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Research on Coupled Energy-Carbon Accounting Modeling and Algorithms in Integrated Energy Systems#

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ABSTRACT

Integrated energy systems (IES) synergize electricity, gas, and heat networks to enhance renewable integration and utilization efficiency, where carbon emissions are intrinsically coupled with multi-energy flows. Precise carbon allocation mechanisms are critical for carbon-aware operations. This paper proposes a unified carbon emission flow (CEF) framework for IES, defining core physical quantities under conservation principles. Explicit CEF models are established for different energy networks. The energy hub formulation integrates these via coupling matrices to map carbon redistribution through converters. Based on the theoretical foundation, a CEF calculation procedure is demonstrated through an integrated electricity-gas-heat case study, to validate the effectiveness of the proposed method. This framework enables precise carbon accounting and decarbonization-oriented IES planning.

Keywords: energy-carbon coupling, carbon emission flow, integrated energy system, energy hub, carbon flow modeling

1. INTRODUCTION

Global climate change is driven largely by CO_2 from energy systems—over 70% of human-made greenhouse gases. China's "Dual Carbon" targets require transforming coal-centric grids into flexible, renewables-friendly infrastructures[1]. Integrated energy systems meet this need by linking distributed resources across electricity, gas, and heat networks for seamless multi-energy coordination[2].

Traditional energy models ignore carbon's journey alongside energy flows, leading to suboptimal planning. The Carbon Emission Flow(CEF) theory changes that by treating electricity, gas, and heat as virtual carbon carriers[3]. In IES, CEF enables a unified view of how carbon moves through conversion and transmission—offering three key innovations for accurate carbon accounting and optimized low-carbon operation: physical-network-granular carbon tracking, cross-energy carrier correlation and dynamic carbon redistribution.

Based on CEF theory, this paper develops an enhanced energy-carbon coupling framework for IES. It extends CEF theory to electricity, gas, and thermal networks through coordinated physical quantity definitions and conservation principles.

This work provides theoretical foundations and computational tools for carbon-aware IES planning, low-carbon dispatch, and equitable carbon responsibility allocation - critical enablers for energy transition.

2. ENERGY-CARBON COUPLED MODELING FOR INTEGRATED ENERGY SYSTEMS

2.1 Overview of Carbon Emission Flow Theory and Fundamental Physical Quantities

To begin with, it is important to clarify the concept of "carbon emission flow". In the power sector, it is defined as a virtual network flow that follows the power flow and represents the carbon emissions required to sustain electricity transmission[4].

Extending this idea to integrated energy systems, carbon emission flow is defined as a virtual flow attached to multiple energy carriers. It quantifies the total carbon emissions associated with delivering energy—electricity, heating, cooling (from the source side), and gas (from the load side)—through various transmission networks.

Based on the above, we introduce the following physical definitions:

(1) Carbon Emission Flow

Carbon emission flow refers to the total amount of carbon emissions associated with energy flowing through a specific branch or node over a given time period. Denoted by F, its unit is consistent with carbon emissions (tCO_2).

(2) Carbon Emission Flow Rate

The carbon emission flow rate is the amount of carbon emission flow per unit time:

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$$R = \frac{dF}{dt} \tag{1}$$

(3) Carbon Flow Density

Carbon flow density is the ratio of the carbon emission flow rate to the corresponding energy flow in a branch:

$$\rho = R/P \tag{2}$$

(4) Carbon Intensity

In integrated energy systems, energy production and consumption are modeled as nodes. We define carbon intensity:

$$e_n = \frac{\sum_{i\in N^+} P_i \rho_i}{\sum_{i\in N^+} P_i} = \frac{\sum_{i\in N^+} R_i}{\sum_{i\in N^+} P_i} \tag{3}$$

The carbon intensity at a node represents the carbon emissions per unit of energy consumed at that node, including both direct local emissions and the upstream emissions from energy production.

2.2 Key principles and assumptions

In power systems, since the origin of electricity cannot be distinguished during transmission, all outgoing flows from a node are considered to proportionally contain contributions from all incoming flows. This is known as the proportional sharing principle (PSP), which is the predominant approach in research and the only approach implemented in practice. However, this principle remains an assumption that can neither be physically proven nor disproven. Though solving the allocation problem of the network does not depend on the state of the system, the feasibility of this method in complex dynamic networks is difficult to validate and may need more testing. This study adopts this method due to its simplicity and accuracy.

By combining this principle with equation(3), we obtain:

$$\rho_j = \frac{R_j}{P_j} = \frac{\sum_{i\in N^+} P_j \frac{P_i}{\sum_{s\in N^+} P_s} \rho_i}{P_j} = \frac{\sum_{i\in N^+} P_i \rho_i}{\sum_{s\in N^+} P_s} = \frac{\sum_{s\in N^+} R_s}{\sum_{s\in N^+} P_s} = e_n \tag{4}$$

This indicates that the carbon emission flow density on every branch emanating from a node equals the carbon intensity at that node.

The other most important principle is the Carbon Flow Balance Principle. As a physical quantity, carbon emissions obey the conservation law. Therefore, carbon flow tracing must follow a balance principle:

$$\sum R_{in} = \sum R_{out} \tag{5}$$

Where R_{in} and R_{out} represent the total input and output carbon flow rates of the energy network.

2.3 Heterogeneous Energy Carrier Analysis

This section presents a comparative analysis of electricity, natural gas, and thermal energy—three representative energy carriers in IES.

Despite differences in physical nature (only natural gas contains carbon chemically), network topology, and energy quality (measured by convertibility), all three carriers can be treated as virtual media for carbon flow. They share the following characteristics:

- Carbon emissions are generated during production or consumption.
- Carbon responsibility can be mapped directly to the flow of energy.
- CEF modeling relies on the corresponding energy flow models.

Besides, a comparison of the three energy networks is summarized below:

Table 2.1 Comparison of three energy networks

Network Type	Energy Carrier	Node Characteristic	Branch Characteristic
Power Network	Electricity, active power	Power conservation, carbon conservation	Power loss
Gas Network	Natural gas	Flowrate & Carbon conservation	Pressure drop
Thermal Network	Hot water	Mass & energy conservation, carbon conservation	Temperature & pressure drop

2.4 Carbon Emission Flow Modeling in Power Networks

Existing literature has established a relatively complete framework for CEF modeling in power networks. The core equations of the model can be summarized as follows:

(1) The carbon flow density of an outgoing branch equals the carbon intensity of its source node:

$$\rho_j = e_n \tag{6}$$

(2) The carbon flow rate loss of a transmission branch equals the product of its carbon flow density and power loss:

$$R_l^{\text{Loss}} = \rho_l P_l^{\text{Loss}} \tag{7}$$

(3) The carbon intensity of a node is determined by both the energy-carbon flow of incoming branches and the generation attached to the node:

$$e_n = \frac{\sum G_n * \rho_n^G + \sum P_i * \rho_i}{\sum G_n + \sum P_i} \tag{8}$$

(4) The carbon flow rate of a load equals the product of the node's carbon intensity and the load's power consumption:

$$R_n^{load} = e_n * P_n^{load} \tag{9}$$

2.5 Carbon Emission Flow Modeling in Natural Gas Networks

Natural gas is inherently a carbon-bearing energy carrier. According to the IPCC, its default carbon emission factor based on combustion heat is approximately 0.202 tCO_2 / MWh. As is the constant:

$$\rho^G = 0.202 \tag{10}$$

(1) Carbon Emissions from Compressors

For electric compressors, the carbon flow rate is the product of power consumption and the carbon intensity of the connected power grid node:

$$R_n^{com} = e_n * P_n^{com} \tag{11}$$

For gas turbines, carbon emissions arise from the combustion of natural gas drawn from the pipeline:

$$R_n^{com} = \rho^G * \tau_n^{com} * B \tag{12}$$

Where: τ_n^{com} is the volumetric flow rate of consumed natural gas under standard conditions (km^3 / h); B is the lower heating value of natural gas (here taken as 10.45 MWh / km^3).

(2) Carbon Flow in Gas Pipelines

The carbon flow rate through a pipeline is directly proportional to the gas flow rate f_p under standard conditions:

$$R_p = \rho^G * f_p * B \tag{13}$$

(3) Carbon Flow Rate of Gas Loads

The carbon flow rate attributed to gas loads is similarly calculated as:

$$R_{load} = f_{load} * \rho^G * B \tag{14}$$

2.6 Energy-Carbon Coupling Modeling of Energy Conversion Processes: Energy Hub Model

The energy hub model extends the concept of a single energy conversion node into a multi-carrier energy hub, while abstracting individual energy supply lines as energy interconnectors. The core of this model lies in constructing an energy coupling matrix that maps the input-output relationships of multiple energy forms—electricity, heat, gas, cooling, and energy storage—enabling the entire system to be represented as a vectorized port model[5].

Before analyzing the overall carbon flow vector of an energy hub, it is necessary to examine the energy and carbon flow characteristics of individual energy conversion components within the hub.

(1) Carbon Flow Modeling of Individual Energy Conversion Devices:

Based on input-output characteristics, conversion devices can be categorized into:

1. Single-input single-output:

$$V_{output} = V_{input} * \eta \tag{15}$$

Let ρ_{input} and ρ_{output} be the carbon flow densities of the input and output, respectively. Based on carbon flow conservation:

$$\rho_{input} * V_{input} = \rho_{output} * V_{output} \tag{16}$$

Then,

$$\rho_{output} = \rho_{input} / \eta \tag{17}$$

2. Single-input multiple-output (CHP as example):

$$V_{O,E}^{CHP} = V_I^{CHP} * \eta_E^{CHP} \tag{18}$$

$$V_{O,Q}^{CHP} = V_I^{CHP} * \eta_Q^{CHP} \tag{19}$$

Carbon flow conservation requires:

$$\rho_I^{CHP} * V_I^{CHP} = \rho_{O,E}^{CHP} * V_{O,E}^{CHP} + \rho_{O,Q}^{CHP} * V_{O,Q}^{CHP} \tag{20}$$

Additional assumptions are needed to determine how carbon is allocated. This study adopts an efficiency-based allocation rule, which is proposed by the GHG Protocol guidance of calculation[5]:

$$\rho_{O,E}^{CHP} / \eta_E^{CHP} = \rho_{O,Q}^{CHP} / \eta_Q^{CHP} \tag{21}$$

(2) Carbon Flow Modeling at the Energy Hub Level

An energy hub consists of multiple coupled conversion units and each of them should be modeled in place. For the energy part:

$$V_o = CV_i \tag{22}$$

Where energy input vector is V_i , energy output vector is V_o , energy coupling matrix is C .

Each energy flow has a corresponding carbon flow, thus a carbon coupling matrix D is defined as:

$$\rho_o = D\rho_i \tag{23}$$

Where ρ_i and ρ_o are the corresponding carbon density of energy vector.

2.7 Energy-Carbon Coupling Modeling of Thermal Networks

The energy hub provides thermal networks with both energy and associated carbon flow density.

(1) Carbon flow rate at branch inlet and outlet:

Given mass flow rate m_k , inlet temperature T_k^{in} ,

$$R_k^{in} = \rho_k c m_k T_k^{in} \tag{24}$$

Where ρ_k is the carbon flow density, c is specific heat.

(2) Carbon loss due to heat loss in the pipe:

$$\Delta T_k = T_k^{in} - T_k^{out} \tag{25}$$

$$R_k^{loss} = \rho_k c m_k \Delta T_k = \rho_k c m_k (T_k^{in} - T_k^{out}) \tag{26}$$

If the pipe represents a thermal load via temperature drop, this carbon loss is treated as the load's carbon emission.

(3) Carbon intensity at nodes:

The carbon flow rate at a node:

$$R_n^{node} = \sum R_k^{out} = \sum \rho_k c m_k T_k^{out} \tag{27}$$

Carbon intensity is then:

$$\rho_n^{node} = \frac{R_n^{node}}{cm_n^{node} T_n^{node}} = \frac{\sum \rho_k cm_k T_k^{out}}{cm_n^{node} T_n^{node}} = \frac{\sum \rho_k m_k T_k^{out}}{m_n^{node} T_n^{node}} \quad (28)$$

(4) Branch carbon density equals carbon intensity of upstream node:

$$\rho_k = \rho_m^{node} \quad (29)$$

Where m is the source node of the branch k.

(5) Carbon flow at sources and loads:

For a heat source:

$$R_{source} = Q_i^{HS} * \rho_i^{HS} \quad (30)$$

For heat loads defined via temperature drop:

$$R_k^{load} = \rho_k cm_k \Delta T_k = \rho_k cm_k (T_k^{in} - T_k^{out}) \quad (31)$$

This chapter presents a simplified theoretical framework for modeling the coupling of energy and carbon flows in integrated energy systems. This approach enables carbon tracking and responsibility allocation, though it simplifies dynamic behaviors and complex constraints.

3. CASE STUDY ANALYSIS AND PRESENTATION

3.1 Structure and Basic Settings of the IES Case Study

(1) Power Network:

The structure of the 6-bus power network used in this case study is shown in the figure below:

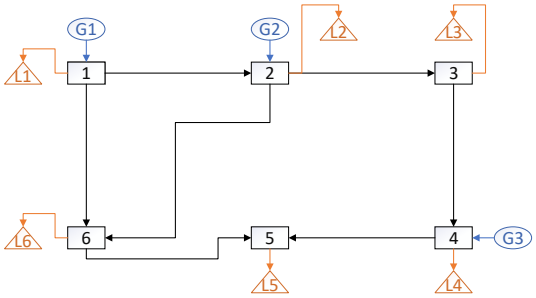

Fig. 3.1 Structure of the power network

Black squares are buses; blue ellipses are generators; orange triangles are loads. The black lines are transmission lines, and the arrows are illustrative only—the real power-flow directions are determined by the calculations.

The generators' capacity and GCI are set as known and can be changed. The data for the power buses and transmission lines are presented in the table below:

Table 3.1 Data for the buses in network

Node	*Type*	$P_b^L(MW)$	$Q_b^L(MW)$	$V_{\max}(p.u.)$	$V_{\min}(p.u.)$
1	PV	200	0	1.07	0.95
2	$V\theta$	300	0	1.07	0.95
3	PQ	280	0	1.07	0.95
4	PV	100	50	1.07	0.95
5	PQ	280	120	1.07	0.95
6	PQ	200	120	1.07	0.95

Table 3.2 Data for the transmission lines in network

From	To	$r(p.u.)$	$x(p.u.)$	$b(p.u.)$	Capacity(MW)
1	2	0.1	0.2	0.04	200
1	6	0.25	0.5	0.04	200
2	3	0.15	0.3	0.06	200
2	6	0.25	0.5	0.02	200
3	4	0.12	0.26	0.05	200
4	5	0.2	0.4	0.08	300
5	6	0.2	0.4	0.06	200

It should be noted that the load connected at bus 5 is the energy hub.

(2) Gas Network:

The 6-node natural gas network structure is shown as:

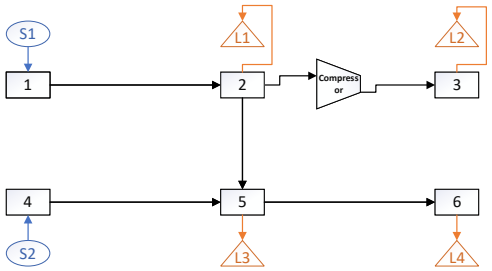

Fig. 3.2 Structure of the gas network

The significance of the graphical representation is analogous to that in power systems; black rectangles are compressors. Note: Load L4 at node 6 corresponds to generator 1 in the power grid, and load L3 at node 5 is the energy hub. Node 1 is set as known-pressure node, while others are known-injection nodes. The data for gas pipelines are presented:

Table 3.3 Data for the gas pipelines

From	To	$K_p(km^3/(h\cdot psi))$	$Capacity(km^3/h)$
1	2	3.426	300
2	3	3.096	300
2	5	1.707	150
4	5	3.053	300
5	6	2.565	200

In this case study, the compressor between node 2 and 3 is a turbo compressor, with its gas consumption rate set to 4.91 km^3/h . The demand of loads is known.

(3) Thermal Network:

The thermal network structure is shown below:

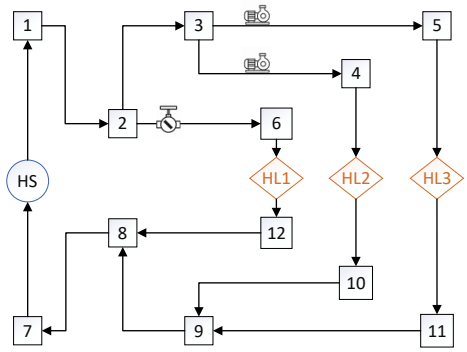

Fig. 3.3 Structure of the thermal network

Node 1 is set as fix-pressure (3Mpa). Others are set as fix-injection, among which node 7 is 300kg/s. The temperature drop at loads are -20, -25, -20 separately. The valve and pumps in the network are also known. The thermal pipeline data are also provided.

(4) Energy Hub:

The energy hub model is shown below:

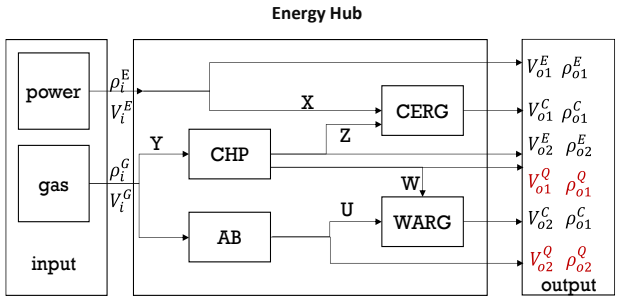

Fig. 3.4 Structure of the energy hub model

The conversion efficiencies of these energy conversion components are set as known.

Based on the above definitions and the carbon flow model of the energy hub, the energy coupling matrix and the carbon flow coupling matrix can be calculated. The output vector V_{o1}^{Q} is the source input of thermal network.

3.2 Presentation and Discussion of Calculation Results

(1) power network

After running the model constructed in MATLAB, the steady-state power flows in the network are as shown below:

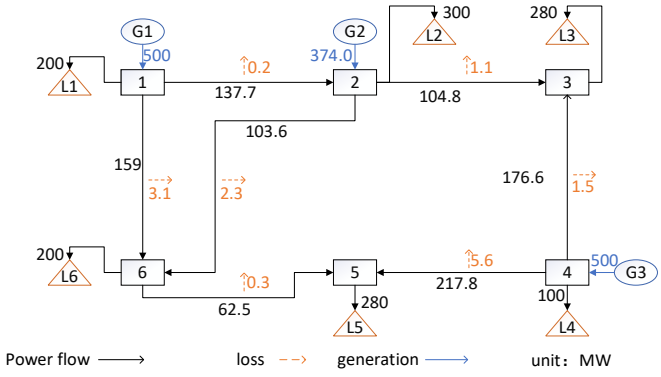

Fig. 3.5 Power flow distribution of the network

The corresponding CEF distribution:

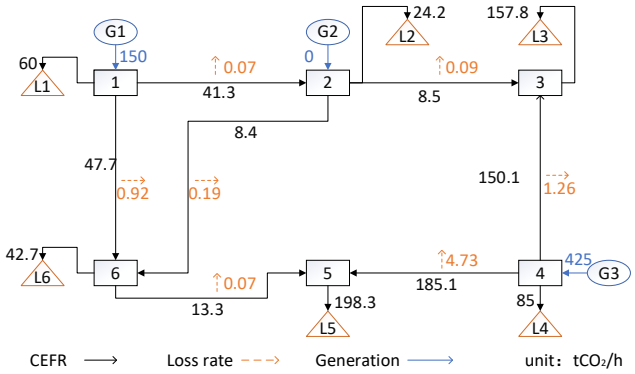

Fig. 3.6 CEF distribution of the network

At buses 1 and 4, the node carbon intensities equal their local generators' emission intensities because they only inject power into the grid and receive no inflows. According to the node carbon intensity formula, these nodes are unaffected by other buses. Bus 2's generator is zero-carbon, but its carbon intensity is nonzero because it draws energy from Bus 1—so its connected load at Bus 2 still carries some carbon flow responsibility originated at Bus 1.

Overall, each node's carbon inflows and outflows balance. Although loads 1 and 6 both consume 200 MW, load 6 has a lower carbon flow rate since it receives some zero-carbon power from Bus 2. Likewise, loads 3 and 5 each draw 280 MW, yet load 5's carbon flow is higher due to greater supply from Coal Plant 4. Finally, load 2's 300 MW yields a much lower carbon flow than load 4's 100 MW, illustrating the strong decarbonization benefit provided by low-carbon generation.

However, if we calculate the grid average emission factor of this system, these transmission characteristics will not be disclosed. The grid average emission factor is 0.42 $\mathrm{tCO_2}/MWh$, adopting this value is unfair for loads connected to clean nodes. For example, the emission flow rate for load 2 would become 125.5 $\mathrm{tCO_2}/h$, much larger than the value 24.2 calculated using CEF method.

(2) Gas network

Since natural gas inherently carries carbon, its carbon flow is directly proportional to its energy flow:

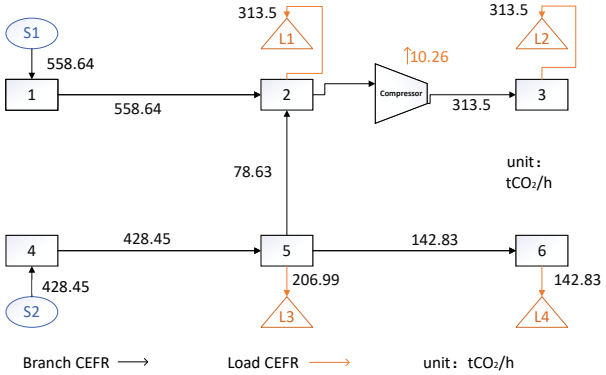

Fig. 3.7 CEF distribution of the gas network

(3) Thermal network

Since this case's heating network uses a single heat source, every node shares the same carbon intensity, equal to the carbon flow density at the energy hub's output thermal port.

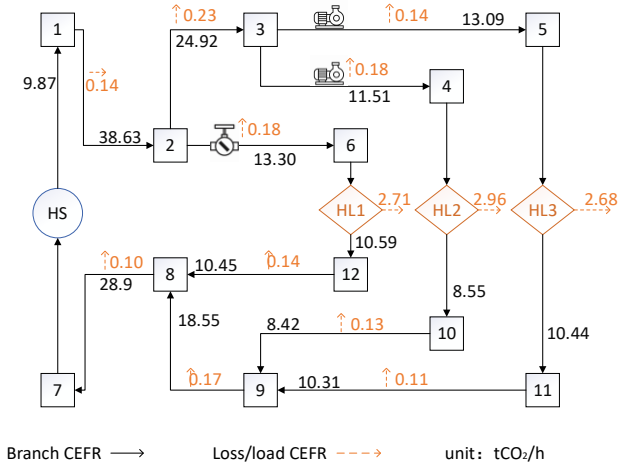

Fig. 3.8 CEF distribution of the thermal network

Unlike conventional approaches that rely on global average emission factors or fixed fuel-based carbon coefficients, the CEF method treats carbon as an "invisible tracer" that moves with energy flows and solves a linear system to track temporal and spatial emission intensities at every node and branch. This enables CEF to capture the effects of network topology, transmission losses, and the dynamic introduction of renewables in real time, while integrating seamlessly with existing power-flow calculations. In contrast, traditional methods ignore such details to yield only regional or annual average emission factors and thus failing to allocate carbon footprints accurately for localized loads or peak events, which can lead to significant errors in responsibility assignment and low-carbon dispatch.

This work completes the precise calculation of carbon emission flows in the coupled power, gas, and heat network case study, validating the accuracy and feasibility of the energy–carbon coupling and carbon flow accounting models developed in Chapter 2.

4. CONCLUSION

This study establishes a unified energy-carbon coupled modeling framework for integrated electricity-gas-heat systems, addressing critical gaps in cross-network carbon flow accounting. By extending CEF theory to heterogeneous energy carriers and defining standardized algorithms, the framework enables granular carbon tracking across multi-energy networks. Crucially, the energy hub model with coupling matrices formalizes carbon flow redistribution through conversion devices.

Validation via a multi-network MATLAB case study confirms the model's efficacy: power networks exhibit non-uniform carbon intensities due to generation mix and flow paths; gas networks show carbon-proportional flows from inherent fuel carbon; thermal networks reflect source-determined carbon homogeneity. This work provides foundational tools for accurate carbon attribution, low-carbon IES scheduling, and policy-driven carbon responsibility allocation, supporting the transition towards a sustainable future.

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REFERENCE

[1] Niu S, Liu Y, Ding Y, Qu W. China's energy systems transformation and emissions peak. Renewable and Sustainable Energy Reviews. 2016;58:782-95.

[2] Song D, Meng W, Dong M, Yang J, Wang J, Chen X, et al. A critical survey of integrated energy system: Summaries, methodologies and analysis. Energy Conversion and Management. 2022;266:115863.

[3] Wang Y, Wang X, Yu H, Huang Y, Dong H, Qi C, et al. Optimal design of integrated energy system considering economics, autonomy and carbon emissions. Journal of Cleaner Production. 2019;225:563-78.

[4] Chongqing K, Tianrui Z, Qixin C, Jianhui W, Yanlong S, Qing X, et al. Carbon Emission Flow From Generation to Demand: A Network-Based Model. IEEE Transactions on Smart Grid. 2015;6:2386-94.

[5] Cheng Y, Zhang N, Wang Y, Yang J, Kang C, Xia Q. Modeling Carbon Emission Flow in Multiple Energy Systems. IEEE Transactions on Smart Grid. 2019;10:3562-74.