

Research on Leak Detection and Location of Natural Gas Pipeline

Ying Yu

Dept of Chemical Engineering
China University of Petroleum-Beijing
Beijing, China
yuyingphd@163.com

Weiting Tang

Department of Chemical Engineering
Texas Tech University
Lubbock, Texas, United States
wttang511@gmail.com

Xinghua Pan

Artie McFerrin Department of
Chemical Engineering
Texas A&M University
College Station, Texas, United States
xhua.pan@gmail.com

M. Nazmul Karim

Artie McFerrin Department of
Chemical Engineering
Texas A&M University
College Station, Texas, United States
nazkarim@mail.che.tamu.edu

Abstract—An evaluation of thermodynamic effects on leak detection and location estimation in natural gas pipelines was provided through non-isothermal process model. To estimate the process states (discretized pressures and mass flow rates) under varying thermodynamic conditions for leak detection and location estimation, a state and parameter estimation method (dual unscented Kalman filter) was proposed for the pipeline model. The dual unscented Kalman filter was adopted to estimate both the states and parameters in the presence of parameter uncertainties. Using the data generated from the non-isothermal model, the proposed new algorithm can detect the leak location efficiently and a real case study was performed to validate it.

Keywords—Dual Unscented Kalman Filter, Parameter Estimation, Leak Detection, Non-Isothermal Modeling, Natural Gas Pipeline

I. INTRODUCTION

Pipelines are one of the most economical transport solutions for natural gas. However, leakage of natural gas from pipelines could harm the environment and cause damage to properties and human lives. In the literature, software methods have been proposed for pipeline leak detection such as the real-time transient method, mass balance-based method[1], and pressure point analysis.[2] However, the real-time transient method requires extensive instrumentation for thermal measurements and the pressure point analysis method cannot accurately estimate the location of a leak. [2–4]

Model-based fault diagnoses use dynamic models to estimate the state of a process.[5,6] To monitor the process, state estimates from model-based estimation techniques are compared with system measurements. Many researchers have conducted modeling and simulation of natural gas flow in pipelines to estimate the flow rate.[7–11] Various simulation methods have been proposed considering non-isothermal conditions and pipeline networks.[12–14]

Although most of the previous studies with model-based natural gas pipeline leak detection utilized isothermal models [15,16], some researchers have attempted to investigate the non-isothermal pipeline flow phenomena, for either steady state or transient state. Osiadacz and Chaczykowski compared the isothermal and non-isothermal pipeline gas flow models.[17] They studied both the steady-state and transient-state flow dynamics. Chaczykowski derived a one-dimensional non-isothermal flow model to study the transient behavior of the fluid flow phenomena.[18] Abbaspour and Chapman studied a non-isothermal transient flow in a natural gas pipeline considering the convective inertia term, friction factor changes with Reynolds number, and compressibility factor as a function of the temperature and pressure.[19] However, a non-isothermal modeling of gas flow in a pipeline with a leak and its influence on the mass flow rate and pressure under non-isothermal conditions has not yet been studied. In this study, leak terms were added to the pipeline model to account for the loss of mass, momentum, and energy. By incorporating the leak terms, thermodynamic changes caused by the leak can be investigated.

In order to apply the model-based fault detection method to locate leaks in pipelines, the flow rate was estimated at nominal conditions. A comparison between state measurement and state estimation was performed to locate the leaks. Owing to the existence of process noises, filtering techniques are required to obtain an accurate estimation of the states.

Researchers have studied filter-based leak detection methods for natural gas pipelines using dynamic models. Benkherouf and Allidina used the extended Kalman filter (EKF) for state estimation.[20] Liu et al. improved the accuracy of state estimation by using an adaptive particle filter to estimate the leak location, and Emara-Shabaik et al. applied a modified EKF for leak estimation.[21,22] Hauge et al. designed an adaptive Luenberger observer in monitoring oil and gas pipelines for leak detection.[23] Model-based leak detection methods for a water pipeline were also developed. [24] Rui et al. developed a model-

based method to locate two leaks from a natural gas pipeline. [25]

The application of filtering or state estimation technique for non-isothermal flow models in gas pipelines has not been demonstrated to the best of our knowledge. In this study, we developed non-isothermal equations of gas flow in pipelines with occurrence of leaks in multiple alternative locations. To estimate the states of the nonlinear non-isothermal process effectively, UKF was proposed and the application of a dual UKF (DUKF) for estimating the state (flow rate) and other parameters that are subject to process and measurement noises was demonstrated. DUKF comprises two UKFs running in parallel to estimate the process states and process parameters, recursively. DUKF takes boundary pressure measurements of the pipeline and estimates the flow rate across the pipeline. The estimation of flow rates from the filter was compared to the simulated measurement of flow rates and the difference was used to determine the leak location by integrating a partial differential equation along the x-axis. The flow discrepancy was then correlated to the leak location under constant boundary conditions. During the application of DUKF, the corresponding thermodynamic parameter in the isothermal model, which can change depending upon different thermal conditions, was estimated and explained in the results section.

II. MATHEMATICAL METHODOLOGY

A. Natural gas pipeline modeling

In this study, we describe a one-dimensional gas flow dynamics through a gas duct by applying the conservation of mass, momentum, and energy to derive the equations. The composition of natural gas is assumed to be 95% methane, 2.5% ethane, 1.6% nitrogen, 0.7% carbon dioxide, and 0.2% propane. The pressure heat capacity (C_P) is assumed to be constant at 2170 J/kg·K. The pipeline is 100 km in length ($L = 100$ km) and 0.6 m in diameter ($D = 0.6$ m). The heat transfer coefficient along the pipeline is assumed to be uniform. The inlet pressure (P_{in}) is 50 bar, and the outlet pressure (P_{out}) is 40 bar. The derivations of the natural gas flow model based on mass balance, momentum balance, and energy balance equations are shown in Appendix A. Compared to previous non-isothermal models reported by other researchers, a leak term Q_L (kg/s) was introduced into the pipeline equations, and its corresponding effect on pressure, mass flow rate, and temperature was considered. The leak term Q_L is defined as the product of a leak flux term (kg/m²·s) and area (m²). The following model equations were used, in which x refers to a location in a pipeline and t refers to the time. The model equations are as follows:

Continuity equation:

$$\frac{\partial \rho v}{\partial x} + \frac{\partial \rho}{\partial t} + \frac{Q_L}{A \Delta x} = 0 \quad (1)$$

Momentum balance equation:

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial \rho v \cdot v}{\partial x} + \frac{\partial P}{\partial x} + \frac{Q_L \cdot v}{A \Delta x} = -\rho g s \sin \theta - \frac{f q^2}{2 D A^2 P} Z R T \quad (2)$$

Energy balance equation:

$$\rho \frac{\partial H}{\partial t} + \rho v \frac{\partial H}{\partial x} - v \frac{\partial P}{\partial x} - \frac{\partial P}{\partial t} = \frac{\rho f v^3}{2 D} - \frac{4 U (T - T_g)}{D} \quad (3)$$

Equation of state:

$$\frac{P}{\rho} = Z R T \quad (4)$$

In the above equations, Q_L is the leak (in terms of mass flow rate) in the pipeline, f is the friction factor, and θ is the inclined angle between the pipeline and the ground, which is set as zero without loss of generality. A is the cross-sectional area of the pipeline, and $q = \rho v A$ is the mass flow rate. Δx is the discretization of the pipeline in the x-axis. Z is the compressibility factor, which is a function of P and T . H is the enthalpy of natural gas and its derivative can be written as $dH = C_P dT + \left\{ \frac{T}{P} \left(\frac{\partial P}{\partial T} \right)_P + 1 \right\} \frac{dP}{P}$. U is the overall heat transfer coefficient between the pipeline and the environment and T_g is the ground temperature, which is assumed to be uniform along the pipeline.

The above equations were rearranged as follows:

$$\frac{\partial P}{\partial t} = \frac{-\frac{1}{A} \frac{\partial q}{\partial x} - \frac{1}{A \Delta x} Q_L + \left(\frac{1}{Z C_P} \frac{\partial Z}{\partial T} + \frac{1}{T C_P} \right) \left(\frac{f q^3 2^2 R^2 T^2}{2 D A^2 P^2} - \frac{4 U (T - T_g)}{D} - \frac{q}{A} C_P \frac{dT}{dx} + \left(\frac{T}{Z} \frac{\partial Z}{\partial T} + 1 \right) \frac{q}{A P} Z R T \frac{dP}{dx} \right)}{\left(\frac{1}{Z R T} - \frac{P}{Z^2 R T^2} \frac{\partial Z}{\partial P} - \left(\frac{\partial Z}{\partial T} \right)^2 \frac{T}{Z C_P} - \frac{2}{Z C_P} \frac{\partial Z}{\partial T} - \frac{1}{T C_P} \right)} \quad (5)$$

$$\frac{\partial q}{\partial t} = -A \frac{\partial P}{\partial x} - \frac{A P}{Z R T} g s \sin \theta - \frac{f q^2}{2 D A P} Z R T - \frac{1}{A} \frac{Q_L}{\Delta x} \left(\frac{q}{P} \right) Z R T \quad (6)$$

$$\frac{\partial T}{\partial t} = \frac{\left(\frac{1}{Z R T} - \frac{P}{Z^2 R T^2} \frac{\partial Z}{\partial P} \right) \frac{\partial P}{\partial t} + \frac{1}{A} \frac{\partial q}{\partial x}}{\left(\frac{P}{Z^2 R T^2} \frac{\partial Z}{\partial T} + \frac{P}{Z R T^2} \right)} + \frac{Q_L}{A \Delta x} \frac{1}{\left(\frac{P}{Z^2 R T^2} \frac{\partial Z}{\partial T} + \frac{P}{Z R T^2} \right)} \quad (7)$$

In these equations, the compressibility of the natural gas (Z) and its derivatives ($\frac{\partial P}{\partial T}$, $\frac{\partial Z}{\partial T}$) were calculated based on the equation proposed by the reference [7].

For the purpose of comparison, the isothermal models use constant compressibility, in which

$$\frac{P}{\rho} = c^2 \quad (8)$$

$|q|$ is used to ensure a positive value of the flow rate in the model development. Equations for the isothermal models are derived as follows [8].

$$\frac{\partial P}{\partial t} + \frac{c^2}{A} \frac{\partial q}{\partial x} + \frac{c^2}{A \Delta x} Q_L = 0 \quad (9)$$

$$\frac{\partial q}{\partial t} + A \frac{\partial P}{\partial x} + \frac{f c^2 q |q|}{2 D A P} + \frac{c^2}{A \Delta x} \left(\frac{q}{P} \right) Q_L = 0 \quad (10)$$

$$\frac{\partial T}{\partial t} = 0 \quad (11)$$

In the results and discussion section, the pipeline isothermal models were used for the state and parameter estimation to provide nominal values of the state variables, which were then compared with the simulated measurement values, to show any occurrence of leaks including their magnitudes and locations.

B. Model simulation and Dual unscented Kalman filter

The Kalman filter is an optimal state estimator for inaccurate and uncertain observations, such as in the presence of process noises and measurement noises. It minimizes the mean square error of the estimated states (or parameters). UKF is a stochastic estimation method for nonlinear systems, which does not require linearization. UKF is applied on the isothermal and non-isothermal models using the system in Equation (12) and Equation (13)

$$x_{k+1} = f_k(x_k) + w \quad (12)$$

$$y_k = Cx + v \quad (13)$$

where x_k represents the state x at time k , which includes the pressure P , mass flow rate q , and temperature T at each discretized nodes. y_k represents the measurement at time k , which refers to pressures at the boundary of the pipeline. The measurement model is an identity mapping model, where the inlet and outlet pressures are the measured variables. The spatial distribution is represented as follows: pressure P at time k and length 0 is $P(k, 0)$ and pressure P at time k and length Δx is $P(k, \Delta x)$. Similarly, P at other spatial intervals are $P(k, 2\Delta x)$, $P(k, 3\Delta x)$... $P(k, L)$. The states x_k (P , q , and T) at each discretized node of the pipeline are calculated as a vector when applying UKF. The location of the leak can be estimated based on the estimation of flow rate from the filter. In the application of the method of lines, the spatial derivatives at all the interior discretized nodes and boundary nodes are approximated by a fully implicit Crank–Nicolson scheme and a second-order central-difference formula, respectively. Fixed boundary conditions were used to solve the partial differential equations, which are set as follows:

$$\frac{\partial P}{\partial t_{x=0}} = 0, \quad \frac{\partial T}{\partial t_{x=0}} = 0, \quad \frac{\partial P}{\partial t_{x=L}} = 0$$

The initial state x_0 for UKF is set as $(P_0, P_0 - \Delta P, P_0 - 2\Delta P, \dots, P_1, q, \dots, q, T_g, \dots, T_g)$, where P_0 and P_1 are the boundary input and output pressures respectively, q is the estimated mass flow rate, and T_g is the estimated ground temperature. ΔP is the sectional pressure drop, which is equal to $(P_0 - P_1)$ divided by the discretization number. The equation derived to identify the location of a leak is based on flow rate discrepancy, which is given as Equation (14) [8,9]. E is the mean value of discrepancies from ten previous measurements.

$$X_L = \frac{L}{1 - \frac{E(q_{in} - q_s)}{E(q_{out} - q_s)}} \quad (14)$$

Where q_{in} and q_{out} represent the artificially measured inlet and outlet flow rate after the leak respectively, and q_s is the estimated flow rate at steady state without the leak. The steady-state flow rate was estimated from DUKF using the measurements of the boundary pressures at both ends of the pipeline, where L is the total length of the pipeline. X_L is the estimated location of the leak. X_L is calculated when the difference between q_{out} and q_s exceeds a certain threshold above the noise level (1.5% in our simulation case). The effects of a leak on the temperature profile across the pipeline were studied by introducing three different magnitudes of leaks in the middle of the pipeline.

C. Estimation of leak location using DUKF

UKF is a powerful state estimation technique for nonlinear systems. In our study, the mass flow rates are the states of a gas flow process in a pipeline. The isothermal and non-isothermal models were compared in terms of the predictions of mass flow rate using DUKF. In the isothermal model, the parameter c is used to define the equation of state in the isothermal model, which is closely related to the thermal properties such as the temperature and compressibility factor, as indicated in Equations (4) and (8).

To determine the effects of the estimated parameter on the estimation of process states (P , q , and T), three different thermal conditions with randomly selected ground temperature, heat transfer coefficient, and inlet temperature were investigated.

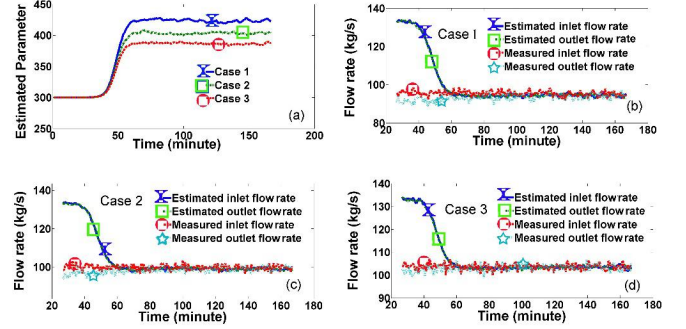


Figure 1. Parameter estimation in three different cases and estimation of flow rate. *Estimated flow rates* were generated based on isothermal model and *measured flow rates* were generated from the non-isothermal model. Case 1, 2, and 3 represent three different thermal conditions.

TABLE I. THREE DIFFERENT THERMAL CONDITIONS

Case	Thermal Conditions		
	Ground temperature (K)	Inlet temperature (K)	Heat transfer coefficient ($J/(m^2 \cdot K \cdot s)$)
Case 1	303	373	1.84
Case 2	273	313	3.84
Case 3	289	343	2.84

In Figure 1 and in all other figures where *measured data* appear, the *measured data* were simulated through the non-isothermal model with process/measurement noises added (white random noise, 1% of the flow rate at steady state), and the *estimated data* were obtained through DUKF. Figure 1(a) shows the estimated parameter (c in Equation (4)) in three different thermal operating cases without leak occurrence. From Figure 1(a), we can conclude that the parameter c can be updated for different thermal operating conditions that matches the flow rate of the non-isothermal model as shown in Figures 1(b), 1(c), and 1(d).

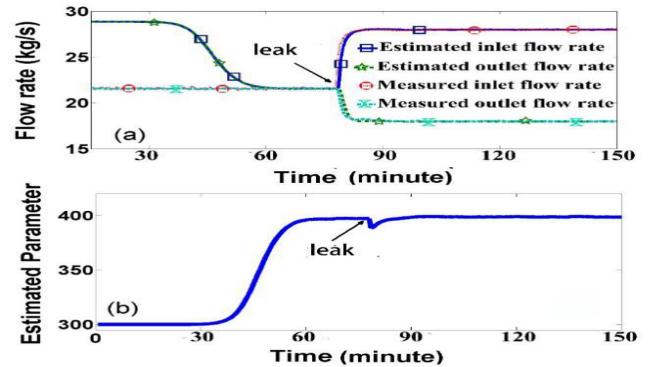


Figure 2. Parameter estimation before and after leak occurrence: (a) matching of flow rate due to parameter estimation, and (b) parameter estimation of c in isothermal model

Figure 2 shows the estimation of the parameter c in the isothermal model with and without leak occurrence. As can be observed from Figure 2(a), the estimated mass flow rate from the isothermal model can match the artificial

measurement before and after the leak through the parameter estimation method using DUKF. The parameter estimation results shown in Figure 2(b) indicate that the estimated parameter c changes from 397.1 to 398.5 owing to the onset of a leak. The parameter c correlates with the thermal state of equation of the gas, and changes as leaks occur. The parameter changes accordingly when a leak causes a change in the temperature profile of the gas flow in the pipeline.

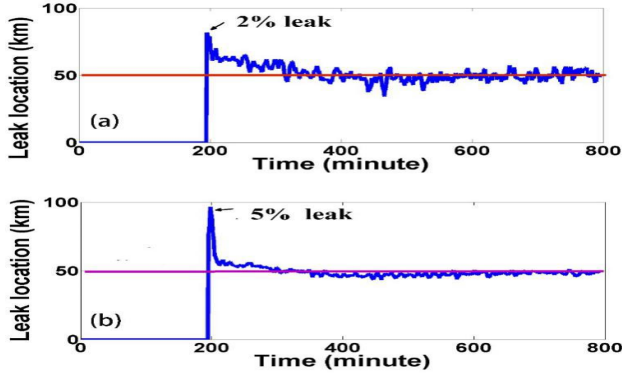


Figure 3. Leak location identification using dual unscented Kalman filter with 2% and 5% leakage

Figure 3 shows the estimation of the leak location using the DUKF algorithm. The location of the leak was calculated based on Equation (14). The figure indicates that by using a simplified pipeline model with parameter update, DUKF is capable of estimating the location of a leak. As shown in Figure 3, the leak location estimation converges faster to a steady-state value for leaks with larger magnitude. To reduce the effects of noises, a moving average window technique was applied, which averaged the data values within a particular time window.

D. Case study

To validate the DUKF algorithm, a case study was performed. we consider a real natural gas pipeline leak case in the Shandong Province, China as an example, referring to it as the SD pipeline hereafter. On March 20, 2019, a leak occurred in the SD natural gas pipeline between NO. 8 and NO. 9 valve Chambers. A steady-state natural gas pipeline flow simulation was performed with and without a leak occurrence. The simulated straight pipeline has a length of 10 km, a pipe diameter of 0.3 m, and a heat transfer coefficient of $1.13 \text{ J}/(\text{m}^2 \cdot \text{K} \cdot \text{s})$. The initial pipeline and ground temperatures were set as 313 K and 289 K, respectively. The inlet and outlet pressures (boundary conditions) were set as 10 bar and 8 bar. We used the DUKF algorithm in our isothermal model to estimate the parameter c in Equations (9).

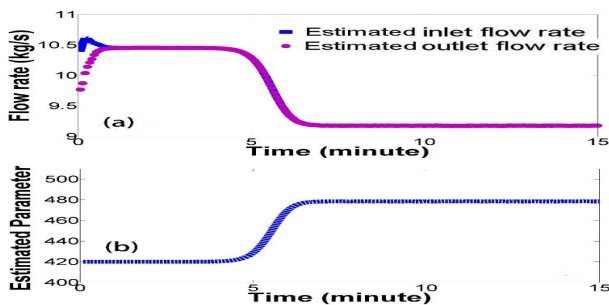


Figure 4. (a) Estimation of flow rate and (b) parameter c in isothermal model

Figure 4 shows the updated flow rate and parameter c using the simulated measurement data. Leak detection was performed. The algorithm calculated the location of the leak at 6.09 km with a 2% leak, whereas the actual location of the leak was at 6.00 km.

III. RESULTS AND DISCUSSION

A. Effect of thermal properties on pressure, flow rate, and temperature distribution of natural gas in the pipeline

The non-isothermal model in this study was built based on the assumption that the friction factor and heat capacity of the natural gas are constant. The ground temperature is constant and is not affected by the gas leak. Three different ground temperatures (T_g) of 273 K, 289 K, and 303 K were applied to study their effects on the mass flow rate, pressure, and temperature profiles.

The effect of thermal properties such as ground temperature variation was studied at steady state. The ground temperature contributes to the heat transfer between the environment and the pipeline. Three different ground temperatures were considered to account for high, medium, and low environment temperature around the pipeline.

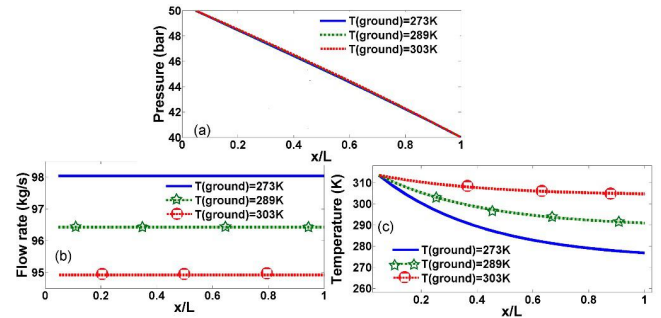


Figure 5. Effect of ground temperature on pressure, flow rate, and temperature distribution of a pipeline at steady state

Figure5 shows the effect of ground temperature on the pressure, flow rate, and temperature profiles for natural gas flow along a pipeline. The inlet temperature was set as 313 K, and the heat transfer coefficient was set as $2.84 \text{ J}/(\text{m}^2 \cdot \text{K} \cdot \text{s})$. Figure5(b) shows the variations of the flow rate at different ground temperatures along the length of the pipeline. The flow rate decreased by 3.31% from a total of 98.2 kg/s when the ground temperature was increased from 273 K to 303 K. A lower ground temperature can increase the mass flow rate because of the increase in gas density. Figure5(c) shows the temperature variations along the length of the pipeline due to different ground temperatures. A significant drop in the temperature along the pipeline was observed at a lower ground temperature.

B. Effect of gas leakage on pressure, flow rate, and temperature distribution

Incorporated leaks into the natural gas pipeline in a non-isothermal situation and their effects on the pressure, mass flow rate, and temperature profiles along the length of the pipeline was studied. Different leak sizes (2%, 5%, and 10% of the total flow rate) and locations ($L/4$, $L/2$, and $3L/4$) were tested.

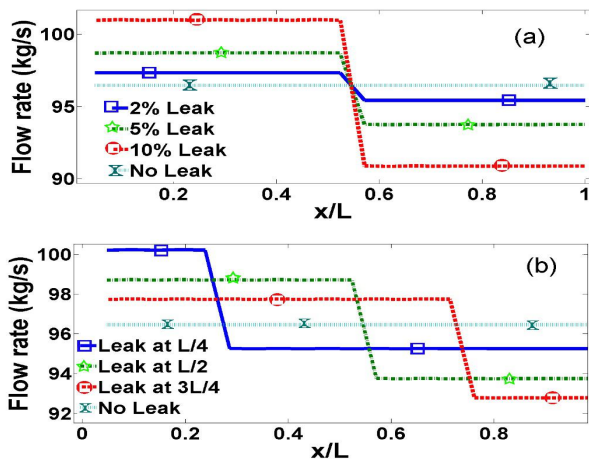


Figure 6. Effect of leak on flow rate with (a) different leak magnitudes at $L/2$ and (b) 5% leak at different locations. L represents the total length of the pipeline

Figure 6 illustrates the change in mass flow rate due to the occurrence of a leak at steady state. After the onset of the leak, the mass flow rate changes at both ends of the pipeline, i.e., at the upstream and downstream of the location of the leak. From Figure 6(a), it can be observed that the upstream flow rate increases, while the downstream flow rate decreases. The difference in mass flow rate between the two ends is equal to the size of the leak. When a leak occurs, the pressure drop across the pipeline will decrease due to the loss of flow rate. Moreover, due to the operating condition of the compressor station, the inlet mass flow rate will increase to satisfy the boundary condition, leading to an increase in the inlet mass flow rate. To maintain a fixed boundary pressure, more natural gas is compressed into the pipeline, which increases the upstream mass flow rate. The downstream flow rate decreases due to the leak. Figure 6(b) shows the variation of flow rate profiles due to leaks occurring at different locations but with the same magnitude. This figure shows that the leak location changes the flow rate profile, which could be used for the purpose of leak location identification. The difference in mass flow rates with and without a leak changes according to the location of the leak. The equation derived to identify the location of a leak is based on flow rate discrepancy.

The effects of a leak on the temperature profile across the pipeline were studied by introducing three different magnitudes of leaks in the middle of the pipeline.

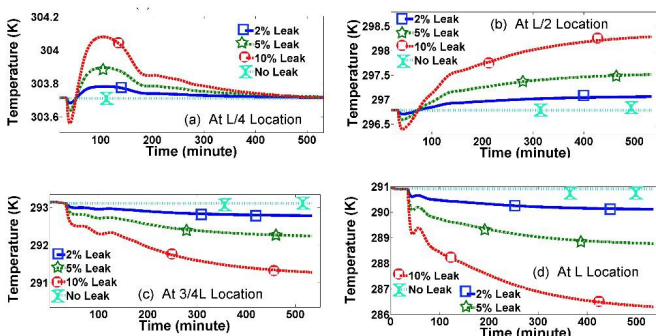


Figure 7. Effect of leak (located at $L/2$) on temperature change of the pipeline at different locations

Figure 7 shows the transient responses of the temperature profiles at different discretized nodes along the

pipeline. As shown in Figure 7, it can be observed that with the onset of a leak, the temperature at a location upstream of the leak point first decreases at a small amount; then, it increases and finally returns to a steady state. The decrease in temperature at the early onset of the leak is due to the pressure drop and the Joule–Thomson effect. Subsequently, the increasing temperature is due to the increased inlet flow rate at high temperature bringing in more energy, and the final steady-state value is reached by heat exchange with the environment.

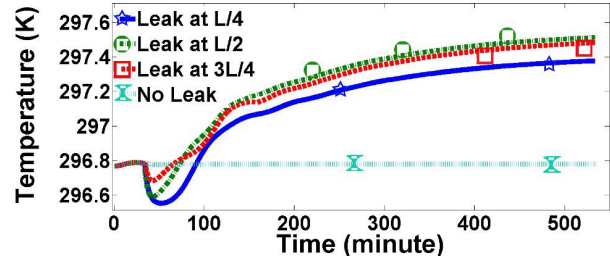


Figure 8. Effect of leak on temperature change (at $x = L/2$) with leak occurring at different locations ($x = L/4, L/2, \text{ and } 3L/4$)

Figure 8 shows the temperature changes at $x = L/2$ due to the leaks introduced at different locations ($x = L/4, L/2, \text{ and } 3L/4$). Leaks with the same magnitude at different leak locations affect the transient behaviors of the temperature profile. The changes in temperature depend on the overall effect of the inlet temperature, ground temperature, and pressure distribution.

IV. CONCLUSIONS

Non-isothermal natural gas flow equations in pipelines were developed to determine the effect of thermodynamic factors. The leak in natural gas pipeline changes the flow rate, pressure, and temperature profiles across the pipeline, depending on the size and location of the leak. The constant parameter c in the isothermal model was estimated for different thermal conditions and leak occurrences. In DUKF with parameter update, the isothermal model can be used as an observer to estimate the gas flow rate under non-isothermal situations at steady state. Using the data generated from the non-isothermal model, the proposed DUKF can detect the leak location efficiently, a real leak case study was used to validate it.

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