

# UNIT COMMITMENT WITH CLUSTERED UNITS AND REPRESENTATIVE PERIODS

Jingbo Wang, Ce Shang\*

Ministry of Education Key Laboratory of Control of Power Transmission and Conversion, Department of Electrical Engineering,  
 Shanghai Jiao Tong University

\* Corresponding Author

## ABSTRACT

Long-term optimized operation has been looking to unit commitment to replace the traditional power flow for more accurate modeling of system operation. Alongside the intuitive advantage of representing not only the production of the generation units of consecutive time periods but also their commitment status, comes the heavier computational burden for the multi-unit long-term model. In fact, representing the commitment status of each unit with a binary variable in the corresponding constraints alone complicates the optimization, let alone other accessory variables, such as those for switch actions. Moreover, unit commitment is usually conducted over a narrow window of time, e.g., one day or one week with an hourly resolution; ordinary unit commitment models that run over an annual load profile will be hindered from even converging to a solution by heavy computation. As such, clustering techniques are proposed to be equipped with the unit commitment in two dimensions, one to select representative periods for the load profile of a long horizon, the other to group homogeneous or similar units. As the former is conducted in the time dimension, it can be named the temporal clustering, and the latter hence named the spatial clustering. As a result, this new method with both is therefore named the tempo-spatially clustered unit commitment. The case study on a 39-node 17-unit system proves the efficacy and efficiency of the proposed unit commitment approach.

**Keywords:** clustering, machine learning, unit clustering, unit commitment, representative days

## NONMENCLATURE

<i>Indexes</i>	
t	Clustered time period index from 1 to T
g	Generators index from 1 to G
c	Clustered units index from 1 to C
l	Lines index from 1 to L
n	Nodes index from 1 to N
$RP^{n(-n)}$	Subset with the first (last) n periods of each representative day
$CI_d$	Cluster index set that records which representative day represents day d
$NL_n^{+(-)}$	Lines set flowing into (out of) node n
$\tau_t$	Weight of time period t
<i>Parameters</i>	
$C_g^{var}$	Variable operating cost [\$/MW]
$C_c^{up}$	Start-up cost [\$/MW]
$C_c^{fix}$	Fix cost [\$/MW]
$C_g^{inv}$	Investment cost of g [\$]
$\bar{p}_g/\underline{p}_g$	Upper/lower bound on production of generator g [MW]
$r_g^{up}/r_g^{down}$	Hourly ramp up/down rate [MW]
$B_l$	Susceptance of line l
$\bar{F}_l$	Transmission capacity of line l [MW]
$d_{n,t}$	load demand [MW]
$\bar{P}_c/\underline{P}_c$	Upper/lower bound on production of clustered generator c [MW]
$R_c^{up}/R_c^{down}$	Hourly ramp up/down rate [MW]
$CN_c$	The number of units clustered into category c
<i>Variables</i>	

$V_{c,t}^{up} / V_{c,t}^{down}$	Integer variable of the number of start-up / shut-down units cluster c
$U_{c,t}$	Integer variable of the number of on-line units in unit cluster c
$p_{g,t}$	Power generation of generator g [MW]
$P_{c,t}$	Power generation of generator cluster c [MW]
$\theta_{n,t}$	Phase angle of bus node n
$F_{l,t}$	Power flow of line l [MW]
$v_{g,t}^{up}$	Binary variable of the generator g start-up / shut-down status
$u_{g,t}$	Binary variable of generator g online status

## 1. INTRODUCTION

The heavy computational burden of general unit commitment mainly comes from the integer variables. Many studies dedicated to reducing the computational complexity of unit commitments are looking at reducing the number of integer or binary variables. Palmintier and Webster proposed a method of clustering similar units [1]. In this model, binary variables of all units are represented by integer variables after clustering, which greatly reduces the number of binary and integer variables, and significantly improve calculation efficiency [2]. However, this model requires the clusters to be clustered under the same node. Du et al. solved this problem and applied this model to scenarios that consider network constraints [3]. But when the time scale of the load data is expanded to the whole year, the calculation time will still increase to an unacceptable level. When applied to the long-term operation, the integer variables need to be relaxed into continuous variables. However, the result of relaxation model usually contains decimals to represent integer variables, such as 0.4 start-up states, which is impossible in reality. And this makes the power of the unit also able to obtain all values between 0 and the maximum power, which in fact, is not completely restricted by commitment status.

Another technique to reduce computation burden is time-period clustering method. This method has been widely used in unit commitment problems. And [4], [5] respectively propose two methods which retrieve the time chronological information disrupted by clustering, making this method more accurate and effective.

Time-domain clustering technique is applied to the model proposed in [3], and a new method named tempo-spatially clustered unit commitment (TSCUC) is proposed in this study. Since it adopts two dimensions to reduce

computational complexity, it can be used for the analysis of large systems over a long time scale.

## 2. TEMPO-SPATIALLY CLUSTERED UNIT COMMITMENT MODEL

For brevity, a single constraint is used to describe the main difference between the general unit commitment (UC) model and the clustered unit commitment (CUC) model proposed in [3]. The contribution of this article is also explained by it.

$$\sum_{t'=t-T^{on}}^t v_{g,t'}^{up} \leq u_{g,t}, \quad \forall t > T^{on} \quad (1)$$

$$\sum_{t'=t-T^{on}}^t V_{c,t'}^{up} \leq U_{c,t}, \quad \forall t > T^{on} \quad (2)$$

$$\sum_{t'=t-T^{on}}^t V_{c,t'}^{up} \leq U_{c,t}, \quad \forall t \notin RP^{T^{on}} \quad (3)$$

Constraints (1) – (3) are the expressions of minimum online time constraints under three different models. Constraint (1) belongs to UC model with binary decision variables  $v_{g,t}^{up}$  and  $u_{g,t}$  meaning the turn on decision and online status of generator g. CUC model replaces all binary variables  $v_{g,t}^{up}$  with an integer variable  $V_{c,t}^{up}$  in (2) when generator g is clustered into unit category c. Discontinuous variables are reduced in this way. The TSCUC model proposed in this study introduces the time clustering into the CUC model; the time chronological information lost during the cluster process, so all constraints related to time continuity can only be considered within the representative period. So (3) only constraints the status when not in the first n periods of each representative period. In addition to the restriction on the time set of the constraints, the weight coefficient  $\tau_t$  also needs to be added to the objective function.

The TSCUC model is formulated as the flowing mixed integer linear programming problem:

$$\min \sum_{t=1}^T \sum_{c=1}^C \tau_t C_c^{up} V_{c,t}^{up} + \sum_{t=1}^T \sum_{c=1}^C \tau_t C_c^{fix} U_{c,t} + \sum_{t=1}^T \sum_{g=1}^G \tau_t C_c^{var} p_{g,t} \quad (4)$$

$$\text{s.t.} \quad p_{g,t} \leq \bar{p}_g \quad \forall g, t \quad (5)$$

$$p_{g,t} - p_{g,t-1} \leq r_g^{up}, \quad \forall t \notin RP^1, g \quad (6)$$

$$p_{g,t-1} - p_{g,t} \leq r_g^{dn}, \quad \forall t \notin RP^1, g \quad (7)$$

$$F_{l,t} = B_l(\theta_{l+,t} - \theta_{l-,t}), \quad \forall l, t \quad (8)$$

$$\underline{F}_l \leq F_{l,t} \leq \bar{F}_l \quad \forall l, t \quad (9)$$

$$f_{n,t} = \sum_{l \in NL_n^+} F_{l,t} - \sum_{l \in NL_n^-} F_{l,t}, \quad \forall n, t \quad (10)$$

$$\sum_{g \in n} p_{g,t} \geq d_{n,t} + f_{n,t}, \quad \forall n, t \quad (11)$$

$$P_{c,t} \leq U_{c,t} \bar{P}_c, \quad \forall c, t \quad (12)$$

$$P_{c,t} \geq U_{c,t} \underline{P}_c, \quad \forall c, t \quad (13)$$

$$P_{c,t} - P_{c,t-1} \leq U_{c,t} R_c^{up}, \quad \forall t \notin RP_1 \quad (14)$$

$$P_{c,t-1} - P_{c,t} \leq U_{c,t-1} R_c^{dn} - V_{c,t}^{up} \underline{P}_c, \quad \forall t \notin RP_1 \quad (15)$$

$$U_{c,t} - U_{c,t-1} = V_{c,t}^{up} - V_{c,t}^{dn}, \quad \forall t \notin RP^1 \quad (16)$$

$$\sum_{t'=t-T^{on}}^t V_{c,t'}^{up} \leq U_{c,t}, \quad \forall t \notin RP^{Ton} \quad (17)$$

$$\sum_{t'=t-T^{off}}^t V_{c,t'}^{dn} \leq CN_c - U_{c,t}, \quad \forall t \notin RP^{Toff} \quad (18)$$

$$\sum_{c=1}^C P_{c,t} \geq \sum_{n=1}^N D_{n,t}, \quad \forall t \quad (19)$$

$$P_{c,t} = \sum_{g \in c} p_{g,t}, \quad \forall t \quad (20)$$

$$U_{c,RP_{Clp-1}^{-1}} = U_{c,RP_{Clp}^1}, \quad \forall p > 1, c \quad (21)$$

The TSCUC model endows most of the framework from the CUC model in [3], expect that the time sets of some constraints are redefined and inter-day constraint is added to create continuity between representative days [5].

The objective function (4) minimizes the total system cost, which includes total start-up cost, fix cost and variable cost. The coefficient  $\tau_t$  which represents the number of periods represented by period t is introduced due to the time domain clustering. The constraints can be divided into three parts. Constraints (5) -- (11) make up the first part which are derived from the dispatch only model with network constraints. Constraint (5) ensures the production of generator under the upper bound. Constraints (6) and (7) enforce the down-ramping limit and the up-ramping limit of generator. Equation (8) and (10) calculate the line power flow and node power flow. Constraint (9) enforce the maximum capacity of transmission lines. Constraint (11) maintains node power balance. The second part of constraints consists (12) -- (19), which come from the unit cluster model. Constraints (12) -- (15) limit the production and ramp rate of unit categories. Equation (16) combines the commitment status and the start-up/shut-down decisions. The minimum time that the units must be online and offline are ensured with Constraints (17) and (18). And constraint (19) ensure the load power balance

of the entire system. The third part of the constraints contains only one equation (20), which is the link constraint connecting the previous two part of constraints.

The time domain of constraints (6) – (7) and (14) – (18) is limited to the representative day. Constraint (21) retrieves the time chronological information that was disrupted after time domain clustering through the index array, ensures the consistency of the on/off states of the adjacent days before clustering.

### 3. CASE STUDY

RTS-39 system was used to test the performance of the proposed TSCUC model. The system contains 17 units with total capacity of 7460 MW distributed on 8 nodes. The total year energy demand is 35.2 TWh and the maximum demand is 6940 MW.

In order to measure the error of the proposed method, the general unit commitment model [6] with DC power flow constraints [7] is used to be reference. And for the purpose of differing the influence of the two reduction methods used in proposed model, the network constraint cluster unit commitment model (CUC) [3] is involved to be another reference. Both CUC and TSCUC cluster the 17 units into 5 unit categories.

#### 3.1 One-week Comparison

Three models are run on the first week of the year and the results show in Table 1. Three, four and five representative days are used to represent the total week demand in TSCUC model. The three rows in the table indicate the total cost, the error relative to the UC model, and the solution time. The result shows TSCUC model greatly reduces the computational time with acceptable objective function error.

TABLE 1 Total cost and computational time (one week)

	UC	CUC	TSCUC 3-rp	TSCUC 4-rp	TSCUC 5-rp
C(M\$)	11.55	11.61	11.49	11.47	11.49
Error	0%	0.52	-0.52%	-0.69%	-0.52%
T(sec)	21.44	3.13	0.69	1.03	1.33

#### 3.2 One-month Comparison

When the commitment period is expanded to one month, and 7, 10 and 14 representative days are chosen in TSCUC model. The results shown in Table 2 indicates that the conclusion drawn above has not changed.

TABLE 2 Total cost and computational time (one month)

	UC	CUC	TSCUC 7-rp	TSCUC 10-rp	TSCUC 14-rp
C(M\$)	52.73	52.96	52.71	52.56	52.89
Error	0%	0.44%	-0.04%	-0.32%	0.30%
T(sec)	679.72	24.93	5.58	3.93	9.64

### 3.3 One-year Case

When the amount of data is expanded to one year, the UC model and CUC model cannot get results in an acceptable time with limited disk space. The result of these models shown in Table 3 is calculated from the results of week by week.

The model can still run with acceptable error and calculation time during the whole year. The model can run continuously during the whole year instead of running in stages.

TABLE 3 Total cost and computational time (one year)

	UC	CUC	TSCUC 12-rp	TSCUC 24-rp	TSCUC 48-rp
C(M\$)	624.66	627.16	627.63	626.29	624.95
Error	0%	0.40%	0.48%	0.26%	0.05%
T(sec)	8379.23	451.27	17.57	160.20	431.28

Figure 1 shows the transformation of error and calculation time with the number of representative periods on the annual data. A good trade-off between CPU time and error can be chosen by adjust the number of representative periods.

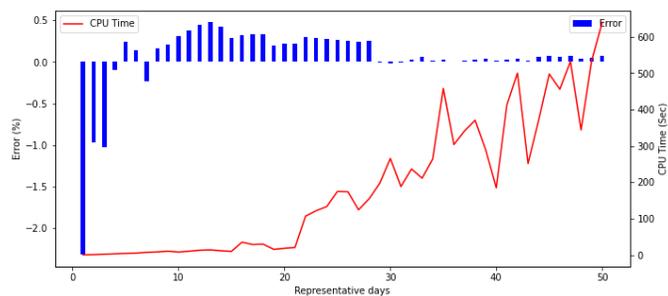


Fig 1 Error and CPU time

## 4. CONCLUSION

Time-period aggregation and unit aggregation are the reduction methods that have been applied in two dimensions for unit commitment which has gradually covered more generation units over longer periods. Both methods entail clustering techniques. This study combines the clustering techniques applied in both dimensions, i.e., the temporal clustering and spatial clustering, and proposes a new unit commitment named

the tempo-spatially clustered unit commitment (TSCUC). The examples based on a 39-node 17-unit system show that TSCUC greatly reduces the computational burden of unit commitment while maintaining accuracy.

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