivin	ng trajectories
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_	Driving state	Average speed (km/h)	Predicted fuel consumption (L/100km)	Actual fuel consumption (L/100km)	RMSE	R2	RE
	Optimal speed1	33.4	9.08	9.12	0.403	0.844	0.057
	Optimal speed2	25.4	6.33	6.32	0.237	0.906	0.039
	Optimal speed3	36.6	9.20	9.55	0.334	0.930	0.051
	High speed1	70.5	16.48	16.44	0.337	0.982	0.036
	High speed2	69.0	14.40	14.31	0.440	0.929	0.053
	High speed3	65.5	13.28	13.20	0.439	0.929	0.047
	Stop-go1	30.7	13.30	13.31	0.403	0.806	0.052
	Stop-go2	32.1	12.60	12.60	0.377	0.894	0.057
	Stop-go3	38.9	11.69	11.68	0.345	0.891	0.047
-		0.368	0.901	0.049			



Fig 8 Prediction results of FuelNet

composition of the input features and the size of the training set in improving the prediction accuracy under different speed conditions. In addition, the five recognized models are compared with the FuelNet based on the same dataset. Finally, we displayed the prediction performance of FuelNet for three types of driving trajectories, which validated its feasibility and superiority in FCP.

Several useful findings can be learned from this study:

• The best results are usually obtained when speed and acceleration are combined to predict future fuel consumption under various speed conditions. The

proposed FuelNet model can accurately predict fuel consumptions in wide speed range of 10 to 80 km/h.

- A comparison among the proposed FuelNet, VSP, VT-Micro, GRNN, RNN, and GRU models showed that the prediction performance of FuelNet is significantly better than that of other models.
- Two case studies were conducted, which concluded that the proposed FuelNet model is suitable for different driving trajectories to predict fuel consumption, which provides a reference for selecting driving behaviors that can save energy and reduce emissions.

Our current approach does not improve the internal structure of the LSTM network, and only used this deep learning method without combine with other neural networks for fuel consumption prediction. Future work will investigate possible techniques to improve the LSTM network and combine with other neural networks for FCP or other applicable problems.

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