

# MODELING GAS TURBINES IN MULTI-ENERGY SYSTEMS: A LINEAR MODEL ACCOUNTING FOR PART-LOAD OPERATION, FUEL, TEMPERATURE, AND SIZING EFFECTS

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## ABSTRACT

So far, the role of fossil fuels in future energy systems is still uncertain. To obtain a deeper understanding of how conversion technologies for fossil fuels act in multi-energy systems (MES), we extended our mixed integer linear programming (MILP) modeling framework for MES, first introduced by Gabrielli *et al.* [1], by adding gas turbines. This work presents the modeling approach for said gas turbines, focusing on the linear description of the efficiency's dependency on load and ambient temperature. Furthermore, the model considers the possibility of selecting either natural gas or hydrogen as fuel, which affects the efficiency as well. A series of simple, proof-of-concept simulations were conducted to show the functionality of the model.

**Keywords:** hydrogen, gas turbine, modeling, mixed integer linear programming, multi-energy system optimization

## NOMENCLATURE

### Abbreviations

MES	Multi-energy system
MILP	Mixed integer linear programming
OEM	Original equipment manufacturer

### Symbols

$\alpha, \beta, \gamma, \delta$	Linear fitting parameters
$\varphi$	Mass ratio fuel - flue gas
$P$	Power output
$F$	Fuel input (in power units)
$N^{tot}$	Turbines installed

$N^{on}$	Turbines operating
$T_x$	Temperature ( $x$ indicating arbitrary index)
$x$	Binary decision whether a turbine is installed
$y$	Binary, time resolved decision whether turbine operates
$c_p$	Specific heat capacity
$LHV$	Lower heating value
$C^{su}, C^{sd}$	Counter for start-up and shut-down, respectively
<i>Indices and sets</i>	
$i$	Turbine capacity
$t$	Time
$k$	Fuel type
amb	ambient
iso	ISO standard conditions
$I$	Set of all available turbine capacities
$T$	Full time horizon (no index)

## 1. INTRODUCTION

The imminent threat of the consequences of climate change and the Paris Agreement [2] increase the importance of investigating future energy systems. Aiming for a system that relies entirely on renewable energy sources is a noble goal; nevertheless, taking supply security and economics into consideration, completely waiving the possibility of utilizing fossil resources and their conversion technologies appears infeasible in times when timely solutions are highly sought for. Gas turbines take a special role in this set of conversion technologies

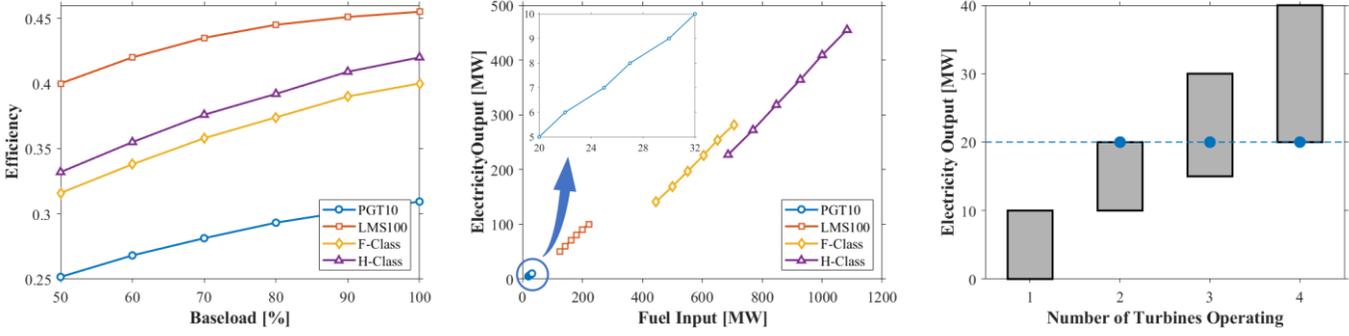


Fig 1: Performance curves for the four considered gas turbines. (L) Efficiency as a function of load factor; (M) Input-Output relation; (R) Electricity output of a PGT10 for varying numbers of operating turbines

as they can utilize hydrogen while also providing back up power in modern electricity grids, where undispachable renewables feature a large share in the total generation.

The optimal design of multi-energy systems is a complex computational problem and mixed integer linear programming (MILP) emerged to be the standard modeling approach today. In order to investigate the role of gas turbines in the design of low or no-carbon energy systems in later research, a linear model in conformity with our modeling framework [1] was developed. Yang *et al.* already presented a temperature dependent gas turbine model in their work. [3] Compared to that, we follow a different approach for the modeling of the temperature dependency. In addition, we consider the influence of different fuels and the dynamic behavior of the turbine.

In this work, we will describe the modeling approach, emphasizing crucial physical assumptions and the model's key features: (i) energy carrier selection (natural gas or hydrogen), (ii) ambient temperature and load dependent efficiency, and (iii) sizing and operation, i.e. selection of capacity, number of installed turbines, number of operated turbines, and load. A piecewise linear cost model based on literature data completes the model.

To show the functionality of the developed model, a cost-effective low-emission energy system, featuring wind turbines, natural gas turbines and hydrogen turbines, will be designed for the Dutch province of Utrecht assuming the availability of zero-emission hydrogen.

## 2. PERFORMANCE AND COSTS

### 2.1 Performance curves

Four discrete sizes have been considered; 10 MW, 100 MW, 250 MW and 400 MW. Data for the 10MW and 100 MW turbines are based on the GE PGT10 and LMS100 turbine models respectively. Data for the heavy duty gas turbines (280 MW (F-class) and 450 MW (H-

class)) are based on the relative efficiency of heavy duty gas turbines [4]. The reference efficiency is taken as an average from Ansaldo, GE, Siemens and Mitsubishi Hitachi Power Systems (MHPS) heavy duty turbines. All data are based on OEM rated gas turbine performance at ISO standard conditions. An operation range of 50-100 % baseload was assumed. Although the efficiency is a non-linear function of the load, the corresponding input-output relation within the considered operation range can be approximated linearly, with the intercept partially accounting for the non-linearity of the efficiency (see Fig 1). Therefore, the power output can be formulated as

$$P_i = \alpha_i \cdot F_i + \beta_i \quad (1)$$

$$i \in I$$

To introduce multiple turbines of equal capacity, the assumption of having equal load at each turbine is crucial. Simple superposition allows to conclude that the slope of the input-output correlation remains unaffected, while the intercept and both the lower and upper limit of fuel input scale linearly with the number of operated turbines at any given moment  $t$ . Introducing this and the time resolution results in equation (2).

$$P_{i,t} = \alpha_i \cdot F_{i,t} + \beta_i \cdot N_t^{\text{on}} \quad (2)$$

$$t \in T$$

Fig 1 (right) shows how the same power output can be achieved with various numbers of turbines. In the example displayed, the desired output of 20 MW can be provided by two, three, or four turbines, i.e.  $N_t^{\text{on}}$  can take 3 values (compare with equation (8)). The trivial solution is the lowest possible number of operating turbines due to higher efficiencies at higher loads. Nevertheless, if the number of start-ups is constrained, this decision becomes non-trivial.

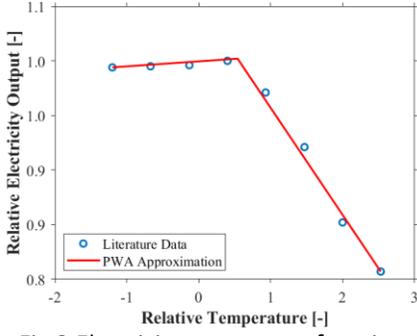


Fig 2: Electricity output as a function of ambient air temperature for a GE LMS100 relative to ISO standard conditions (15 °C, 100 MW)

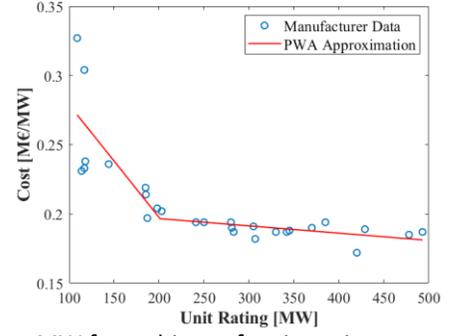
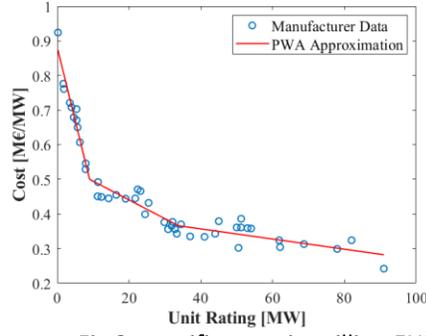


Fig 3: specific costs in million EUR per MW for turbines of various sizes. (L) Capacity < 100 MW; (R) Capacity 100-500 MW

## 2.2 Temperature dependency

The formulation of the temperature dependency is based on the GE LMS100 turbine. It is assumed that all turbines show the same relative temperature dependency at all loads. Fig 2 shows the relative power output (relative to ISO standard conditions) as a function of the relative ambient temperature. The behavior can be very well approximated by two linear segments, breaking at a relative ambient temperature of 0.4 (or an absolute ambient temperature of 6 °C). This information allows to formulate the correction factor  $f$ .

$$f = \gamma \cdot \frac{T_{\text{amb}}}{T_{\text{iso}}} + \delta \quad (3)$$

where  $\gamma$  and  $\delta$  are stepwise constant, reflecting the discontinuity at 6 °C ambient temperature. The correction factor  $f$  can be calculated in a pre-processing step since the temperature profile is not affected by the optimization's decision variables.

## 2.3 Fuel effect

Gas turbines operating on hydrogen have been shown to be technically feasible but are not yet state of the art due to the small role of hydrogen in the current energy system, and partially due to few technical issues like flame speed and NO<sub>x</sub> emissions. Nevertheless, having at least an approximate model is indispensable to investigate future energy systems. Detailed data [5] for gas turbines re-engineered to operate using hydrogen were compared with natural gas based performance curves to calculate a correction factor, which adjusts the turbine efficiency. Due to a lack of published data, the correction factor was calculated at 100 % baseload and assumed to be constant across the full operating range.

## 2.4 Investment Cost

In order to get representative market values rather than focusing on a specific model, the data available [6] was analyzed regarding specific costs of turbines up to 500 MW (see Fig 3).

## 3. MODEL FORMULATION

### 3.1 Performance

This section describes the final formulation of the model including all constraints. Constraints on a system level, like energy balances, are not reported here. The electric power output is described by

$$P_{i,t}^{\text{el}} = (\alpha_i \cdot x_i \cdot y_{i,t} \cdot F_i + \beta_i \cdot N_{i,t}^{\text{on}}) \cdot f \quad (4)$$

where  $x_i$  and  $y_{i,t}$  form a double bilinearity with  $F_i$ . This is resolved by a big-M approach as described in [1]. The heat output is indirectly determined via an energy balance (equation (5))

$$P_{i,t}^{\text{heat}} = F_{i,t} - P_{i,t}^{\text{el}} - P_{i,t,k}^{\text{loss}} \quad (5)$$

where the losses are defined as heat exchange between the stack and its surrounding

$$P_{i,t,k}^{\text{loss}} = \underbrace{\varphi_k \cdot \frac{F_{i,t}}{LHV_k}}_{\text{mass flow of flue gas}} \cdot c_p^{\text{flue gas}} \cdot \Delta T \quad (6)$$

$$k \in \{\text{natural gas, hydrogen}\}$$

The selection of turbine capacity is determined by  $x_i$  and unconstrained, i.e. all four turbines can be installed in parallel. The number of turbines per capacity level is constraint user defined minimum and maximum values

$$N_{\min} \leq N_t^{\text{tot}} \leq N_{\max} \quad (7)$$

and the number of turbines operating at any given time  $t$  is constrained by the actual fuel input

$$\frac{x_i y_{i,t} F_{i,t}}{F_i^{\max}} \leq N_{i,t}^{\text{on}} \leq \frac{x_i y_{i,t} F_{i,t}}{F_i^{\min}} \quad (8)$$

Finally, the fuel input is constrained by the operation range (50-100% baseload) and the total number of turbines

$$F_i^{\min} \leq F_{i,t} \leq F_i^{\max} \cdot N_t^{\text{tot}} \quad (9)$$

### 3.2 Maximum number of start-ups

To quantify the number of start-ups, the introduction of two separate variables, a counter for start-ups  $C_t^{\text{SU}}$  and a counter for shut-downs  $C_t^{\text{SD}}$ , is necessary to avoid non-linearities through absolute values. The following set of equations (10)-(13) make  $C_t^{\text{SU}}$  and  $C_t^{\text{SD}}$  count increases and decreases in the number of operating turbines, respectively.

$$N_t^{\text{on}} - N_{t-1}^{\text{on}} = C_t^{\text{SU}} - C_t^{\text{SD}} \quad (10)$$

$$C_t^{\text{SU}}, C_t^{\text{SD}} \in \{0, N^{\text{tot}}\}$$

$$C_t^{\text{SU}} + C_t^{\text{SD}} \leq N^{\text{tot}} \quad (11)$$

$$C_t^{\text{SU}} \leq N^{\text{tot}} - N_{t-1}^{\text{on}} \quad (12)$$

$$C_t^{\text{SD}} \leq N_{t-1}^{\text{on}}$$

Finally,  $C_t^{\text{SU}}$  can be constrained as

$$\sum_t C_t^{\text{SU}} \leq C_{\max}^{\text{SU}} \cdot N^{\text{tot}} \quad (13)$$

## 4. RESULTS

The problem was formulated in MATLAB R2018b using YALMIP [7] and solved with Gurobi v8.1 on a server featuring 2 Intel Xeon Silver 4110 2.1 GHz processors using 10 threads. In the single-objective analyses, it was found that the system relies entirely on wind turbines and natural gas turbines due to the lower price of the fuel when optimizing for costs. When optimizing for emissions, a strong emphasis on wind turbines and hydrogen turbines was found due to the lower emissions of hydrogen turbines. To better exploit the flexibility of the model, a cost optimization was conducted while limiting

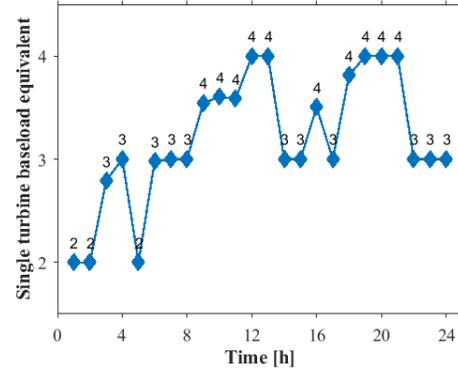


Fig 4: Power output of the 250 MW hydrogen turbines for one day presented as equivalents of the baseload of a single turbine. The labels show the number of turbines operating at each hour

the CO<sub>2</sub> emissions to the value obtained in the previous optimization for emissions, i.e. the lowest CO<sub>2</sub> emissions achievable. As expected, a middle-ground was found with one 10 MW natural gas turbine and four 250 MW hydrogen turbines. Fig 4 shows the number of operated hydrogen turbines and their power output for one day. It can be observed that high-load operation is favored whenever possible. Furthermore, the number of start-ups, 5 in total or 1.25 per turbine, is clearly lower than the set maximum of 2 start-ups per turbine and day.

### ACKNOWLEDGEMENT

ACT ELEGANCY, Project No 271498, has received funding from DETEC (CH), FZJ/PtJ (DE), RVO (NL), Gassnova (NO), BEIS (UK), Gassco AS and Statoil Petroleum AS, and is cofunded by the European Commission under the Horizon 2020 programme, ACT Grant Agreement No 691712.

### REFERENCE

- [1] Gabrielli P, Gazzani M, Martelli E, Mazzotti M. Optimal design of multi-energy systems with seasonal storage. *Appl Energy* 2018;219:408–24.
- [2] United Nations. Paris Agreement. 21st Conf Parties 2015.
- [3] Yang Y, Zhang S, Xiao Y. An MILP (mixed integer linear programming) model for optimal design of district-scale distributed energy resource systems. *Energy* 2015.
- [4] Van Den Bergh K, Delarue E. Cycling of conventional power plants: Technical limits and actual costs. *Energy Convers Manag* 2015.
- [5] Chiesa P, Lozza G, Mazzocchi L. Using Hydrogen as Gas Turbine Fuel. *J Eng Gas Turbines Power* 2005.
- [6] Gas Turbine World. GTW Handbook 2016.
- [7] Lofberg J. YALMIP : a toolbox for modeling and optimization in MATLAB, 2005.