

# An Optimal Dispatch Model with Dynamic Power Curve for Integrated Energy System

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**Abstract**—With multiple energy sources, diverse energy demands, and heterogeneous socioeconomic factors, energy systems are becoming more and more complex and multifaceted. Therefore, it is also becoming more and more challenging to improve the efficiency, security and sustainability for such complicated systems. To address these challenges, we propose a general optimal dispatch model for integrated energy systems. Two interesting and challenging decisions of any such model is how it takes power curves of equipment into account and how it deals with nonlinearity. We use a Gaussian Process (GP) to estimate the dynamic power curves, and then we linearize the nonlinear program using special ordered sets of type 2 and solve it using CPLEX. To demonstrate the practicality of the proposed approach, we combine real world and simulated scenarios to perform extensive experiments.

**Keywords**—Virtual integrated energy system, renewable energy, MILP, optimal dispatch model, dynamic power curve

## I. INTRODUCTION

Energy is the basis for human survival and a prerequisite for social development and progress. In recent years, with the promotion of concepts such as integrated energy systems and energy internet, the energy industry is evolving towards improving efficiency, security and sustainability [6] [7].

One way to study energy systems is to look at it from the optimal dispatch (OD) point of view, answering the question of what combination of energy supply should be used in order to satisfy the demand, under operational constraints, and optimizing some objective(s). The OD problem is usually formulated as a mixed integer nonlinear program (MINLP), in which the different types of optimal objectives and constraints are defined. The objectives can

be single- or multi-objective [11]–[14], and the constraints can include energy balance constraints, ramp rate constraints, equipment constraints and so on [15]–[17]. The MINLP could be solved by a deterministic method [19]–[21], or a stochastic method such as Particle Swarm Optimization (PSO) or Genetic Algorithms (GA) [22] [23].

In this paper, we propose an optimal dispatch model of an integrated energy system. We apply a Gaussian Process (GP)[25] [26] to model the dynamic power curve of equipment, and then linearize the resulting mixed integer nonlinear program using special order sets of type 2 (SOS2), and solve the resulting linear program. In order to study arbitrarily complex integrated energy systems, we combine actual and simulated scenarios, set appropriate baselines, and analyze the resulting virtual system, thus showing the efficacy of our approach. The proposed dispatch model can be applied to planning, operations and dispatch, as well as energy trading.

GP is a non-parametric Bayesian method commonly used to model a complex nonlinear system robustly. Prior applications of GP in energy research include [18], who use GP to express the nonlinear power curve of wind turbines for equipment monitoring; [36], who use GP for forecasting the power load; and [37], who use GP to propose some models for the prediction control of power systems. Here we use GP to model the equipment input/output constraints of a dispatch problem. GP is well suited for this application for the following reasons. First, it is robust and not prone to over-fitting plaguing parametric methods. Second, it supports online learning and deals with uncertainty, which is ideal for reflecting the complicated and dynamic input/output relation of equipment. Third, it can capture the changes in equipment efficiency as it depreciates. Finally,

since it is a non-parametric Bayesian method, it can express the power curve of all equipment uniformly, instead of having to build one model per equipment like parametric methods.

We apply our method to the example of combining an actual photovoltaic plant, an actual combined heating and power station, and a set of simulated storage batteries. As mentioned above, we convert each nonlinear relation with the constraints into linear form using SOS2, and solve the model using CPLEX. The results show that we can effectively take the variability of renewable energy into account in an integrated energy system, and achieve the optimization of heating and power scheduling.

The remainder of this paper is organized as follows. Section II describes the proposed optimal dispatch model and describes how to use GP to model the dynamic power curve of equipment. Section III presents the example, and analyzes the results. The conclusions and future work are described in Section IV.

## II. OPTIMAL DISPATCH MODEL

The model considered here describes the coordination and cooperation of different equipment in an integrated energy system in order to optimize the efficiency of energy utilization. We will later apply this general model to a virtual integrated energy station (see Figure 2).

### List of symbols

P	Power
Q	Quantity of a certain raw energy
C	Selling/purchasing price
R	Rewards for using a specific type of energy
L	User loads
F	On-off state variable, either 0 or 1
D	On-to-off state variable, either 0 or 1
U	Off-to-on state variable, either 0 or 1
T	Number of time periods
t	A certain time period $t = 0, \dots, T - 1$
N	Number of equipment types
$n_i$	Number of units for i-th equipment type

### Subscripts

ge	Gas engine
hrsg	Heat recovery steam generator
chp	Combined heating and power
gsb	Gas steam boiler
pv	Photovoltaic
wg	Wind generator
stg	Storage
u	User
s	Source
i, j	The i-th type j-th unit equipment

a → b Indicates an energy flows in a pipeline from a to b

### Superscripts

e	Electricity energy
c	Cooling energy
st	Hot steam energy
sm	Hot smoke energy
g	Gas
oo	On-off
ou	Start-up
od	Shut-down

in/out Input or output of a certain energy to an equipment

### A. Model Objective

The model built here can deal with different objectives. Here we give the example of the maximization of profit, the total revenue minus total costs. The total revenue at the t-th time period is from selling hot steam to users and electricity to both users and the local grid:

$$C_u^{e,in}[t] \cdot L_u^e[t] + C_u^{st,in}[t] \cdot L_u^{st}[t] + C_s^{e,in}[t] \cdot P_s^{e,in}[t] \quad (1)$$

The total costs at t-th time period is:

$$\text{Materials costs: } C_s^{g,out}[t] \cdot Q_s^{g,out}[t] + C_s^{e,out}[t] \cdot P_s^{e,out}[t]$$

$$\text{On-off costs: } \sum_{i=1}^N \sum_{j=1}^{n_i} C_{i,j}^{oo}[t] \cdot F_{i,j}[t]$$

$$\text{Start-up costs: } \sum_{i=1}^N \sum_{j=1}^{n_i} C_{i,j}^{ou}[t] \cdot U_{i,j}[t]$$

$$\text{Shut-down costs: } \sum_{i=1}^N \sum_{j=1}^{n_i} C_{i,j}^{od}[t] \cdot D_{i,j}[t]$$

In addition to revenue and cost, bonus will be rewarded for using renewable energy in the first place - this can represent the direct incentives of using renewable energy, or be used to internalize the environment benefits of renewable energy. In summary, the profits over all T time periods is total revenue minus total cost plus total rewards, or:

$$\begin{aligned} & \sum_{t=0}^{T-1} (C_u^{e,in}[t] \cdot L_u^e[t] + C_u^{st,in}[t] \cdot L_u^{st}[t] + C_s^{e,in}[t] \cdot P_s^{e,in}[t] \\ & - C_s^{g,out}[t] \cdot Q_s^{g,out}[t] - C_s^{e,out}[t] \cdot P_s^{e,out}[t] \\ & - \sum_{i=1}^N \sum_{j=1}^{n_i} C_{i,j}^{oo}[t] \cdot F_{i,j}[t] \\ & - \sum_{i=1}^N \sum_{j=1}^{n_i} C_{i,j}^{ou}[t] \cdot U_{i,j}[t] \\ & - \sum_{i=1}^N \sum_{j=1}^{n_i} C_{i,j}^{od}[t] \cdot D_{i,j}[t]) \\ & + R_{pv}[t] P_{pv}^{e,out}[t] + R_{wg}[t] P_{wg}^{e,out}[t], \quad (2) \end{aligned}$$

which is an objective to be maximized in the integrated energy system.

### B. Constraints of the Model

The constraints that our model considers can be divided into several categories, depending on what is relates to: startup-shutdown, equipment and operation specific,

source and user, and connection.

In the following subsections, we describe in detail only a small subset of those constraints.

1) *Startup-Shutdown Constraints*: For most equipment, it is necessary to consider the cost related to its startup-shutdown transitions. For example, here we explain the startup-shutdown constraints for CHP. The same goes for the rest of the equipment. All state variables of a CHP over T time periods must satisfy the following conditions:

$$0 = F_{\text{chp}}[0] - F_{\text{chp}}^0 - U_{\text{chp}}[0] + D_{\text{chp}}[0], \quad (3)$$

$$0 = F_{\text{chp}}[i+1] - F_{\text{chp}}[i] - U_{\text{chp}}[i+1] + D_{\text{chp}}[i+1] \\ \text{for } i=0, \dots, T-2, \quad (4)$$

$$1 \geq D_{\text{chp}}[i] + U_{\text{chp}}[i] \text{ for } i=0, \dots, T-1, \quad (5)$$

where  $F_{\text{chp}}^0$ , taking value either 0 or 1, denotes the initial on-off state before  $t = 0$ .

2) *Gas Engine Constraints*: Gas engine (GE) consumes natural gas and converts its chemical energy into electricity, while producing hot smoke. The energy of hot smoke can be utilized a second time by a heat recovery steam generator (HRSG) to generate hot steam for the user. Here we assume that the maximum and minimum power of a gas engine are denoted by  $P_{ge}^{\text{max}}$  and  $P_{ge}^{\text{min}}$  respectively, and electricity-heat ratio by  $r_{ge}$ , then we have:

$$F_{ge}[i] P_{ge}^{\text{min}} \leq P_{ge}^{\text{e,out}}[i] \leq F_{ge}[i] P_{ge}^{\text{max}} \quad (6)$$

$$P_{ge}^{\text{sm,out}}[i] = r_{ge} P_{ge}^{\text{e,out}}[i], \text{ for } i=0, \dots, T-1 \quad (7)$$

For an equipment, its input and output should satisfy a certain relation, which is often unknown beforehand. Suppose we have that

$$Q_{ge}^{\text{g,in}}[i] = f_{ge}(P_{ge}^{\text{e,out}}[i]) \text{ for } i=0, \dots, T-1, \quad (8)$$

where  $f_{ge}(\cdot)$  represents a certain relation between  $P_{ge}^{\text{e,out}}$  and  $Q_{ge}^{\text{g,in}}$ . One common way to obtain is to apply polynomial-fitting techniques. However, the nonlinearity of the equipment input/output relation makes the problem difficult to solve. Therefore we use special ordered set of type 2 (SOS2) [32]–[34] to specify integrality conditions and approximate the nonlinear relation by piece-wise linear relations, thus converting the nonlinear programming problem into a linear one.

### C. Power Curve Estimation

A key part in the model is the relation between input and output of an equipment, which is usually represented as a power curve. A power curve, a set of functions to describe thermal efficiency, fuel mass flow rate, energy recovered from the water jacket, and exhaust gas enthalpy, is usually expressed in terms of a variable partial load (PL). PL is the running load as a percentage of nominal load and reflects the relation between PL and energy efficiency. All of these data are required for setting constraints of equipment in the MINLP model.

1) *Power Curve Modeling*: Studies referring to [30] express the conversion of energy as coupling matrix and regard the efficiency of equipment as constants. Others use a parametric regression method such as a linear or polynomial function to model a power curve. Both

approaches have their shortcomings. First, a power curve is a variable function of PL, not a constant. Second, if we take the valve point effect into consideration, a power curve is non-convex and not smooth. A simple parametric method can not model such complicated data well. A complex parametric method might fit the model better, but it is prone to over-fitting. Lastly, there are dozens of equipment in an integrated energy system, and they can not be modeled by one or several uniform parametric models.

Also, the power curve of equipment is not static but dynamic during the entire lifetime period and depends on the environment.

Based on the above discussions, a power curve model should have the following characteristics:

- Robust and not prone to over-fitting.
- Support online learning and deals with uncertainty.
- Be able to model the power curve of all equipment uniformly.

In this paper we adopt the GP Regression model to estimate the dynamic power curve.

2) *Gaussian Process for Power Curve Regression*: A Gaussian Process (GP) is a Bayesian, non-linear machine learning method. We use GP to solve the probabilistic regression and classification problems [25] and define  $p(f)$  as a distribution over functions by GP.

A GP is parameterized by a mean function,  $u(x)$ , and a covariance function  $K(x, x')$ . For  $x$ , we obtain the expression of GP as follow:

$$f(x) = \text{GP}(u(x), K(x, x')). \quad (9)$$

For more information about GP, refer to [31].

Assuming we have a data set D of n observations, where  $D = \{(x_i, y_i)_{i=1}^n\} = (x, y)$ , captured by equipment sensors, where x represents PL (partial load), and y represents energy efficiency, we have the following regression model [38]:

$$y_i = f(x_i) + \varepsilon_i \quad (10)$$

$$f \sim \text{GP}(\cdot | 0, K) \quad (11)$$

$$\varepsilon_i \sim N(\cdot | 0, \sigma^2) \quad (12)$$

If the prior on f is a GP, likelihood is Gaussian, then the posterior on f is also a GP.

$p(y^* | x^*, D) = \int p(y^* | x^*, f, D) p(f | D) df$  (13)  
 $x^*$  could be any value in the PL domain,  $y^*$  is the corresponding predicted value of the energy efficiency. (x,y) represents the power curve which can be sequentially learned by Gaussian process regression model.

To obtain the latest power curve of the equipment, we update the GP model as follows. First we train the model using the initial data set, and obtain the kernel function and its parameters, with which the power curve can be modeled. Then, when new data comes in, the model can be retrained, and the updated power curve based on the retrained GP model is then used in the optimal dispatch model.

3) *Results & Analysis of Gaussian Process*: In this part, we analyze the effectiveness of the GP model on real data captured from a GSB with a maximum load of 10 tons, by comparing it with several parametric methods including spline interpolation and polynomial fitting on the GSB dataset which contains 96 samples. The regression of high degree is prone to over-fitting, therefore we select the polynomial regression of degree 1, 2 and 3, and the spline

Algorithm	RMSE	Algorithm	RMSE
GPR	0.001198	1 degree polynomial	0.001641
1 degree spline	0.001373	2 degree polynomial	0.001604
2 degree spline	0.002855	3 degree polynomial	0.001262

TABLE I: Results of different algorithms

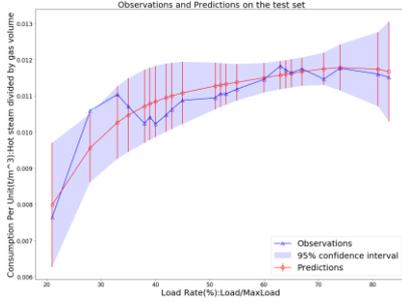


Fig. 1: Power Curve of GSB

regression of degree 1 and 2 to compare. We equally and randomly divide data into 3 parts, and then use the 3-fold cross-validation to evaluate the performance of the Gaussian Process Regressor and the selected methods.

We use Bayesian optimization to perform model selection and hyperparameter inference, by which we find that the Radial Basis Function (RBF), RBF(length scale bounds=(1,1e3)) with noise level  $\alpha$  of  $10^{-8}$  achieves the best performance. When using these algorithms for comparison, we first get an interpolation or fitting model on the training set, then apply the model to the test set. Table I presents the root mean squared error (RMSE) of these algorithms.

From Table I, we can see that GPR performs better than the interpolation and fitting methods. The GPR also gives the uncertainty of the prediction which is shown by the confidence intervals in Figure 1 from which we can observe most of the ground-truth values are within the 95% confidence interval (CI) region, and the uncertainty of the prediction is not fixed.

The prediction in the region with less observations has more uncertainty, and when we obtain more observations in the corresponding region the uncertainty decreases. The experimental results demonstrate that GPR is an appropriate method for power curve modeling and can obtain better performance than many traditional parametric methods.

#### D. Solution of the Model

Above all, the optimal dispatch of an integrated energy system boils down to a nonlinear optimization problem, with the nonlinearity due to the nonlinearity between the inputs and outputs of equipment. We use a technique called special ordered set of type 2 (SOS2) [32]–[34] to convert the nonlinear relationships between inputs and outputs into linear relationships, and then the problem becomes a mixed integer linear programming, which can be solved by using optimization software such as CPLEX.

### III. RESULT & ANALYSIS

In this section, we apply our model to the example of combining an actual combined heating and power station (station A in Figure 2), an actual photovoltaic plant (station B in Figure 2), and a set of simulated storage batteries. The

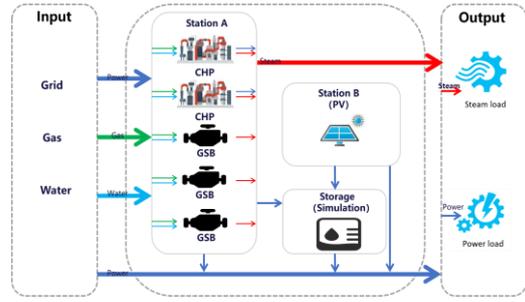


Fig. 2: Virtual integrated energy system

results show that we can effectively take the variability of renewable energy into account, and achieve the optimization of heating and power scheduling.

The depreciation expense and efficiency loss cost of batteries was considered to be 0.3 yuan/kWh.

In our experiments, external power grid acts as both a seller and buyer of electricity, at the prices listed in Table II.

#### A. Comparison and Analysis

1) *Comparison of virtual integrated energy system with baseline:* Here we use the independent operation of A and B as the baseline, and compare the gross profit of the virtual integrated energy system with this baseline. The increase of gross profit is presented in Table III, in yuan, the unit of Renminbi. We use the actual data of user load and power production from a randomly chosen date. Analysis of other days yield similar results. For simplicity, here we only show the results of optimal dispatch related to electricity.

In case 1, A and B stations are independent and there is no storage. This is the actual case today and forms the baseline of our comparison, with gross profit of 43555 yuan/day. In this case, station A relies on the grid for electricity when its output is insufficient to meet the demand. B sells all the electricity generated to the grid, and so in this case we will not show B in the dispatch analysis. See the first part of Figure 3, the positive y-axis is the electricity supply (in this case, purchased from grid, CHP1 or CHP2 generated; again, note that in this case the PV generated is excluded from the figure because B sells all the electricity generated to the grid), and the negative y-axis is the electricity demand (in this case, end user demand).

In case 2, A and B stations are integrated together as if it is one energy system, and they are optimized together, resulting in a gross profit of 45037 yuan a day, an increase of 3.40% over the baseline. This shows that there is value in optimizing the integrated energy system, even without storage. In particular, in this case, A's reliance on the grid is decreased by the presence of B's PV solar generated electricity. Surplus electricity can still be sold to the grid. See the second part of figure 3, the positive y-axis is the electricity supply (in this case, purchased from grid, CHP1 or CHP2 generated, or PV generated), and the negative y-axis is the electricity demand (in this case, end user demand, or selling back to the grid).

In case 3, we consider storage in the integrated energy system, resulting in a gross profit of 45736 yuan a day, an increase of 5.01% over the baseline. The comparison results show that when the electricity price is low, the storage devices store the surplus energy instead of selling to the

Time	Sell to grid	Purchase from grid
00:00 - 08:00	0.38	0.41
08:00 - 12:00	0.5	0.65
12:00 - 21:00	0.7	1.0
21:00 - 24:00	0.38	0.41

TABLE II: Power price tariff

	Gross profit (yuan/day)	increase wrt Case 1
Case 1: Independent and no storage	43555	-
Case 2: Integrated and no storage	45037	3.40%
Case 3: Integrated and with storage	45736	5.01%

TABLE III: Comparison of gross profit grid. For example, see Figure 3, at 12pm, the CHPs generates more electricity than the demand, so the surplus is stored in the storage. Compared to case 2, this shows that storage plays an important role in balancing the tiered pricing. See the third part of Figure 3, the positive y-axis is the electricity supply (in this case, purchased from grid, CHP1 or CHP2 generated, PV generated, or storage discharge), and the negative y-axis is the electricity demand (in this case, end user demand, selling).

2) *Energy Market Prices*: Here we analyze case 3 further to see how the operation scheduling of the integrated energy system changes as pricing changes. We propose a term called 'profit index', which denotes the net profit of an equipment for producing one unit electricity power (kWh). The net profit is calculated by subtracting raw material costs and levelized cost of electricity (LCOE) from the revenue when producing electricity of one kilowatt hour. For instance, when the CHP generates one kilowatt hour electricity power, a large amount of hot steam power is also produced at the same time. Based on the electricity-steam ratio  $r_{chp}$ , we know the total revenue is equal to  $C_u^{e,in} * 1 + C_u^{st,in} / r_{chp}$ . Assume that the natural gas produces electricity power for one kWh is  $Q_{chp}^{g,in}$ , we obtain the formula for calculating the 'profit index' of a CHP,

$$C_u^{e,in} * 1 + \frac{C_u^{st,in}}{r_{chp}} - Q_{chp}^{g,in} \cdot C_s^{g,out} - LCOE_{CHP}. \quad (14)$$

Applying (14) to our scenario, we get the profile index, for more details, see TABLE IV. Knowing the profit indices of equipment in the integrated energy system, we can anticipate that during off-peak periods, we should purchase electricity from the grid and either sell it directly to consumers or store it in the storage battery. However, during the mid-peak and on-peak periods, using electricity from the storage and two CHPs will be our first priority since their profit indices are much higher than that of on-peak purchasing from grid. Furthermore, if the LCOE of the storage increases by 0.05 yuan, which makes its profit index less than that of on-peak, the integrated energy system will not use the storage battery at all. An interesting fact we can observe is that when the price we purchase from grid in off-peak periods increases by more

CHP1	0.1093 ~ 0.1712	Off-peak	0.2543
CHP2	0.0774 ~ 0.1269	Mid-peak	0.016
Storage	-0.3236	On-peak	-0.3618

TABLE IV: Profit indices of some key equipment vs purchasing from grid

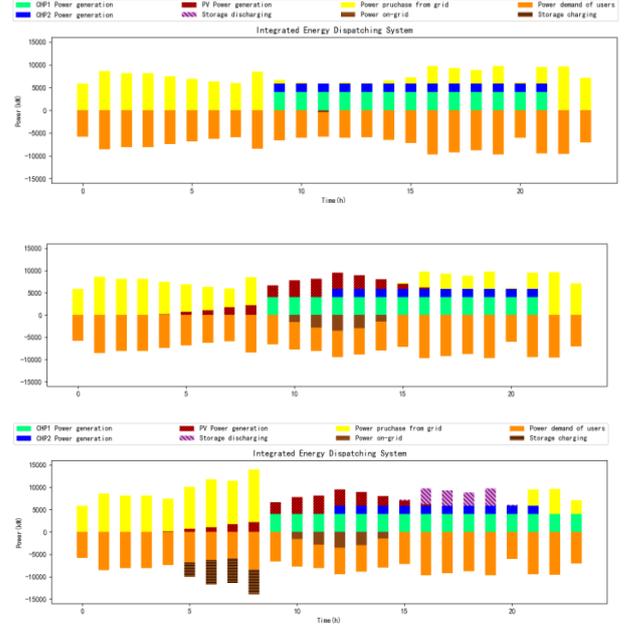


Fig. 3: Optimal Dispatch, Cases 1-3

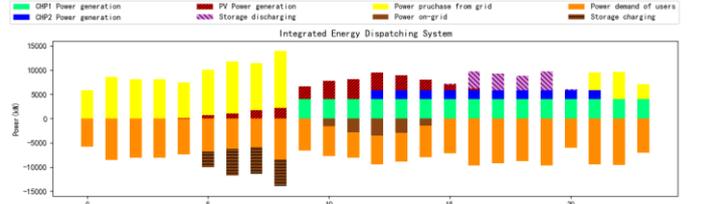


Fig. 4: Operation under profit index

than 0.08 yuan/kWh, the CHPs start operating. All phenomena, obtained by comparing their profit indices, can be validated from our experiments. Figure 4 shows the optimal dispatch of electricity of the integrated energy system, from which we can find that the first CHP should keep operating after 9pm, after the grid increases its electricity price. For the objective of max profit, the model will always prioritize using the equipment which has the highest profit index.

#### IV. CONCLUSIONS AND FUTURE WORK

In this paper we propose an optimal dispatch model of an integrated energy system, where equipment are modeled using a dynamic power curve based on Gaussian Process. We design experiments for studying arbitrarily complex integrated energy systems, by combining actual and simulated scenarios, setting appropriate baselines, and analyzing the resulting virtual system.

The results obtained were analyzed from three aspects, which are minimizing costs, improving performance and adopting the change of price. The analysis indicates that the model of integrated energy system has an important role in evaluating and increasing the use of sustainable energy and technologies.

We plan to replace the historic data of load (steam and power) used here with a prediction model, and perhaps obtain a relationship between the uncertainty of the prediction model used and the uncertainty of the optimization result. We also plan to extend our model to cover more types of energy inputs, models of different types

of equipment and coupling amongst them, and more diverse energy demands. In such a scenario, we anticipate that the results would be even better, since there are a lot more synergies that are possible. This would also allow us to see if our model is computationally feasible or not, and allow us to further fine tune the model. Also, we can extend our current work to study security and sustainability.

Although the equipment power curve model discussed in this paper is for equipment, it can easily be generalized to an energy hub which takes integrated energy as input and output [27] [29] [30]. When we generalize the Gaussian process regression to the energy hub model, the model is not just a power curve but a hyper-surface in the high dimensional space. An energy hub can be seen as a super equipment containing multiple equipment.

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#### REFERENCES

- [1] Fumo, Nelson, Pedro J. Mago, and Louay M. Chamra. Emission operational strategy for combined cooling, heating, and power systems. *Applied Energy* 86.11 (2009): 2344-2350.
- [2] Fu, L., et al. Laboratory research on combined cooling, heating and power (CCHP) systems. *Energy Conversion and Management* 50.4 (2009): 977-982.
- [3] Arteconi, Alessia, Caterina Brandoni, and Fabio Polonara. Distributed generation and trigeneration: Energy saving opportunities in Italian supermarket sector. *Applied Thermal Engineering* 29.8-9 (2009): 1735-1743.
- [4] Hernandez-Santoyo, Joel, and Augusto Sanchez-Cifuentes. Trigeneration: an alternative for energy savings. *Applied Energy* 76.1-3 (2003):219-227.
- [5] Perry, Simon, Jiri Klemes, and Igor Bulatov. Integrating waste and renewable energy to reduce the carbon footprint of locally integrated energy sectors. *Energy* 33.10 (2008): 1489-1497.
- [6] Helm, Dieter. Energy policy: security of supply, sustainability and competition. *Energy policy* 30.3 (2002): 173-184.
- [7] Tanaka, Kanako. Review of policies and measures for energy efficiency in industry sector. *Energy policy* 39.10 (2011): 6532-6550.
- [8] Dincer, Ibrahim. Renewable energy and sustainable development: a crucial review. *Renewable and sustainable energy reviews* 4.2 (2000):157-175.
- [9] Klemes, Jiri, et al. Sustainability in the process industry: integration and optimization (Green Manufacturing & Systems Engineering). McGrawHill Professional, New York, 2010.
- [10] Foo, Dominic CY. Process integration for resource conservation. CRC Press, 2016.
- [11] Chen, Po-Hung, and Hong-Chan Chang. Large-scale economic dispatch by genetic algorithm. *IEEE transactions on power systems* 10.4 (1995):1919-1926.
- [12] Xia, X., and A. M. Elaiw. Optimal dynamic economic dispatch of generation: A review. *Electric power systems research* 80.8 (2010): 975-986.
- [13] Ciomei, Irina, and Elias Kyriakides. Recent methodologies and approaches for the economic dispatch of generation in power systems. *International Transactions on Electrical Energy Systems* 23.7 (2013):1002-1027.
- [14] Piperagkas G S, Anastasiadis A G, Hatzigiorgiou N D. Stochastic PSO-based heat and power dispatch under environmental constraints incorporating CHP and wind power units[J]. *Electric Power Systems Research*, 2011, 81(1): 209-218.
- [15] Wang, C., and S. M. Shahidehpour. Ramp-rate limits in unit commitment and economic dispatch incorporating rotor fatigue effect. *IEEE Transactions on Power Systems* 9.3 (1994): 1539-1545.
- [16] Swarup, K. S., and S. Yamashiro. Unit commitment solution methodology using genetic algorithm. *IEEE Transactions on Power Systems* 17.1(2002): 87-91.
- [17] Han, X. S., H. B. Gooi, and Daniel S. Kirschen. Dynamic economic dispatch: feasible and optimal solutions. *IEEE Transactions on Power Systems* 16.1 (2001): 22-28.
- [18] Pandit, Ravi, and David Infield. Gaussian process operational curves for wind turbine condition monitoring. *Energies* 11.7 (2018): 1631.
- [19] Handschin, Edmund, et al. Optimal operation of dispersed generation under uncertainty using mathematical programming. *International Journal of Electrical Power & Energy Systems* 28.9 (2006): 618-626.
- [20] Ren, Hongbo, and Weijun Gao. A MILP model for integrated plan and evaluation of distributed energy systems. *Applied Energy* 87.3 (2010):1001-1014.
- [21] Morais, Hugo, et al. Optimal scheduling of a renewable micro-grid in an isolated load area using mixed-integer linear programming. *Renewable Energy* 35.1 (2010): 151-156.
- [22] Bagherian, Alireza, and SM Moghaddas Tafreshi. A developed energy management system for a microgrid in the competitive electricity market. *PowerTech, 2009 IEEE Bucharest. IEEE*, 2009.
- [23] Wang, Jiang-Jiang, You-Yin Jing, and Chun-Fa Zhang. Optimization of capacity and operation for CCHP system by genetic algorithm. *Applied Energy* 87.4 (2010): 1325-1335.
- [24] Karki, Shankar, et al. Efficiency improvements through combined heat and power for on-site distributed generation technologies. *Cogeneration and Distributed Generation Journal* 22.3 (2007): 19-34.
- [25] Rasmussen, Carl Edward. Gaussian processes in machine learning. *Advanced lectures on machine learning*. Springer, Berlin, Heidelberg, 2004. 63-71.
- [26] Seeger, Matthias. Gaussian processes for machine learning. *International journal of neural systems* 14.02 (2004): 69-106.
- [27] Geidl M. Integrated modeling and optimization of multi-carrier energy systems. ETH Zurich, 2007.
- [28] Arnold M, Negenborn R R, Andersson G, et al. Distributed predictive control for energy hub coordination in coupled electricity and gas networks. *Intelligent Infrastructures*. Springer, Dordrecht, 2010: 235-273.
- [29] Geidl M, Andersson G. Optimal power flow of multiple energy carriers[J]. *IEEE Transactions on Power Systems*, 2007, 22(1): 145-155.
- [30] Geidl M, Koeppel G, Favre-Perrod P, et al. Energy hubs for the future. *IEEE power and energy magazine*, 2007, 5(1): 24-30.
- [31] Robert C. Machine learning, a probabilistic perspective. 2014.
- [32] Beale E M L, Forrest J J H. Global optimization using special ordered sets. *Mathematical Programming*, 1976, 10(1): 52-69.
- [33] Beale E M L, Tomlin J A. Special facilities in a general mathematical programming system for non-convex problems using ordered sets of variables. *OR*, 1970, 69(447-454): 99.
- [34] de Farias Jr I R, Johnson E L, Nemhauser G L. A generalized assignment problem with special ordered sets: a polyhedral approach. *Mathematical Programming*, 2000, 89(1): 187-203.
- [35] EPA. CHP benefits. <https://www.epa.gov/chp/chp-benefits>, retrieved Nov. 2018.
- [36] Yang Y, Li S, Li W, et al. Power load probability density forecasting using Gaussian process quantile regression[J]. *Applied Energy*, 2018,213: 499-509.
- [37] Hachino T, Takata H, Fukushima S, et al. Model predictive control of electric power systems based on gaussian process predictors[J]. *Journal of Automation and Control Engineering* Vol. 2015, 3(5): 70.
- [38] Ghahramani, Zoubin. "A Tutorial on Gaussian Processes (or why I dont use SVMs)." *MLss2011. Comp. Nus. Edu. Sg* (2011).