

# Integrating Taguchi method and Neural network to optimize and predict the geometry of unileg thermoelectric generator and performance

Tzu-Hsuan Huang

Department of Aeronautics and  
Astronautics  
National Cheng Kung University  
Tainan 701, Taiwan  
b0939104553@gmail.com

Wei-Hsin Chen

Department of Aeronautics and  
Astronautics  
National Cheng Kung University  
Tainan 701, Taiwan  
weihsinchen@gmail.com

*Abstract*—When waste heat is recovered from vehicle exhaust gas at medium and high temperatures, using a unileg thermoelectric module (TEM) can effectively reduce thermal stress and increase its lifespan. To simultaneously maximize power generation and minimize the thermal stress of a unileg TEM, this study combines the Taguchi method and ANOVA to design the TEM, and a new objective function of the output power/thermal stress (P/S) ratio is conducted to find its maximum value. The Taguchi method considers four geometrical factors, including TE leg geometry, TEM height, copper thickness, and ceramic thickness, with four levels. The analysis suggests that the optimal geometric design considers the Rectangular geometry as the TE leg, a TEM height of 1 mm, a copper substrate thickness of 0.8 mm, and a ceramic thickness of 0.9 mm, yielding a P/S value of  $0.01255 \text{ W}\cdot\text{MPa}^{-1}$  with the maximum output power and thermal stress of 2.9 W and 232.32 MPa, respectively. The optimized case increases the P/S value by 1.29%. The ANN in this study achieves the average error values of 1.61% and 0.35% for the predictions of dual objective functions by multiple-step optimization. The relative errors of predicted P/S value and conversion efficiency by ANN under the Taguchi method suggestion optimal combination are 2.8% and 0.55%, respectively.

*Keywords*—Thermoelectric generator; Unileg; Thermal stress; Taguchi Method; Artificial Neural Network, Waste heat recovery

## I. INTRODUCTION

Thermoelectric generators (TEGs) are well suited for waste heat recovery applications [1]. Only a temperature difference is required to enable TEG to generate electricity. TEGs provide the following advantages, zero pollution, long equipment life, compact size, silent operation, high reliability, and no moving parts. When a TEG is implemented, vehicle exhaust gas or industrial waste heat can be used as the heat source. However, the TEG experiences thermal stress, causing a lessened TEG lifespan. Previous literature shows that using unileg-designed TEM can effectively reduce

thermal stress and expand the device's lifespan in medium-high temperature environments [2].

The TEM mainly consists of a ceramic substrate, solder layer, copper electrode, and p, n-type semiconductor legs. The coefficients of thermal expansion of each TEM component are different. Therefore, when there is a temperature gradient across the TEM and the TEM is restricted by outward conditions, it can't deform freely, posing stress to be generated inside the TEM. This is known as thermal stress. The Taguchi method is an effective approach to improving TEM performance, which can analyze the influence of different factors to design suitable conditions for a given TEM [3]. Artificial intelligence (AI) is a computer algorithm that imitates human intelligence, and it has been widely used in medical, automotive, business, and other applications [4]. The Taguchi method can be combined with AI in the research of thermoelectric generators

At present, no research study that optimizes the geometry of unileg thermoelectric legs in the combination of machine learning to predict and analyze thermoelectric performance and thermal stress. Therefore, this study aims to combine the Taguchi method, ANN, and unileg TEG to optimize and predict the geometry and efficiency of thermoelectric generators.

## II. PAPER STRUCTURE

### A. Physical model

The physical model of a unileg TEM [5] is considered in this study. For the simulation, only one pair of n-type semiconductors and nickel terminals are used for analysis. The original length, width, and height of the thermoelectric legs are  $4 \text{ mm} \times 4 \text{ mm} \times 4.6 \text{ mm}$ , respectively. The nickel solder thickness is 0.2 mm on both sides of the semiconductor. The geometry of the nickel terminal is cylindrical, with a diameter of 1 mm and a height of 5 mm. The substrate material is copper, and its dimensions are  $10 \text{ mm} \times 4 \text{ mm} \times 0.8 \text{ mm}$ . The ceramic ( $\text{Al}_2\text{O}_3$ ) is used as an insulating material and placed on the outermost of the TEM. The TEM model includes four different thermoelectric leg

geometries with the same volume and cross-sectional area. This study uses skutterudite as the thermoelectric material.

### B. Boundary conditions

For the thermal boundary conditions, the hot and cold surface temperatures are fixed at 873 K and 303 K, respectively. The following assumptions were made to simplify and decrease the handling of the physical model. (1) the heat flux and temperature between the element and the electrode are continuous, (2) the contact resistance and the Thomson effect are neglected, (3) the heat transfer ( $Q_h$ ) and current on the side of the element are ignored, and (4) the thermal radiation effect is ignored.

For the thermal stress analysis, the multiphysics coupling involving a static structural system is applied. The cold surface is assumed to be fixed. The boundary conditions details of TEM are shown in Fig. 1a.

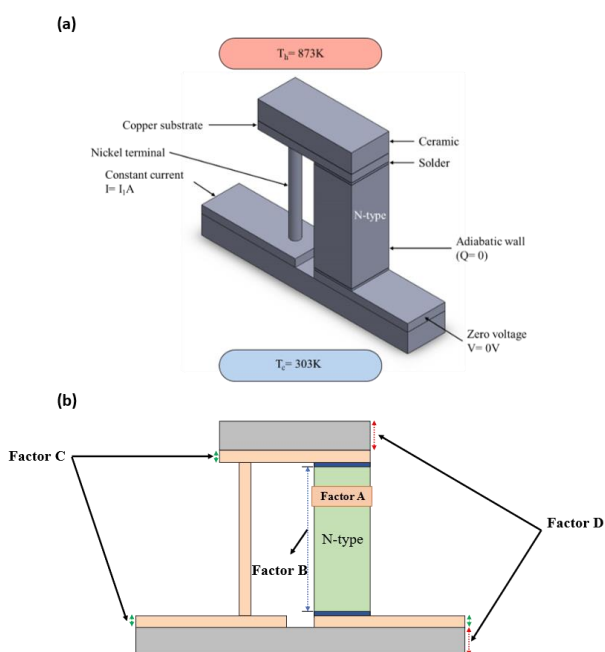


Fig. 1. (a) Boundary conditions and (b) the factors of Taguchi method in thermoelectric module.

### C. Numerical method and validation

In this study, the commercial software ANSYS 2020 R3 is used to analyze the performance of TEM. Coupling the temperature field and static structure to evaluate the thermal stress and lifespan improvement of unileg TEMs. The thermoelectric system and thermal stress module for this study and the literature result are compared, and the average relative error of validations are less than 7.7% and 5%, respectively.

### D. Optimization method

The Taguchi method is a technique that can reduce experimental time and cost. The purpose is to find the best system factor combination, retain the objective function's robustness, and stabilize the design target quality. The indicator of the quality measurement is defined by the signal-to-noise (S/N) ratio. The signal-to-noise (S/N) ratio can be used to evaluate the sensitivity of parameters to target

behavior. Reducing thermal stress and increasing output power at the same time is an important target for TEMs. In this study, the objective function has used the ratio of maximum output power and maximum thermal stress, P/S value ( $W \cdot MPa^{-1}$ ), and the optimization target is the maximum ratio value. In the design of the TEM, four control factors are considered: Factor A is the geometry of the TE leg, Factor B is the TEM height, Factor C is the copper substrate thickness, and Factor D is the ceramic thickness. In the optimization process, the shape of TE legs is chosen to be either Rectangular, Cylindrical, Triangular, or trapezoid geometry. The typical orthogonal array L16 ( $3^4$ ) with 16 runs for the Taguchi method is shown in Table 1.

TABLE I. CONTROL FACTORS AND LEVEL SETTINGS FOR TAGUCHI

Factor	Control parameter	Level			
		1	2	3	4
A	Geometry of TE leg	Rectangular	Cylindrical	Triangular	Trapezoidal
B	TEM height (mm)	1	2.5	4	5.5
C	Cu Substrate thickness (mm)	0.8	0.9	1	1.2
D	Ceramic thickness (mm)	0.7	0.8	0.9	1

### E. Data analysis

The commercial software PolyAnalyst is a statistical data analyzer. In this study, the ANN is based on resilient propagation (Rprop) networks computed with PolyAnalyst. The Rprop is an adaptation and improvement over the back propagation technique. Rprop neural networks involve multiple input, hidden, and output layers. Rprop is the best algorithm in terms of the convergence speed, accuracy, and robustness of training parameters. The convergence speed of Rprop is faster than the back-propagation networks. Both back-propagation and Rprop techniques work in a similar way. They use repeated calculations to determine the effect of each weight in the network while minimizing the difference between the network's output and the sample's target output. In this study, the P/S value and energy conversion efficiency are target outputs. The ANN data source comes from the Taguchi method's orthogonal array data.

## III. RESULTS AND DISCUSSION

### A Optimized TEM performance with the Taguchi method

To find the best combination of TEM designs to increase the output power and reduce the generation of thermal stress, the Taguchi method is used in this study for geometry optimization of the TEM. The Taguchi method can significantly reduce the number of experiments and operation time. The Taguchi method considered four factors (TE leg geometry, TE leg height, copper substrates, and ceramics thicknesses) and four levels in this study. The

objective function P/S value results of the 16 experimental runs are shown in Fig. 2a.

The S/N ratio is calculated for each case. The results show that the S/N ratios of Case 5 and Case 12 are -38.14 and -52.75, respectively. Fig. 2b shows the S/N ratio of the 16 cases, and the distribution of the average S/N ratio of 16 cases is shown in Fig. 2c. The sensitivity of the objective function P/S value to the 4 factors can be obtained by the maximum average S/N ratio of the factor minus the minimum average S/N ratio.

To obtain the mean S/N ratio of Factor A at level 1, which need to calculate the mean value for case 1 (-38.2949), case 2 (-43.1717), case 3 (-48.0564), and case 4 (-51.1895). Factor B, Factor C, and Factor D of each level follow this calculation method to get the mean S/N ratio. The higher the sensitivity of a factor to the P/S value, the more significant the factor's effect on the P/S value. Fig. 3a employs the radar chart to represent the influence values of the four factors in the Taguchi optimization method in this study. The influence of the factors on the P/S value is ranked as Factor B > Factor A > Factor C > Factor D. The results show that the TEM height has the most significant influence on the P/S value. On the contrary, the influence of ceramic thickness on the P/S value is fairly insignificant.

ANOVA is a method used to evaluate the effect of each control factor on the objective function. In this study, the factor is considered statistically significant when the p-value of the factor is less than the  $\alpha$ -value  $\leq 0.05$ . The contribution of each factor to the total variance is expressed as a percentage (PC%). The analysis results of ANOVA are shown in Table 2. The F-test value of Factor B is 492.02, which is the largest among the four factors. This implies, in turn, that Factor B's influence on the system is the largest, and its PC value is 92.09%. The lowest F-test value is Factor D of 5.33, which indicates that the impact of Factor D on the system is very small, and its PC value is 0.99%. The influence analyzed by ANOVA is ranked as Factor B (TEM height) > Factor A (geometry of TE leg) > Factor C (copper substrate thickness) > Factor D (ceramic thickness). The results show that the effect of Factor B on the system is much greater compared to the other three factors, while the effect of factor D is relatively insignificant. Overall, the influence rank of the factors obtained by the ANOVA method is aligned with the Taguchi method.

TABLE II. ANALYSIS OF VARIANCE FOR THE FOUR FACTORS

Factor	Sum of squares	f	F-value	p-value	PC (%)
A	$S_A=13.025$	3	20.25	0.0171	3.79
B	$S_B=316.504$	3	492.02	0.0002	92.09
C	$S_C=10.726$	3	16.67	0.0225	3.12
D	$S_D=3.427$	3	5.33	0.1015	0.99
Error	0.643	3			0.01

The optimal combination of the four factors for the P/S ratio can be obtained from the results shown in Fig. 2c. The best combination follows the geometry of the Rectangular TE leg (Level 1), the TEM height at 1 mm (Level 1), the

copper substrate thickness at 0.8 mm (Level 1), and the ceramic thickness at 0.9 mm (Level 3). Because this combination case does not appear in the 16 cases, this optimal combination is predicted additionally. The P/S value of the best combination is  $0.012552 \text{ W}\cdot\text{MPa}^{-1}$ . Accordingly, the Taguchi method successfully optimizes the operating parameters to improve the P/S value. The optimized case has a 1.29 % increase in P/S value compared to Case 5.

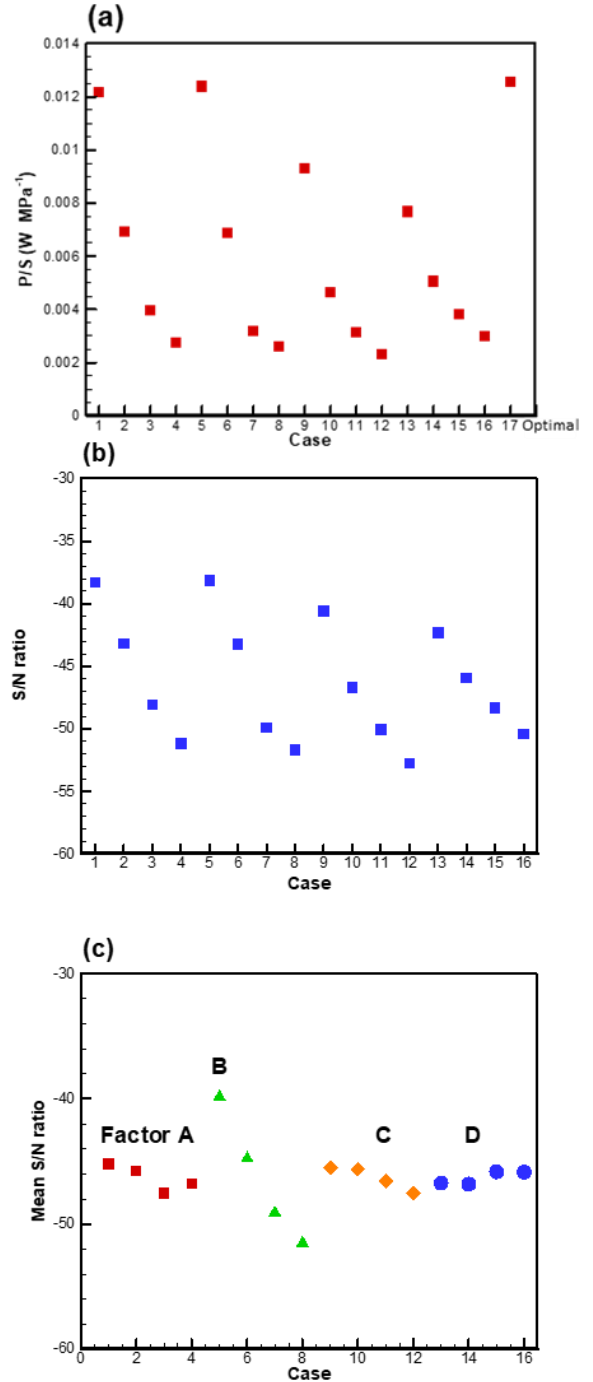


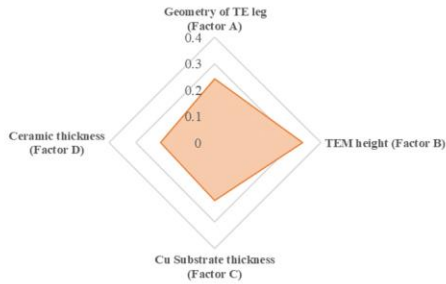
Fig. 2. (a) Profiles of (a) S/N ratio and (b) mean S/N ratios of factors.

### B Data analysis in artificial neural network

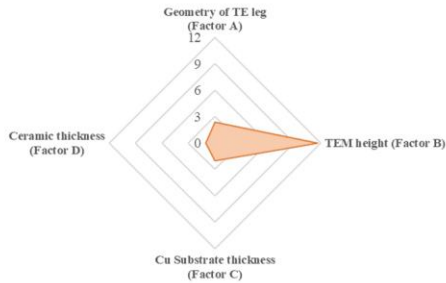
The ANN model is adopted to predict data in this study, in which the analysis data from the Taguchi method is used as the database for the ANN. The sensitivities of the four

factors in ANN to P/S value and conversion efficiency are shown in Fig. 3b and Fig. 3c. For the P/S value, the rank of sensitivity of the four factors is Factor B (TEM height) > Factor A (geometry of TE leg) > Factor C (copper substrate thickness) > Factor D (ceramic thickness). The Taguchi method, ANN, and ANOVA have the same sensitive rank. The rank of factor sensitive for conversion efficiency is Factor B > Factor D > Factor C > Factor A.

(a) Taguchi method- P/S value



(b) ANN- P/S value



(c) ANN- Efficiency

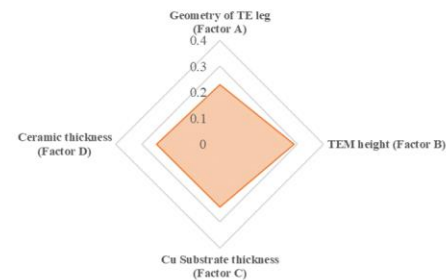


Fig. 3. Radar charts of effect of factors in (a) Taguchi method- P/S value, (b) ANN- P/S value, and (c) ANN- efficiency.

To obtain more accurate prediction data, the predicted case increases the levels of Taguchi method Factors B, C, and D from Level 4 to Level 10, in which the number of cases is 4000, namely, the Factor B (1-5.5), factor C (0.8-1.25), factor D (0.7-1.15). The data of the ANN model database is divided into training data and test data in a ratio of 7:3. The activation function of the output layer and hidden layer is the sigmoid function. Before the training, the effects of different hidden layers and the number of neurons on prediction are tested, including two hidden layers with 50 neurons (30, 20), two hidden layers with 90 neurons (70, 20), three hidden layers with 90 neurons (70, 15, 5). The predicting fit quality of the three case tests are 94.6% ( $R^2 = 0.946$ ), 99.9% (0.999), 99.8% (0.998), respectively. The results showed that the best  $R^2$  value (0.999) occurred in the second test.

In this study, the training process is divided into multiple steps to improve the model database to increase prediction

accuracy. In the first step, 4000 combination cases of data are predicted by ANN using 16 data from the Taguchi method. The five combinations (Excellent, Good, Average, Fair, Poor) are extracted from the obtained prediction results and used simulation software (Ansys) to analyze the correct results. Then, the error value between the simulation results and the predicted data is evaluated. The five correct data calculated by Ansys are added to the database (21 data) and used in the next prediction. Using the same method to correct the model accuracy until the mean error value is smaller than 2%. Fig. 4a and Fig. 4b show the relative error evaluation results of P/S value and conversion efficiency for multiple-step improvements. The average error value of the prediction model of the P/S value decreases from 8.33% in the first step to 1.61% in the fifth step. For the prediction of conversion efficiency, the average error value decreases from 24.98% in the first step to 0.36% in the fifth step. The results show that after the fifth step of correction, the model has high reliability, and the total number of data entries in the database is 37. This means that the database has more entries, and the prediction results are more accurate.

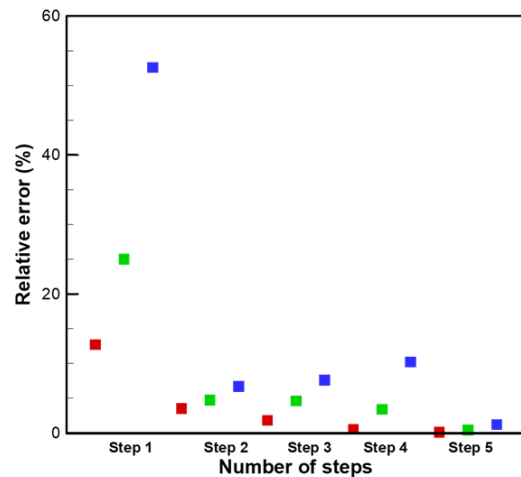
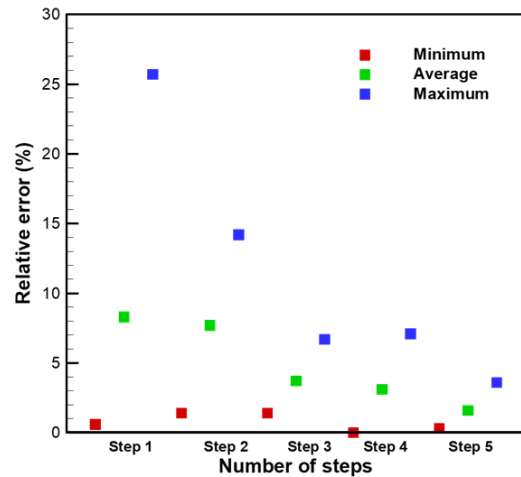


Fig. 4. The multi-step average relative error of prediction versus simulation of (a) P/S value and (b) efficiency.

The linear regression analysis is used to verify the ANN's reliability, as shown in Fig. 5a and Fig. 5b. The results show that the ANN model has very high  $R^2$  values by linear analysis. This means that the ANN model can precisely

predict the two objective functions. Finally, the results from the Taguchi runs, the Taguchi optimal case, and the ANN predicted optimal case are shown in Table 6. In addition, the data predicted by the improved ANN model and the correct data analyzed are compared. The results show that the ANN prediction 18 cases error values of P/S ( $< 8\%$ ) and conversion efficiency ( $< 2.1\%$ ). The relative errors of predicted P/S value ( $0.01222 \text{ W}\cdot\text{MPa}^{-1}$ ) and conversion efficiency (3.78%) by ANN under the Taguchi method suggestion optimal combination (optimal case) are 3.79% and 0.54%, respectively. For the ANN independent prediction optimal combination, the best combination includes the geometry of TE leg as a Rectangular, the TEM height at 1 mm, the copper substrate thickness at 0.8 mm, and the ceramic thickness at 0.95 mm. The optimal combination recommended by ANN has closed to the Taguchi method optimal combination. The relative errors of P/S value and conversion efficiency by the ANN to predict the optimal combination are 1.28% and 0.19%, respectively, which indicates that the ANN model has excellent accuracy.

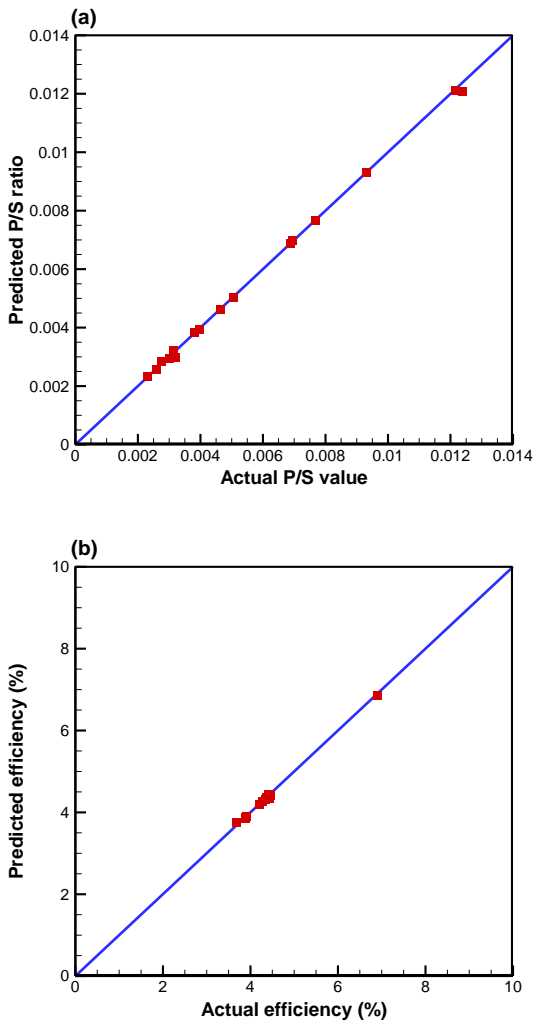


Fig. 5. Regression analysis of (a) P/S value and (b) conversion efficiency between predicted and real value in ANN for step 5.

#### IV. CONCLUSIONS

This work is the first to combine the Taguchi method and machine learning ANN to optimize the performance and reduction the thermal stress distribution of unileg TEM in waste heat recovery. The Taguchi method analyzes four factors (geometry of TE leg, TEM height, copper, and ceramic thickness) for the effect of the new objective function P/S value (output power/thermal stress) and finding its maximum value. The Taguchi method and ANOVA show the importance of the four factors, and ranked them based on their level of influence. The result shows that the influence of TEM height is much larger than the other three factors. The optimal combination has the geometry of TE leg as a Rectangular, the TEM height at 1 mm, the copper substrate thickness at 0.8 mm, and the ceramic thickness at 0.9 mm, and the best P/S value is obtained ( $0.012552 \text{ W}\cdot\text{MPa}^{-1}$ ). The optimized case increases the P/S value by 1.29%. In ANN, this study improves the prediction accuracy of the ANN model through multiple-step improvement. The average error value of the predicted double objective function was reduced to 1.61% and 0.36% in the fifth step improvement. This reflects that the more improvements to the database, the prediction results are more accurate. The relative errors of ANN prediction for P/S value and conversion efficiency under the optimal combination case are 2.8% and 0.55%, respectively. The relative errors of predicted P/S value and conversion efficiency by the ANN independent prediction optimal combination are 1.28% and 0.19%. In addition, the Taguchi method, ANOVA, and ANN have the same sensitive rank for the P/S value. Overall, this study successfully uses the Taguchi method to find the optimal combination of TEM geometries for unileg TEMs. Then, the results of the Taguchi method are brought into ANN, and the prediction model is successfully established. These results reveal that ANN can accurately predict dual objective functions and is a promising tool for designing unileg TEMs.

#### REFERENCES

- [1] Ji D, Wei Z, Mazzoni S, Mengarelli M, Rajoo S, Zhao J, et al. Thermoelectric generation for waste heat recovery: Application of a system level design optimization approach via Taguchi method. *Energy Conversion and Management*. 2018;172:507-16.
- [2] Shittu S, Li G, Zhao X, Ma X. Review of thermoelectric geometry and structure optimization for performance enhancement. *Applied Energy*. 2020;268.
- [3] Ji D, Hu S, Feng Y, Qin J, Yin Z, Romagnoli A, et al. Geometry optimization of solar thermoelectric generator under different operating conditions via Taguchi method. *Energy Conversion and Management*. 2021;238:114158.
- [4] Chen W-H, Tsai Z-L, Chang M-H, You S, Kuo P-C. Geometry optimization and pressure analysis of a proton exchange membrane fuel cell stack. *International Journal of Hydrogen Energy*. 2021;46:16717-33.
- [5] Nemoto T, Iida T, Sato J, Sakamoto T, Hirayama N, Nakajima T, et al. Development of an Mg<sub>2</sub>Si Unileg Thermoelectric Module Using Durable Sb-Doped Mg<sub>2</sub>Si Legs. *Journal of Electronic Materials*. 2013;42:2192-7.