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Low-carbon Allocation for Urban Multi-operator Energy Systems: A Coordinated Planning Approach Based on Feasible Region Projection#

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ABSTRACT

With accelerating climate change and policy-driven mandates for carbon neutrality, cities are prioritizing low-carbon transitions in energy systems. In response to this urgent need, this paper proposes a low-carbon-oriented planning framework for urban multi-operator integrated energy system (UMIES), integrating multi-operator coordination, carbon emission cost and system feasibility. Firstly, a novel low-carbon/economic feasible region model is developed to harmonize multi-operator resource allocation with carbon reduction goals, and the low-carbon-oriented optimal allocation model is further developed. Then, the coordinated planning approach based on feasible region projection is proposed, which enables the multi-operator optimal allocation realized in a privacy-protected, iteration-free and calculation-distributed fashion. Case studies validate the method's efficacy in balancing economic viability, carbon mitigation, and privacy protection, offering a practical pathway for sustainable urban energy planning.

Keywords: low-carbon planning, urban energy system, multi-operator coordination, feasible region.

1. INTRODUCTION

Cities are responsible for close to 70% of global CO2 emissions associated with energy consumption, with the proportion being as high as 80% in North America[1]. Low-carbon development in urban areas is therefore key to addressing global energy shortages and climate deterioration. For various energy sectors in cities, effective coordination has become a crucial strategy for resource integration, systematic energy saving and deep decarbonization. This situation pushed urban energy systems to evolve from decentralized to coupled, and led to the formation of the Urban Multi-operator Integrated Energy System (UMIES). However, this interdependence significantly increases the difficulty of equitable resource allocation and efficient operation management, which means that the advanced digital tools are required to address the computational complexity[2].

In order to promote the integration and utilization of urban energy resources, there has been considerable research on the allocation methods for UMIES. Ref.[3] proposes a decentralized resource allocation approach, considering the integration of both heat, electricity, and various types of hybrid energy devices. Ref.[4] proposes a mixed game-based two-stage planning model for distribution system with multi-operator integrated energy microgrids. With low-carbon orientation, Ref.[5] establishes a bilevel allocation model of shared energy storage station for UMIES accounting for carbon emission reduction. These allocation methods are typically constructed as bilevel model, combined with the optimization of operational processes[7]. However, the lack of efficient rules for information interactions in multilayer models, resulting in privacy leakage and intensive iteration, has hindered the practical application of multi-operator coordination, which is often neglected by existing research.

In this regard, the feasible region (FR) method provides effective tool for the analysis of UMIES with the advantages of global analysis, privacy protection, and standardized characterization[8]. Several studies have been carried out to construct and analyze the FR of integrated energy systems. A convex hull-based FR for electricity-gas integrated energy system is constructed in the Ref.[9]. Further, Ref.[10] proposes the concept of integrated energy system security region base on FR. Ref.[11] realizes the complete characterization of the steady-state FR for electricity-gas integrated systems. However, traditional FRs are difficult to completely characterize non-operational features such as economy and low carbon, while lacking the mechanism for coordination of isomers, making them less adapted to multi-operator and multi-layer programming problems.

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Thereby few studies apply FR to optimize allocation for energy systems, and its potential for multi-operator coordination needs to be further excavated.

Based on the above research, this paper studies region-based optimal allocation method in UMIES. The contribution of this paper is twofold:

1) A novel FR modeling approach considering multi-operator and non-operational features is developed. Based on this, the optimal allocation model of UMIES is developed to determine the optimal capacity of the coupled energy devices and the lowest carbon operation of the system.

2) A novel multi-operator coordinated planning approach based on FR equivalent projection is proposed, which supports the energy governance in organizing various operators to conduct coordinated planning with a privacy-protected, iteration-free and calculation-distributed fashion.

2. FRAMEWORK OF UMIES

Fig.1 illustrates the typical framework of UMIES, which consists of several energy operators and a unified energy governance. At the physical level, urban energy operators receive energy inputs at energy supply stations and distribute them through urban energy distribution networks, such as power distribution networks (PDN), gas distribution networks (GDN), and district heating networks (DHN), to meet the energy demands of end-users; Energy governance is responsible for investing and managing the energy hub (EH), which consists of energy coupling devices such as Combined heat and power units (CHP) and Electric Boilers (EB), to coordinate the energy conversion among various distribution networks. At the information level, the bidirectional communication is established between the unified energy governance and any energy operator. The necessary operational data is provided by the energy operators to support the planning assignment of energy governance. Correspondingly, the energy governance allocates the coordinated planning scheme to guide the operators to optimize the operation.

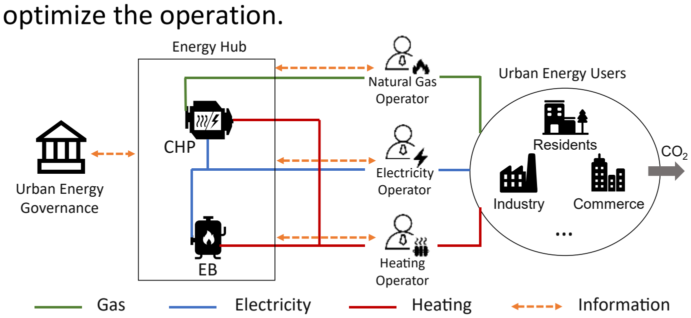


Fig. 1 Framework of UMIES

From the multi-operator perspective as illustrated in this framework, the following considerations need to be discussed in the coordination of UMIES, which determine whether the optimal allocation of resources with a low carbon orientation is practically feasible.

1) For energy operators, user load and other private data are required to be adequately protected during the coordination. Moreover, different operators need to focus on optimizing their own resources, which implies the necessity for distributed computing.

2) For the energy governance, it is necessary to balance investment economics and low carbon goals while the operational feasibilities of various distribution networks are met. Therefore, a coordination mechanism, which considers multiple stakeholders and multiple technical features, is required to ensure the globally optimal performance of the system.

3. MODELING FOR UMIES BASED ON LOW-CARBON/ECONOMIC FEASIBLE REGION

In this section, we propose a modeling approach for the low-carbon feasible region (LFR) of each operator and the economic feasible region (EFR) of energy governance. Accordingly, the low carbon-oriented optimal allocation model of UMIES is further developed.

3.1 Low-carbon feasible region of energy operators

From the perspective of operating feasibility and security, FR is applied to portray the operational characteristics of the operators' network. It is defined as a set of all the feasible operating points satisfying the security constraints[10]. Taking the GDN as an example, its FR is defined as Eq.(1), encompassing all feasible combinations of flow and pressure that simultaneously satisfy flow and pressure limits, second-order cone relaxation Weymouth constraints, and nodal balance constraints.

$$\Omega^{\mathrm{G,FR}}=\left\{W^{\mathrm{G}}\left|\begin{array}{l}0\le F_{ij}\le F_{ij,\max}\\ p_{i,\min}\le p_i\le p_{i,\max}\\ (F_{ij})^2\le(K_{ij}p_i)^2-(K_{ij}p_j)^2\\ \sum\limits_{n:n\to i}F_{ni}-\sum\limits_{k:i\to k}F_{ik}=-F_i^{\mathrm{Source}}\\ \qquad+(F_i^{\mathrm{Load}}-F_i^{\mathrm{Shed}})+F_i^{\mathrm{Hub}}\end{array}\right.;\forall i,j\in I_{\mathrm{G}}\right\} \tag{1}$$

Where, $W^{\mathrm{G}}=(x^{\mathrm{G}},y^{\mathrm{G}})$ denotes the feasible operating points of GDN, in which x^{G} represents the device-side contacted variables between the GDN and the EH, and y^{G} indicates the internal variables of the GDN; I_{G} represents the set of nodes; F_{ij} represents branch flow; p_i represents nodal pressure; K_{ij} is the Weymouth parameter for natural gas pipelines; F_i^{Source}, F_i^{Load}, F_i^{Shed}, and F_i^{Hub} correspond to the nodal supply flow, load flow, load shedding, and interaction flow with EHs, respectively.

Furthermore, the carbon emissions cost (CEC) has been introduced to establishes a link between low-carbon technical features and economics. It measures the total amount of carbon emissions produced by all carbon sources in the system at a penalty price The definition of the CEC in the GDN is as follows.

$$E_g = \sum_{i \in I_{\text{G}}} c^{\text{Source,gCO}_2} F_i^{\text{Source}} \tag{2}$$

Where, $c^{\text{Source,gCO}_2}$ is the carbon emission factor per unit of carbon emission source in the GDN.

Based on FR and CEC, the LFR of energy operator is further proposed to characterize the feasible space from the low-carbon and secure perspectives. In addition to satisfying security constraints, all feasible operating points within the LFR must keep CEC within an acceptable limit. Once the LFR has been determined, low-carbon boundaries and all secure feasible operating scenarios for one operator are fixed, which means that the low-carbon technical feature and secure operational characteristics of the network are fully inscribed. Similarly, the LFR of GDN, PDN and DHN can be expressed in the following unified form.

$$\Omega^{m,\text{LFR}} = \left\{ W^m, \alpha^m \middle| \begin{array}{l} H_m(W^m) \le 0 \\ E_m(W^m) \le \alpha^m \end{array} \right\}, m \in \text{M} \tag{3}$$

Where, $H_m(W^m) \le 0$ represents security constraints of energy operator; $E_m(W^m) \le \alpha^m$ represents carbon emission constraints, in which α^m is a CEC constrained auxiliary variable; m identifies the different energy operator networks and M represents the set of all networks.

3.2 Economic feasible region of energy governance

The FR of the energy governance is defined as the joint feasible space of all the coupled devices within the managed EH, denoted as Eq.(4).

$$\Omega^{\text{EG,FR}} = \left\{ W^{\text{EG}} \middle| \begin{array}{l} H_{eg}(W^{\text{EG}}) \le 0 \\ R_{eg}(W^{\text{EG}}) = 0 \end{array} \right\} \tag{4}$$

Where, $W^{\text{EG}} = (F^{\text{Hub}}, P^{\text{Hub}}, Q^{\text{Hub}})$ denotes the feasible operating points, equivalently expressed in terms of device-side contacted variables as $W^{\text{EG}} = (x^{\text{G}}, x^{\text{E}}, x^{\text{H}})$; $H_{eg}(W^{\text{EG}}) \le 0$ represents the operation constraints of the coupled devices; $R_{eg}(W^{\text{EG}}) = 0$ established the relationship between electric power, heat flow, and gas flow for different networks as Eq.(5), using CHP unit and EB as typical devices.

$$\begin{aligned} P^{\text{CHP}} &= \eta^{\text{CHP,tP}} F^{\text{CHP}} \\ Q^{\text{CHP}} &= \eta^{\text{CHP,tH}} F^{\text{CHP}} \\ Q^{\text{EB}} &= \eta^{\text{EB,tH}} P^{\text{EB}} \end{aligned} \tag{5}$$

Where, $\eta^{\text{CHP,tP}}$, $\eta^{\text{CHP,tH}}$, and $\eta^{\text{EB,tH}}$ represent the energy conversion coefficient.

For coupled devices to be allocated in EH, not only operating feasibility but also investment feasibility should be met. There, the EFR of the energy governance considering economic investment is proposed, defined as follows.

$$\Omega^{\text{EG,EFR}} = \left\{ W^{\text{EG}}, z^{\text{EG}} \middle| \begin{array}{l} L_{eg}(z^{\text{EG}}) \le 0 \\ H_{eg}(W^{\text{EG}}, z^{\text{EG}}) \le 0 \\ R_{eg}(W^{\text{EG}}) = 0 \end{array} \right\} \tag{6}$$

Where, $z^{\text{EG}} = (z^{\text{EG,loc}}, z^{\text{EG,cap}})$ represents the feasible investment points, in which $z^{\text{EG,loc}}$ and $z^{\text{EG,cap}}$ corresponds to the installed location and capacity of the EH's devices, respectively; $L_{eg}(z^{\text{EG}}) \le 0$ indicates the investment constrains, which include limitations on the maximum capacity of the devices; $H_{eg}(W^{\text{EG}}, z^{\text{EG}}) \le 0$ means that the operating boundaries of the devices is determined simultaneously by the power operating points and the installed capacity to be invested.

It should be noted that the investment constraints $L_{eg}(\bullet)$ and the security constraints $H_e(\bullet)$, $H_h(\bullet)$, and $H_{eg}(\bullet)$ for the PDN, HDN, and EH are modeled in detail in the Ref.[5]. and Ref.[6] Thereby the specific form of these constraints is not introduced additionally in this paper.

3.3 Modeling for Optimal Allocation of UMIES

Based on the LFRs and EFR, the optimal allocation model of UMIES is established to achieve a balance between low carbon emissions and economic viability. The objective is to minimize the comprehensive cost of investment and low-carbon operation, and the energy governance's EFR and the energy operator's LFRs are included in the constraints, as shown follow.

$$\begin{aligned} &\min C_a(z^{\text{EG}}) + \sum_{m \in \text{M}} \alpha^m \\ &s.t. \quad (x^{\text{G}}, x^{\text{E}}, x^{\text{H}}, z^{\text{EG}}) \in \Omega^{\text{EG,EFR}} \\ &\qquad\ (x^m, y^m, \alpha^m) \in \Omega^{m,\text{LFR}}, \forall m \in \text{M} \end{aligned} \tag{7}$$

Where, $C_a(\bullet)$ represents the annualized allocation cost of the coupled devices, denoted as Eq.(8).

$$C_a = \frac{\gamma(1+\gamma)^{y_r}}{(1+\gamma)^{y_r} - 1} (c^{\text{EG,loc}} z^{\text{EG,loc}} + c^{\text{EG,cap}} z^{\text{EG,cap}}) \tag{8}$$

Where, γ is the discount rate; y_r is the payback period of the coupled devices; $c^{\mathrm{EG,loc}}$ represents the site construction cost and $c^{\mathrm{EG,cap}}$ represents the unit capacity investment cost of the coupled devices.

4. COORDINATED PLANNING APPROACH BASED ON FEASIBLE RIGION PROJECTION

In this section, we propose a coordinated planning method for UMIES's model based on feasible region projection (FRP-based), making the optimal allocation realized in a privacy-protected, iteration-free and calculation-distributed fashion.

4.1 Equivalent Projection of LFRs

From an external observation, the geometric feature of LFR is a closed polyhedron in the operational space. The equivalent projection (EP) has been developed to describe the external characteristics of LFR.

In the EP model, the device-side contacted variables x^m and the CEC auxiliary variable α^m are retained as the observed variables portfolio to characterize equivalent projection region (EPR), while the internal variable y^m is eliminated, as defined by Eq.(9). Fig. 2 illustrates the relationship between the original LFR and the processed EPR.

$$\Phi^{m,\mathrm{EPR}} = \left\{(x^m,\alpha^m) \middle| \exists y^m, (x^m,y^m,\alpha^m) \in \Omega^{m,\mathrm{LFR}} \right\} \tag{9}$$

The above model illustrates that the EPR consists of all feasible contact variables (x^m,α^m). For any defined set of (x^m,α^m) in the EPR, there exists at least one feasible value of the internal variable y^m such that (x^m,y^m,α^m) is a feasible point of LFR.

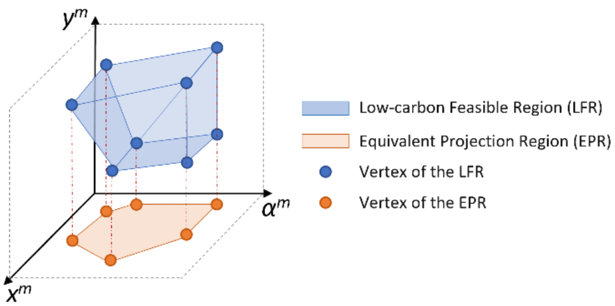


Fig. 2 Equivalent projection of LFR

For comparison, the dimension of the LFR constituted by (x^m,y^m,α^m) is $\mathbb{R}^{N_x+1}\times\mathbb{R}^{N_y}$. In general, the number of internal controllable devices is significantly larger than the number of coupled devices, which means that $N_y \gg N_x+1$. Accordingly, the EP model achieves dimensionality reduction observation of complex LFRs by selecting low-dimensional contact variables $(x^m,\alpha^m)\in\mathbb{R}^{N_x+1}$, while preserving α^m to depict the low-carbon technical features, and eliminating internal variable y^m to support privacy protection.

Additionally, there are already well-established methods that support the generation of display representations of EPRs from raw FRs, such as the Progressive Vertex Enumeration (PVE) algorithm and the Fourier-Motzkin Elimination algorithm in Ref.[12]. And the former is employed in the computation experiments in this paper.

4.2 Coordinated planning based on EP

According to the proposed framework, the energy governance performs the optimal allocation of coupled devices based on the EPRs submitted by the operators. Therefore, the original planning model shown in Eq.(7) can be equivalently decomposed into the following bi-level optimization model.

- Upper-level coordinated allocation:

$$\begin{aligned} &\min C_a(z^{\mathrm{EG}}) + \sum_{m\in[M]} \alpha^m \\ &s.t.\quad \begin{aligned} &(x^{\mathrm{E}},x^{\mathrm{G}},x^{\mathrm{H}},z^{\mathrm{EG}}) \in \Omega^{\mathrm{EG,EFR}} \\ &(x^m,\alpha^m) \in \Phi^{m,\mathrm{EPR}},\quad \forall m\in[M] \end{aligned} \end{aligned} \tag{10}$$

- Lower-level distributed optimization:

$$\begin{aligned} &\min E_m\left(\hat{x}^m,y^m\right) \\ &s.t.\quad \begin{aligned} &H_m(\hat{x}^m,y^m) \le 0 \\ &E_m(\hat{x}^m,y^m) \le \hat{\alpha}^m \end{aligned} \end{aligned} \tag{11}$$

Where, $\hat{x}^m$ and $\hat{\alpha}^m$ denotes the optimal coordination instruction for the upper-level model.

Obviously, the coordinated planning framework avoids inefficient iterations, which is the open-loop and easy-to-practice business process. Simultaneously, both EPRs computation and lower-level optimization can be implemented in parallel by distributed computers deployed in different operators.

The implementation of the coordination is divided into three stages according to business process, i.e., equivalent projection, coordinated allocation, and distributed optimization, as shown in Fig.3. It is described in detail as follows.

Stage 1. Equivalent projection. According to Eq.(9), the operators calculate their respective EPRs $\Phi^{m,\mathrm{EPR}}$ from the LFRs $\Omega^{m,\mathrm{LFR}}$ in parallel and submit the EPRs to the energy governance.

Stage 2. Coordinated allocation. According to Eq.(10), the energy governance performs the optimal allocation for EH's coupled devices combining its EFR $\Omega^{\mathrm{EG,EFR}}$ and EPRs Φ^m. The optimal allocation scheme

z^* and the optimal coordination instruction $(\hat{x}^m, \hat{\alpha}^m)$ are obtained and distributed to the corresponding operator.

Stage 3. Distributed optimization. According to Eq.(11), each operator solves the distributed operation optimization model in parallel based on fixed coordination instruction $(\hat{x}^m, \hat{\alpha}^m)$ to obtain the optimal low-carbon operating point (x^{m*}, y^{m*}) and calculates the corresponding CEC E_m^*, respectively.

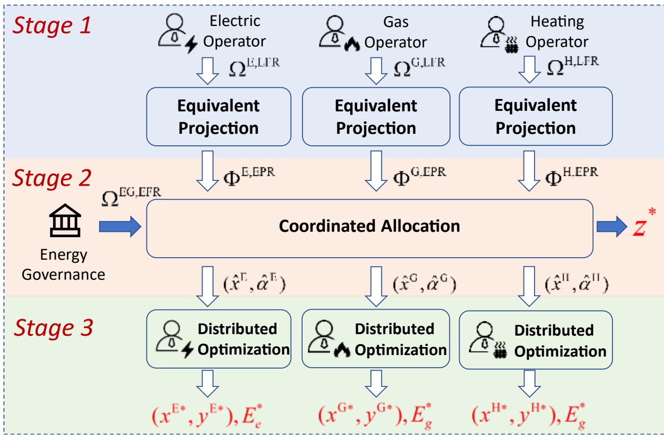


Fig. 3 Coordinated planning framework for UMIES

5. CASE STUDIES

The standard distribution-level integrated energy system, containing IEEE bus-33 PDN, IEEE bus-20 GDN, and Bali bus-32 DHN, are employed to validate the FRP-based method proposed in this paper. The optimized allocation of the energy governance's EH is carried out on this multi-operator network topology. Typical source-load scenarios required for planning are generated by clustering from real-world operational data. And the unit carbon emission price, the unit CHP installed cost, and the unit EB installed cost, are set to be 63 CNY/tCO2, 7000 CNY/kW, and 782 CNY/kW, respectively.

For comparison, three cases are set up. And all optimization processes involved in the follow cases are implemented using the Gurobi solver.

Case1: Coordinated optimization based on the centralized method, considering EH;

Case2: Coordinated optimization based on the FRP-based method, considering EH;

Case3: Autonomous operation of each operator's network without EH involvement in coordination.

The optimal allocation scheme of UMIES is shown in Tab.1. The same optimal allocation schemes are obtained by the centralized method and the FRP-based method. And the primary investment cost are both 809.34 × 10^4CNY and 219.21 × 10^4CNY.

Tab. 1 Optimal allocation result

Method	Coupled device	Optimal location	Optimal capacity (kW)	Primary investment cost (10^4CNY)
FRP-based method	CHP	GDN - 18 PDN - 32 DHN - 3	1156.20	809.34
	EB	PDN - 19 DHN - 3	2093.74	219.21
Central method	CHP	GDN - 18 PDN - 32 DHN - 3	1156.20	809.34
	EB	PDN - 19 DHN - 3	2093.74	219.21

Tab. 2 shows the comparison of various costs. The Annualized investment cost obtained by the above two methods is both 162.33 × 10^4CNY. And the annualized operation cost obtained by the FRP-based method is 47.54 × 10^4CNY, which is 1.56% different from that obtained by the centralized method. The above results indicate that the allocation scheme obtained by the FRP-based method is globally optimal, excluding subtle computational errors originating from the PVE algorithm.

Tab. 2 Comparison of various costs

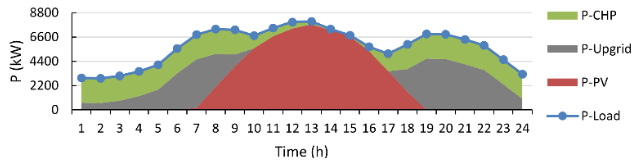
Method	Annualized investment cost (10^4CNY)	Annualized operation cost (10^4CNY)	Total annual cost (10^4CNY)
FRP-based method	162.33	47.54	209.87
Central method	162.33	46.81	209.14

A comparative analysis in Tab.3 demonstrates FRP-based method superiority over centralized method in scalability and privacy preservation. For planning problem, FRP-based method reduces passed variables by 98%, total variables by 98%, and constraints by 17%, while achieving faster solution times. This streamlined architecture enhances data privacy through minimal parameter transmission and improves computational efficiency for large-scale optimization tasks.

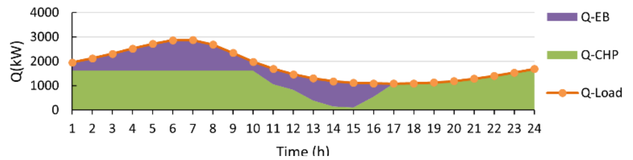
Tab. 3 Method performance comparison

Method	Num of passed variables	Scale of planning problem		Solution Time (s)
		Num of constrains	Num of variables	
FRP-based method	1152	33737	1224	24.67
Central method	59904	40479	59976	28.46

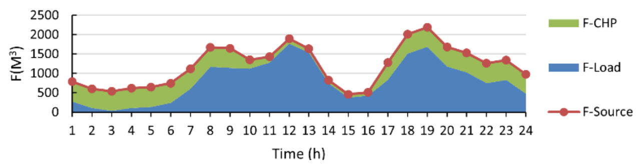
Fig.4 demonstrates effective carbon reduction through optimal coordinated operation. The power generation structure reveals a preference for lower-carbon CHP units in the electricity operator to reduce the purchase of high-carbon power. And during peak PV hours (11:00-16:00), zero-carbon solar power is utilized replacing carbon-containing power to meet load demand. Simultaneously, surplus PV power is converted via EB, fulfilling 37.6% - 90.8% of thermal demand and reducing gas-fired heating by 34.7% - 93.6%.



(a) The operation result of the electricity network



(b) The operation result of the heating network



(c) The operation result of the gas network

Fig. 4 Optimal operation results for operators' networks

Fig. 5 shows the comparison of the CEC for each operator's network with/without EH in the system, i.e., case1 and case3. Compared to case3, the CEC for gas, electricity, and heating networks in case1 decreased by 5.13%, 10.54% and 31.66%, respectively. These also demonstrate that multi-energy coordination can effectively promote carbon emission reduction.

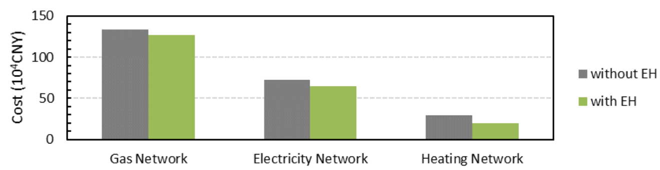


Fig. 5 Carbon emission costs with/without EH

6. CONCLUSIONS

This paper studies the low-carbon-oriented allocation for UMIES. The results confirm that the proposed FRP-based method ensures global optimality, low-carbon performance, privacy protection, and computational efficiency. Compared with traditional methods, this method has excellent scalability and lower planning problem complexity. This approach also provides a governance-led, multi-operator coordination mechanism that promotes the practical application of multi-operator energy systems in achieving low-carbon urban development.

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