Numerical simulation of the effects of temperature-dependent thermal conductivity and viscosity on temperature and velocity fields

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ABSTRACT

The present study numerically investigates the effects of the variations of thermal conductivity and viscosity with temperature on velocity and temperature fields. The simulation is performed in a two-dimensional steady channel flow. The velocity profile is first validated against its analytical solution for the case of constant properties. A good agreement between numerical and analytical solutions is observed. From a physical point of view, it is revealed that by increasing the temperature in liquids, the fluid elements near the high-temperature wall are moving faster compared to those adjacent to the low-temperature one.

Keywords: temperature-dependent properties, channel flow, numerical solutions, laminar flow

1. INTRODUCTION

The variations of viscosity and thermal conductivity of fluids should be taken into account if more accurate results in thermofluid problems are aimed. Numerous numerical and/or experimental investigations have explored the effects of temperature dependency of viscosity and thermal conductivity, see [1-6].

The main objective of this paper is to numerically study the effects of temperature-dependent viscosity and thermal conductivity properties in the fluid, and provide a tensor notation that is not dependent on a specific coordinate and different coordinates for various problems could be used.

2. PROBLEM DEFINITION AND GOVERNING EQUATION

In this article, a general tensor notation of governing equations with temperature-dependent viscosity and

thermal conductivity physical properties is presented. A full-numerical channel flow has been conducted.

The governing equations for the current problem are equations of mass, momentum, and heat transfer:

$$\nabla \cdot \mathbf{V} = \mathbf{0},\tag{1}$$

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V}\right) = \nabla \cdot \mathbf{S} + \mathbf{k}, \tag{2}$$

$$\rho c_{p} \left(\frac{\partial T}{\partial t} + (\mathbf{V} \cdot \nabla) T \right) = \nabla \cdot (\lambda \nabla T) + \dot{q}, \quad (3)$$

where ρ , V, c_{P} , k, λ and \dot{q} are density, velocity vector and heat capacity, body force, thermal conductivity, and generation term, respectively. As a common assumption for incompressible flows, the viscous dissipation term in Eq. (3) is neglected. The stress tensor S for Newtonian fluids can be expressed as:

$$\mathbf{S} = -p \,\mathbf{E} + \mu \big(\nabla \,\mathbf{V} + \mathbf{V} \,\nabla \big), \tag{4}$$

in which p^{p} and μ^{μ} are pressure and dynamic viscosity, respectively. In this work, thermal conductivity and viscosity variations are considered linear functions of temperature as used in [7]:

$$\mu = \mu_0 - \mu_1 T , \qquad (5)$$

$$\lambda = \lambda_0 + \lambda_1 T \,. \tag{6}$$

By substituting Eq. (4) into Eq. (2) yields:

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$$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} \right) = -\nabla p$$
$$+ \frac{d \mu}{dT} \left[(\nabla T \cdot \nabla) \mathbf{V} + \nabla (\nabla T \cdot \mathbf{V}) \right]$$
$$+ \mu \Delta \mathbf{V} + \mathbf{k}$$
(7)

$$\rho \left(\frac{\partial T}{\partial t} + (\mathbf{V} \cdot \nabla) T \right) = \frac{d\lambda}{dT} \nabla T \cdot \nabla T$$

$$+ \lambda \Delta T + \dot{a}$$
(8)

 $+\lambda\Delta T + \dot{q}.$

The above-coupled equations are to be solved simultaneously. Note that this form of governing equations is general and can be opened in any coordinate of desire. Here a channel flow as a case study is examined. The flow is assumed to be steady, twodimensional, and fully developed. The above non-linear coupled equations for the case referenced here are numerically solved by the conventional finite difference scheme. The second-order central finite difference equations are solved by a pseudo-time stepping approach. The time-stepping was being done until the steady-state was achieved.

For validation purposes, a comparison between numerical and analytical solutions for the case of constants properties is performed and a good agreement is observed in Fig. 1 below.



Fig.1: Comparison of the numerical and analytical solutions

3. RESULTS AND DISCUSSION

In this section, the results of the current simulation are presented. The numerical solution of the full nonlinear equations for the velocity and temperature fields is also reported. Physically speaking, it is evident that with increasing the temperature in liquids, the viscosity decreases so that the fluid elements adjacent to the high-temperature wall is moving faster than those adjacent to the low-temperature wall.

In this work, the effect of temperature on the viscosity is considered, thus, it is expected that the symmetry of the velocity profile is altered and the location of the maximum velocity moves from the channel center towards the high-temperature wall. Additionally, the mass flow rate \dot{m} throughout the channel is increased, as compared to the case of isothermal walls at a lower temperature. From Fig. 2 the changes in the velocity fields can be easily seen.



Fig. 2: Velocity field obtained for different values of lower wall temperature



Fig. 3: Temperature profile obtained for $T_L = 20$



Fig. 4: Temperature profile obtained for $T_L = 30$



Fig. 5: Temperature profile obtained for $T_L = 40$

Fig. 3, 4, 5, and 6 show the temperature fields at different temperatures of the lower wall $T_L = 20$, $T_L = 30$, $T_L = 40$ and $T_L = 50$, respectively.



Fig. 6: Temperature profile obtained for $T_L = 50$

4. CONCLUSION

In this paper, the effect of the temperature-dependent viscosity and thermal conductivity on the velocity and temperature profiles in a steady, two-dimensional channel flow is numerically investigated and validated with the analytical solution for the case of constant viscosity and thermal conductivity. The variations of viscosity and thermal conductivity with temperature are considered linear functions.

Results show that the maximum value of velocity in the channel is moving from the center to the hightemperature wall which here the lower wall temperature is increasing.

REFERENCE

[1] Kafoussias, N. G., and E. W. Williams. The effect of temperature-dependent viscosity on free-forced convective laminar boundary layer flow past a vertical isothermal flat plate. Acta Mechanica 110, no. 1 (1995): 123-137.

[2] Elbashbeshy, E. M. A., and M. A. A. Bazid. The effect of temperature-dependent viscosity on heat transfer over a continuous moving surface. *Journal of Physics D: Applied physics* 33, no. 21 (2000): 2716.

[3] Pal, Dulal, and Hiranmoy Mondal. Effect of variable viscosity on MHD non-Darcy mixed convective heat transfer over a stretching sheet embedded in a porous medium with non-uniform heat source/sink. *Communications in Nonlinear Science and Numerical Simulation* 15, no. 6 (2010): 1553-1564.

[4] Emerman, Steven H., and D. L. Turcotte. Stagnation flow with a temperature-dependent viscosity. *Journal of Fluid Mechanics* 127 (1983): 507-517.

[5] Goudarzi, K., and M. Ahmadinejad. Thermodynamic analysis of an efficient Rankine cycle powered by jacket water, oil engine and exhaust gas waste heat of internal combustion engine with the approach of selection of appropriate working fluid. Gas 7.8 (2020): 5.

[6] Delfani, F., Rahbar, N., Aghanajafi, C., Heydari, A., & KhalesiDoost, A. Utilization of thermoelectric technology in converting waste heat into electrical power required by an impressed current cathodic protection system. Applied Energy, 302, (2021): 117561.

[7] Panahi-Kalus, H., M. Ahmadinejad, and A. Moosaie. The effect of temperature-dependent viscosity and thermal conductivity on velocity and temperature field: an analytical solution using the perturbation technique. Archives of Mechanics 72.6 (2020): 555-576.