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# Robust optimal scheduling method for integrated heating/electricity community energy systems with flexible heating loads of buildings under energy prices uncertainties

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## ABSTRACT

This paper proposes a two-stage robust optimal scheduling method for the integrated community energy system (ICES) with flexible heating loads of buildings. At the first stage, consumers in buildings optimize their heating loads to minimize their heating costs. The thermal dynamics of buildings with controllable indoor radiators are modeled using the Resistor-Capacitor thermal network. At the second stage, the ICES operator seeks to maximize its profit by optimizing the schedules of energy generation and supply. Moreover, a robust optimization is used at the second stage to cope with the energy prices uncertainties from the energy markets. Numerical studies show that the proposed optimal scheduling method can reduce the heating costs of consumers in buildings while ensuring the ICES's profit under energy prices uncertainties.

**Keywords:** Buildings, integrated community energy system, prices uncertainties, resistor-capacitor thermal network, robust optimization

## NONMENCLATURE

| | |
|---|---|
| *Abbreviations* | |
| ICES | Integrated community energy system |
| RC | Resistor-Capacitor |
| RO | Robust optimization |
| CHP | Combined heat and power |
| *Symbols* | |
| $T$ | Index of time |
| $N$ | Number of consumers |
| $Ce_t^{uo}$ / $Ch_t^{uo}$ / $Cg_t^{uo}$ | Operator's electricity/heat/gas purchasing price at time $t$ ($/KWh) |
| $P_t^e$, $P_t^g$, $P_t^h$ | Operator's purchasing amounts of electricity/gas/heating at time $t$ (kW) |
| $Ce_t^{sale}$ / $Ch_t^{sale}$ | Operator's electricity/heat sale price at time $t$ ($/KWh) |
| $P_{n,t}^{e,l}$/ $P_{n,t}^{h,l}$ | Electricity/ heating load of consumer n at time $t$ (kW) |
| $Cw^{i,j}$ | Heat capacity of the wall between zone $i$ and zone $j$ (J/K) |
| $Rw^{i,j}$ | Thermal resistance of the wall between zone $i$ and zone $j$ (K/kW) |
| $T^j$ | Temperature of node $j$ |
| $Tw^{i,j}$ | Thermal resistance of the wall between zone $i$ and zone $j$ |
| $R^{i,j}$ | Identifier of sunlit wall |
| $\alpha^{i,j}$ | Radiative heat absorption coefficient of the wall between zone $i$ and zone $j$. |
| $Qrad^{i,j}$ | Radiative heat flux density on the wall (W/m²) |
| $Aw^{i,j}$ | The area of wall between zone $i$ and zone $j$ (m²) |
| $Cr^i$ | Heat capacity of the zone $i$ (J/K) |
| $Rwin$ | Thermal resistance of the window (K/W) |
| $Q^{int}$ | Internal heat gain of the zone (kW) |
| $T_t^{out}$ | Outdoor temperature at time $t$ (°C) |
| $Cp$ | The specific heat capacity of water [J /(kg·°C] |
| $T_s/T_r$ | Supply/Return water temperature (°C) |
| $T_t^r$ | Indoor temperature at time $t$ (°C) |
| $m_t^r$ | Water flow rate of the radiator at time $t$ (kg/s) |
| $P_t^{hp}$ | Power consumption of heat pumps at time $t$ (kW) |
| $P_t^{chp}$ | Power generation of the CHP unit at time $t$ (kW) |
| $\eta_e/\eta_h$ | Conversion efficiency of gas into electricity and heating through the combined heat and power (CHP) unit |
| $\varepsilon_r^f$ | Total error of energy price prediction |

## 1. INTRODUCTION

The integrated community energy system (ICES) has been extensively used to supply heating for buildings in residential communities because it saves space and reduces noise emissions for heating supply. Moreover, with multiple energy generation and conversion equipment (e.g., combined heat and power (CHP) unit and heat pump) [1], the ICES operator can optimize the schedules of energy purchase (i.e., electricity, natural gas, and heating purchased from energy markets) and energy generation efficiently and economically [2].

Thanks to the thermal inertia of buildings [3], the indoor temperature can vary within a certain range while keeping the consumers' comfort levels. Thus, with respect to heating prices, consumers in buildings can adjust the heating loads through the radiators in an optimal way to reduce their heating costs. Therefore, optimal scheduling of heating loads of buildings in the ICES can benefit both the consumers in buildings and the ICES operator. However, since the real-time energy prices are hard to be accurately predicted, the ICES operator could experience potential risks in the day-ahead energy purchase strategy. To cope with the uncertainties of the energy prices, the robust optimization (RO) method is used in this paper. Compared to scenario-based stochastic programming, RO is based on worst-case analysis which can be described as a min-max problem and reduce much computation work.

To address the above issues, a two-stage RO method for the ICES with flexible heating loads of buildings is proposed to fully explore the heating inertia of buildings while ensuring the ICES's profit under energy prices uncertainties.

## 2. PROBLEM DESCRIPTION

The framework of energy scheduling of the ICES with flexible buildings is shown in Fig. 1. As can be observed, the ICES operator purchases electricity, gas, and heating at the market prices released from energy markets and determines the optimal sale prices of energy delivered to buildings. To cope with the energy prices uncertainties, the RO method is used for the ICES operator. Consumers in buildings can optimize the heating loads by adjusting the water flow rates in their radiators based on the heating sale prices from the ICES operator.

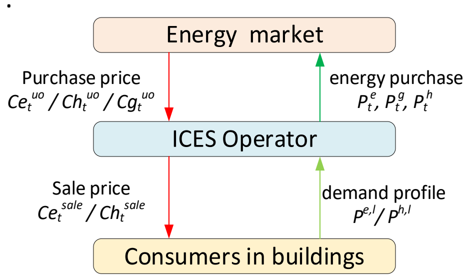



Fig. 1. Framework of energy dispatch

## 3. MATHEMATICAL MODELING

### 3.1 First Stage-consumers in buildings

*1) Objective*

At the first stage, consumers in buildings optimize heating loads to minimize the total heating cost, as shown in (1):

$$\min \sum_{n=1}^{N}\sum_{t=1}^{T}(Ce_t^{sale}P_{n,t}^{e,L}+Ch_t^{sale}P_{n,t}^{h,L}) \tag{1}$$

*2) Constraints of thermal balance in buildings*

The thermal dynamics of a typical heating zone in one building is modelled with Resistor-Capacitor (RC) thermal network, as shown in Fig. 2. As can be observed, there are two kinds of nodes (i.e., room node and wall node) considered in the RC network. The nodes are connected to each other with thermal resistance to transmit heat and are grounded with thermal capacity to preserve heat.

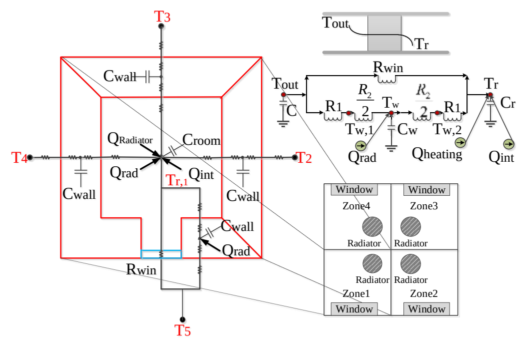



Fig. 2. RC model of one heating zone

The relationship among the outdoor temperature, heat gains of the heating zone, and the wall temperatures can be formulated using the RC model with the wall and room nodes. Then, the thermal balance of the walls of the heating zone shown in Fig. 2 is formulated as,

$$\begin{cases} Cw^{1,2}\dfrac{dTw^{1,2}}{dt}=\dfrac{T^r-Tw^{1,2}}{Rw^{1,2}}+\dfrac{T^2-Tw^{1,2}}{Rw^{1,2}}+r^{1,2}\alpha^{1,2}Aw^{1,2}Qrad^{1,2} \\ Cw^{1,3}\dfrac{dTw^{1,3}}{dt}=\dfrac{T^r-Tw^{1,3}}{Rw^{1,3}}+\dfrac{T^3-Tw^{1,3}}{Rw^{1,3}}+r^{1,3}\alpha^{1,3}Aw^{1,3}Qrad^{1,3} \\ Cw^{1,4}\dfrac{dTw^{1,4}}{dt}=\dfrac{T^r-Tw^{1,4}}{Rw^{1,4}}+\dfrac{T^4-Tw^{1,4}}{Rw^{1,4}}+r^{1,4}\alpha^{1,4}Aw^{1,4}Qrad^{1,4} \\ Cw^{1,5}\dfrac{dTw^{1,5}}{dt}=\dfrac{T^r-Tw^{1,5}}{Rw^{1,4}}+\dfrac{T^5-Tw^{1,5}}{Rw^{1,4}}+r^{1,5}\alpha^{1,5}Aw^{1,5}Qrad^{1,5} \end{cases} \tag{1}$$

$$Cr\frac{dT^{room}}{dt}=\sum_{j=2}^{5}\frac{Tw^{1,j}-T^r}{Rw^{1,j}}+\frac{T^{out}-T^r}{Rwin} +Q_1^{load}+Q^{\mathrm{int}}+\tau^{win}A^{win}Q^{win} \tag{2}$$

Consumers can adjust heating loads with the radiator by controlling the water flow rate, as shown in (4). Buildings' overall heating load is described in (5).

$$Q_{i,t}^{load}=c_p*m_t^r*(T_s-T_r) \tag{3}$$

$$P_{1,t}^{h,L}=\sum_{i=1}^{I}Q_{i,t}^{load} \tag{4}$$

*3) Indoor temperature constraints of buildings*

$$T^{r\_\min}\le T_t^r\le T^{r\_\max} \tag{6}$$

*4) Flow rate constraints of radiators*

$$m^{r,\min}\le m_t^r\le m^{r,\max} \tag{7}$$

### 3.2 Second Stage-the ICES operator

After the consumers in buildings decide their electricity and heating demand at the first stage, the ICES operator seeks an optimal energy scheduling strategy at the second stage. To cope with the uncertainties of energy prices, the RO method is used for the ICES operator to maximize its profit under the worst-case scenario, rendering the following mathematical formulation.

$$\begin{aligned}&\min_{W}\max_{X} -(Ce_t^{UO}P_t^e + Cg_t^{UO}P_t^g + Ch_t^{UO}P_t^h)\\&+\sum_{t=1}^{T}[\sum_{n=1}^{N}(Ce_t^{sale}P_{n,t}^{e,L} + Ch_t^{sale}P_{n,t}^{h,L})]\end{aligned} \tag{8}$$

Where

$$W=\{Ce_t^{UO}, Ch_t^{UO}$$

$$Ce_t^{UO\_\min} \le Ce_t^{UO} \le Ce_t^{UO\_\max} \quad :\alpha_{1,t}^L,\alpha_{1,t}^U \tag{9}$$

$$-\varepsilon_r^f \le \sum_t (Ce_t^{UO} - Ce_t^{UO\_f}) \le \varepsilon_r^f \ :\alpha_2^L,\alpha_2^U \tag{10}$$

$$Ch_t^{UO\_\min} \le Ch_t^{UO} \le Ch_t^{UO\_\max} \quad :\alpha_{3,t}^L,\alpha_{3,t}^U \tag{11}$$

$$-\varepsilon_r^f \le \sum_t (Ch_t^{UO} - Ch_t^{UO\_f}) \le \varepsilon_r^f \ :\alpha_4^L,\alpha_4^U\} \tag{12}$$

$$X=\{(P^e,P^h,P^g,H^{CHP},P^{CHP},P^{HP},H^{HP})|$$

$$P_t^e + P_t^{chp} = P_t^{hp} + \sum_{n=1}^{N} P_{n,t}^{e,L} + P_t^{e,L\_other} \tag{13}$$

$$P_t^{chp} = \eta_e P_t^g \tag{14}$$

$$H_t^{chp} + H_t^{hp} + P_t^h = \sum_{n=1}^{N} P_{n,t}^{h,L} + P_t^{h,L\_other} \tag{15}$$

$$H_t^{chp} = \eta_h P_t^g \tag{16}$$

$$H_t^{hp} = \eta_{hp} P_t^{hp} \tag{17}$$

$$0 \le P_t^e \le P_t^{e,\max} \tag{18}$$

$$0 \le P_t^g \le P_t^{g,\max} \tag{19}$$

$$0 \le P_t^h \le P_t^{h,\max} \tag{20}$$

$$0 \le P_t^{hp} \le P_t^{hp,\max}\} \tag{21}$$

The objective shown in (8) is to maximize the ICES operator's profit, where the first term is energy purchasing cost and the second term is energy selling income; *W* is the uncertainty set of energy purchasing prices. The range of energy prices' variations are shown in (9) and (11), where the minimum values of energy prices are 10% lower than the predicted energy prices and the maximum values are 10% higher. To describe the deviation from the energy price forecasts, the total errors of energy prices are assumed to be less than $\varepsilon_r^f$, as shown in (10) and (12). $\alpha_{1,t}^L$, $\alpha_{1,t}^U$, $\alpha_2^L$, $\alpha_2^U$, $\alpha_{3,t}^L$, $\alpha_{3,t}^U$, $\alpha_4^L$, $\alpha_4^U$ are dual variables of constraints (9)-(12). *X* are the constraints of electricity and heating balance in ICES, as shown in (13)-(21).

### *3.3 Solution method of the RO problem*

According to the method proposed in [4], the energy purchase ($P_t^e$, $P_t^g$, $P_t^h$), $\forall t$ are considered constant at first. Then the first term in (8) can be transformed into the following linear term:

$$\min_{W} -(Ce_t^{UO}P_t^e + Cg_t^{UO}P_t^g + Ch_t^{UO}P_t^h) \tag{22}$$

And according to the dual theorem, its dual linear programming (LP) can be obtained as below:

*1) Objective*

$$\begin{aligned}\max\sum_{t=1}^{N_T}\ &(Ce_t^{UO\_\min}\alpha_{1,t}^L + Ce_t^{UO\_\max}\alpha_{1,t}^U + \alpha_2^L(\sum_t Ce_t^{UO\_f} - \varepsilon_r^f)\\&+\alpha_2^U(\sum_t Ce_t^{UO\_f} + \varepsilon_r^f) + Ch_t^{UO\_\min}\alpha_{3,t}^L + Ch_t^{UO\_\max}\alpha_{3,t}^U\\&+\alpha_4^L(\sum_t Ch_t^{UO\_f} - \varepsilon_r^f) + \alpha_4^U(\sum_t Ch_t^{UO\_f} + \varepsilon_r^f))\end{aligned} \tag{23}$$

*2) Constraints*

$$\alpha_{1,t}^L + \alpha_{1,t}^U + \alpha_2^L + \alpha_2^U = -p_t^e \tag{24}$$

$$\alpha_{3,t}^L + \alpha_{3,t}^U + \alpha_4^L + \alpha_4^U = -p_t^g \tag{25}$$

$$\alpha_{1,t}^L \ge 0, \alpha_{1,t}^U \le 0, \alpha_{3,t}^L \ge 0, \alpha_{3,t}^U \le 0, \forall t \tag{26}$$

$$\alpha_2^L \ge 0, \alpha_2^U \le 0, \alpha_4^L \ge 0, \alpha_4^U \le 0 \tag{27}$$

As the ICES operator is allowed to change energy schedules, the worst-case energy scheduling problem at the second stage can be reformulated as follows:

$$\begin{aligned}&\sum_{t=1}^{T}[\sum_{n=1}^{N}(Ce_t^{sale}P_{n,t}^{e,L} + Ch_t^{sale}P_{n,t}^{h,L})] + \max\sum_{t=1}^{N_T}\ (Ce_t^{UO\_\min}\alpha_{1,t}^L\\&+Ce_t^{UO\_\max}\alpha_{1,t}^U + \alpha_2^L(\sum_t Ce_t^{UO\_f} - \varepsilon_r^f) + \alpha_2^U(\sum_t Ce_t^{UO\_f} + \varepsilon_r^f)\\&+Ch_t^{UO\_\min}\alpha_{3,t}^L + Ch_t^{UO\_\max}\alpha_{3,t}^U + \alpha_4^L(\sum_t Ch_t^{UO\_f} - \varepsilon_r^f)\\&+\alpha_4^U(\sum_t Ch_t^{UO\_f} + \varepsilon_r^f))\\&s.t.\ (13)\text{-}(21)\quad(24)\text{-}(27)\end{aligned} \tag{28}$$

It is clear that (28) is still a LP, which ban be solved easily.

## 4. CASE STUDIES

System in Fig.3 is used as a test case. The building studied in here has 20 floors, with 2 consumers in each floor. And each consumer has 4 heating zones, which are 6m long, 6m wide, 3m high.

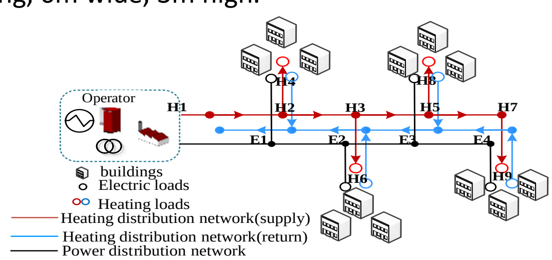



Fig. 3. Schematic of the electricity and heating distribution networks of the system

The relevant thermal parameters of one heating zone are shown in Tab. 1 [3]. The data of outdoor environment (i.e., outdoor temperature and solar radiation) and energy prices are from [3].

Tab.1 buildings parameters

| $Rw^{1,j}$(K/W) | Rwin(K/W) | Cr(J/K) | $Cw^{1,j}$(J/K) | $A^{win}$($m^2$) |
|---|---|---|---|---|
| 0.06 | 0.02 | 2.5e+5 | 7.9e+5 | 4 |

As shown in Fig. 4, buildings adjust water flow rates according to the outdoor temperature and the heating sale price at the first stage. As can be observed, the valleys of heating sale price lead to the peaks of water flow rate (e.g., at 10:00, 16:00). On the contrary, the peaks of heating sale price lead to the valleys of water flow rate (e.g., at 11:00,17:00).

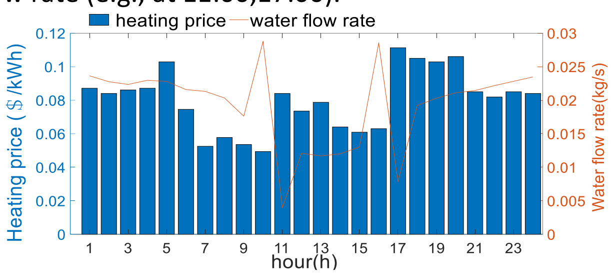
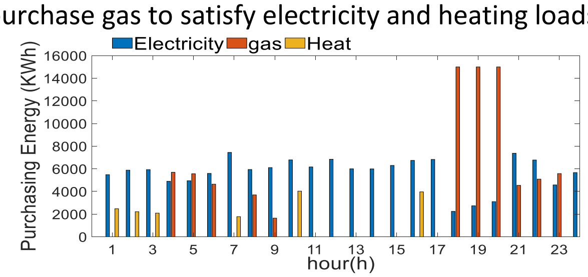
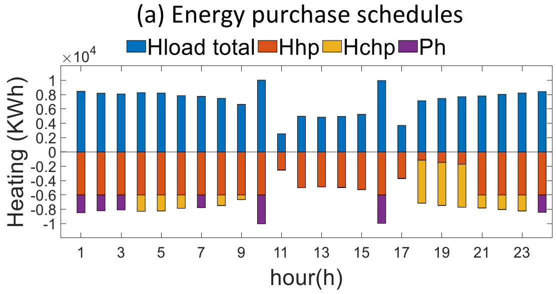



Fig. 4. Flow rate of the radiator of one zone

As can be observed in Fig. 5 (a), the ICES operator can optimize the purchase schedules of electricity, heating and gas based on energy prices. Since the heat generation efficiency of the heat pump is much higher than that of CHP, the ICES operator tends to dispatch heat pump to cover heating loads most of the time. The ICES operator can also purchase heating from the upper energy system directly (e.g., 01:00~03:00, 10:00) or dispatch CHP (e.g., 04:00~06:00, 18:00~23:00) to supply heat, as shown in Fig.5 (b). However, when the electricity price peaks at 18:00~20:00, the ICES operator prefers to purchase gas to satisfy electricity and heating loads.




(a) Energy purchase schedules




(b) Heating supply schedules

Fig. 5. Scheduling results of the ICES operator

To investigate the influence of energy prices uncertainties, four different scenarios are employed in this paper.

Scenario 1: The uncertainties of electricity and heating prices are both considered.

Scenario 2: Only consider the uncertainties of electricity prices.

Scenario 3: Only consider the uncertainties of heating prices.

Scenario 4: The uncertainties of electricity and heating prices are not considered.

Tab.2 Energy cost of one consumer and the profit of the ICES operator

| Scenario | Energy cost of one consumer ($) | Profit of operator ($) |
|---|---|---|
| 1 | 15.60 | 8507.7 |
| 2 | 15.60 | 8596.2 |
| 3 | 15.58 | 9568.9 |
| 4 | 15.58 | 9706.9 |

As can be observed in Tab. 2, with the increase of energy prices uncertainties into consideration, the energy cost of one consumer is almost the same but the profit of the ICES operator will decrease. Compared to scenario 4, the profit of the ICES operator in scenarios 1,2,3 decrease 12.35%, 11.44%, 1.42% respectively. The profit of the ICES operator in scenario 3 is 10.17% more than that in scenario 2, which demonstrates that the uncertainties of electricity prices have more influence on the profit of the ICES operator.

## 5. CONCLUSION

In this paper, we propose a two-stage optimization model for the ICES with flexible heating loads of buildings. At the first stage, consumers adjust heating loads based on heating price from the operator. At the second stage, the RO method is adopted to optimize the energy schedules under uncertainties. Simulation results show that the proposed optimal scheduling method can reduce the heating costs of consumers while ensuring the ICES's profit under energy prices uncertainties.

## REFERENCE

[1] Xiaolong Jin, Qiuwei Wu, Hongjie Jia. Local flexibility markets: Literature review on concepts, models and clearing methods. Applied Energy, 2018; 154: 329–340.

[2] W Lin, X Jin, Y. Mu, et al., "A two-stage multi-objective scheduling method for integrated community energy system," Appl. Energy, vol. 216, pp. 428-441, 2018.

[3] T. Jiang, Z. Li, X. Jin, et al., "Flexible operation of active distribution network using integrated smart buildings with heating, ventilation and air-conditioning systems," Appl. Energy, vol. 226, pp. 181–196, 2018.

[4] Falk, J.E. A linear max—min problem. Mathematical Programming 5, 169–188 (1973)