

Model Predictive Control of a Substation of a District Heating System for Enhanced Environmental and Economic Performance

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ABSTRACT

This paper describes the design and optimization of the model-based predictive control (MPC) of a substation in the district heating system of the VUB campus. A properly controlled energy exchange improves the economy of the entire system while minimizing the pollutant emissions and fossil fuel consumption; the main goal of the European energy plan until 2030. The designed MPC modifies the flow rates in the substation, based on temperature evolutions and continuously provides an optimal return temperature in the network, while dispatching the desired amount of thermal power to the building. Compared to the installed proportional integral (PI) control, a reduction in substation return temperature of 10 °C can be obtained. The environmental and economic benefits of this reduced substation return temperature were assessed via a techno-economical model of the condensing boiler in the DHS, showing a 13% increase in overall efficiency, 11% of operational savings and a reduction of 5 ton in CO₂ emissions, when compared with the experimental data of December 2020.

Keywords: Model predictive control (MPC), district heating (DH), substation design, modeling of plate heat exchanger, techno-economical modeling of condensing boiler

NONMENCLATURE

Abbreviations

CAPEX	Capital expenditures
CB	Condensing Boiler
LHV	Lower Heating Value
MILP	Mixed Integer Linear Programming
MIMO	Multi Input Multi Output
MPC	Model Predictive Control
NG	Natural Gas
NPV	Net Present Value

OC	Operating cost
PBP	Payback Period
SISO	Single Input Single Output
WCC	Water Compensated Control

Symbols

B	Benefits	€
b	Plate width	m
C	Cost	€
c_p	Specific heat capacity	J/kgK
D	Free distance between plates	m
η	Efficiency	%
H	Heat transfer coefficient	W/m ² K
I	Identity matrix	
L	Length of the plate	m
M	Molar mass	kg/mol
\dot{m}	Mass flow rate	kg/s
N	Number of control volumes	
n	Paying years	
\dot{Q}	Thermal power	W
r	Weighting factor flow rates	
ρ	Mass density	kg/m ³
S	Set of feasible solutions	
T	Temperature	°C
t	Time	s
\dot{W}	Electric power	W
Y	Number of plates	
Q	Weighting matrix	
z	Discount rate	

Subscripts

ch tr	Chemical treatment
cond	Condenser
elec	Electric
fg	Flue gas
h	High temperature
l	Low temperature
prim	Primary
R	Return from substation
sec	Secondary

sens Sensible
Mpo,Msri Measured temperatures at
primary and secondary sides

1. INTRODUCTION

The benefits of the low return temperature from heating substation are prominent in the DH technology [1, 2]. The low return temperature from the substation increases the ΔT across the distribution network [1]. An increase of ΔT of 10 °C results in approximately 55% reduction in required pumping power, and depending on the heat generation method fuel saving can be achieved between 0.1% to 14% [3]. This also reduces the CO₂ emission to the atmosphere.

To gain the advantage of low temperature return, a condenser is installed at the heat production facility which increases the production by recovering the heat from flue gases [4, 5]. In addition to condenser, an absorption heat pump is also added in networks where temperatures are not optimized which increases the CAPEX of the DH network [5, 6]. This generates the demand of temperature optimization and control in the DH system [1, 2, 7].

In [3], the radiator supplying temperature set-point is optimized against supply temperature of the network to achieve low substation return. The potential of using optimized secondary supply temperature set-point by keeping in view the low substation return is also studied in [8, 9]. In [8], an advanced adaptive approach is adopted in which both the radiator temperature supply curve and the secondary flow rates are optimized to yield the lowest primary return. In the discussed approaches, output-feedback based PI control is implemented for the substation design which tracks the optimized heating curve. The optimization of control curve is performed based on the simplified steady-state models which may not be adequate in context of network flexibility as discussed in [10]. The model-based design approaches are required for efficient and robust performances of energy systems [11],[12]. In [13], a dynamical model of plate heat exchanger is developed which describes the thermal behavior of the fluids involved in the heat transfer process. The model was nonlinear and linearized to design a PI controller and its gains are tuned using frequency domain analysis. In [11], implemented a model-based robust H_∞ design for plate heat exchanger and showed the robust performance in terms of output tracking in comparison with installed PI controller.

The benefits of applying advanced control approaches such as MPC [14] in DH systems are well understood [10, 15, 16]. In [15], a simplified building

heating system model is developed and implemented with MPC to demonstrate a case study of 95-flat communal heating system. In [17], a MILP based MPC is designed for space heating system of building. The MILP calculates the reference trajectories of control inputs and set-points for MPC which subsequently ensures the tracking of set-points simultaneously. In [18], potential in cost and energy savings by replacing a WCC controlled radiator system with a linear MPC controller is investigated.

In the above design approaches, SISO framework is considered which regulates the primary side flow at substation level to maintain the desired heating curve. The objective to achieve low primary return is handled by optimizing the control set-points of radiator supply. However, these optimization of control curves is not common in DH system and traditional outdoor based generation of control set-points is in use [3, 9]. This has also been the case in the VUB heating network, where the supply curve is based on the outdoor temperature and subsequently controlled by a PI controller. However, it does not give a low return which can be suitable for DH system to gain the economic and environmental benefits as discussed in [3, 5]. Hence, in this work, the designed MIMO-MPC is developed based on the first-principle based dynamical modelling of a plate heat exchanger in contrast to the existing steady-state approaches. The model is validated with the experimental data and then implemented with the constrained nonlinear model predictive control (MPC). The operational limits both in terms of the substation temperatures and flow rates are considered as constraints in the formulated MPC framework. As a result, the designed MPC enables the operation close to the operational design constraint while tracking of supply and return temperatures of the substation. The quantitative analysis has also been made between the existing PI controller and with the new proposed MPC controller to satisfy the heat demand of 1 MW. Moreover, the economic and environmental gain of underlying DH system are also quantified and compared in closed loop simulation by employing the techno-economical model of CB.

2. MODELLING OF DISTRICT HEATING SYSTEM

The model of district heating system comprised of subsystems such as condensing boiler, heating substation, pumps, pipes, valves, and control valves mainly [3, 8]. However, the efficiency of the DHS depends on the design of the substation [3, 8]. Hence, the dynamical model of the main component of

substation such as plate heat exchanger is considered for control design.

2.1 Substation Model

The design of the substation is shown in Fig.(1). The plate heat exchanger is responsible for the controlled energy exchange between the primary side and secondary side of the network. Hence, the control-oriented form of the heat exchanger is developed to assist the model-based design of the substation. The heat demand of the building is fixed to 1 MW.

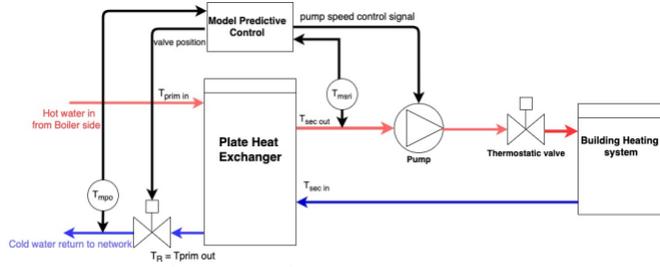


Fig.1. Design configuration of VUB heating substation; here: MPC based control design is selected to optimize the flow rates for the tracking of temperature set-points.

The mathematical model of plate heat exchanger is comprised of nonlinear partial differential equations (PDEs) which describes the energy and heat transfer as hot and cold streams passes through the heat exchanger [11, 19]

$$\frac{\partial T_h}{\partial t} = \frac{\dot{m}_{\text{prim}}}{\rho Y b d} \frac{\partial T_h}{\partial x} + \frac{h}{\rho c_p d} (T_l - T_h) \quad (1)$$

$$\frac{\partial T_l}{\partial t} = -\frac{\dot{m}_{\text{sec}}}{\rho Y b d} \frac{\partial T_h}{\partial x} + \frac{h}{\rho c_p d} (T_h - T_l) \quad (2)$$

The nonlinear PDEs Eqs.(1) and (2) cannot be used directly to design the model-based control design framework [11, 19]. Hence, these equations have been reduced to nonlinear ordinary differential equations (ODEs) by using finite volume and integral method [11] as:

$$\frac{\partial T_{h,j}}{\partial t} = \frac{N \dot{m}_{\text{prim}}}{\rho Y b d L} (T_{h,j+1} - T_{h,j}) + \frac{h}{\rho c_p d} (T_{l,j} - T_{h,j}), \quad (3)$$

$j = 2, \dots, N.$

$$\frac{\partial T_{l,j}}{\partial t} = \frac{N \dot{m}_{\text{sec}}}{\rho Y b d L} (T_{l,j-1} - T_{l,j}) + \frac{h}{\rho c_p d} (T_{h,j} - T_{l,j}), \quad (4)$$

$j = 2, \dots, N$

The nonlinear ODEs in Eqs. (3) and (4) can be written into a control-oriented configuration for $N=10$, as

$$\dot{x} = f(x(t), u(t)) \quad (5)$$

where $x \in \mathbb{R}^{20 \times 1}$ represents the hot and cold-water temperature distribution in the channels, $u \in \mathbb{R}^{2 \times 1}$ correspond to the primary and secondary mass flow rates as \dot{m}_{prim} and \dot{m}_{sec} , respectively. The comparison of the temperature $T_{h,0}$ and $T_{l,0}$ is shown in Fig.(2) and a good match between the model output and the measured data can be observed, indicated by a small relative error of less than 8%. Hence, Eqs. (3) and (4) will be used for the MPC-based control design of VUB heating substation shown in Fig.(1).

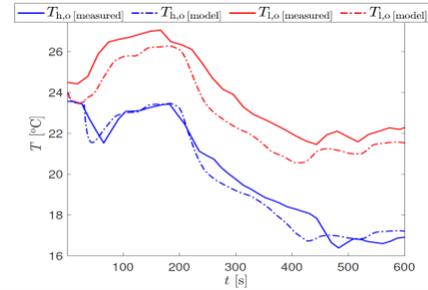


Fig.2. Temperature evolution in heat exchanger; here: a comparison of nonlinear model Eqs. (3) and (4) vs experimental data in [19] is shown.

2.2 MPC design of substation

A constrained nonlinear MPC is designed for district heating substation as shown in Fig.(1). The operational constraints of substation are directly formulated in defined optimal control problem of MPC [14], as

Minimize:

$$\int_{t_i}^{t_i+t_f} V(t, x(t), u(t)) dt + W(x(t_i + t_f))$$

subject to

$$\dot{x} = f(t, x(t), u(t)), \quad t \in [t_i, t_i + t_f]$$

$$5 \leq u_1 \leq 17 \text{ kg/s}, 7 \leq u_2 \leq 17 \text{ kg/s}$$

$$10 \leq x_1 \leq 50 \text{ }^\circ\text{C}$$

$$10 \leq x_{20} \leq 80 \text{ }^\circ\text{C}$$

$$x(t_i + T) \in S$$

(6)

The dynamical state constraints and input constraints are set to provide the heat demand of 1 MW while tracking the desired reference value x^r . $S = [x_1, \dots, x_{10} \in 42; x_{11}, \dots, x_{20} \in 70]$. V is the running cost which characterizes the main control objectives such as minimization of tracking errors of supplying and return temperatures with minimal control inputs u_1 and u_2 ,

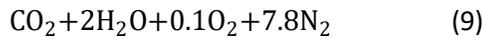
$$V(x, u) = \|x - x^r\|_Q^2 + \|u\|_r^2 \quad (7)$$

$$W \text{ represents terminal cost yielding final objective as } W(x, u) = x^T Q x \quad (8)$$

where $Q = 100 \times I(20)$ and $r = 10$. It is difficult to obtain the analytical solution of the formulated optimal control problem given in Eq. (6). Hence, the numerical solution is obtained by converting an optimal control problem into multiple shooting nonlinear optimization problem [20], and solved by using optimization tool CasADi [21].

2.3 Techno-economic model of condensing boiler

The steady-state model of condensing boiler [4] is considered. The modeling of condensing boiler is divided in three parts such as combustor modeling, gas to water heat transfer (boiler) and condensing flue gas to water heat transfer (condenser), respectively, [4]. The combustion is carried out by considering the methane CH_4 as a fuel gas and dry air as a fuel oxidizer $\text{CH}_4 + 10(0.21\text{O}_2 + 0.79\text{N}_2) \rightarrow$



and 5% of excess air is considered for smooth combustion. The composition of the flue gas can be obtained using Eq. (9). The steady-state analysis of boiler is accomplished by establishing the energy balance. The heat duty of boiler (\dot{Q}_{boiler}) and the exit temperature of water ($T_{w,\text{out}}$) can be obtained from following equations, as discussed in [6]

$$\begin{aligned} \dot{Q}_{\text{HHV}} &= \dot{Q}_{\text{fg,sens}} + \dot{Q}_{\text{loss,wall}} + \dot{Q}_{\text{boiler}} + \dot{m}_v L_v \\ \dot{Q}_{\text{boiler}} &= \eta_{\text{boiler}} \dot{Q}_{\text{LHV}} \\ \dot{Q}_{\text{boiler}} &= \dot{m}_w c_{p_v} (T_{w,\text{out}} - T_{w,\text{in}}) \end{aligned} \quad (10)$$

The condenser is modeled for 6 MW natural gas fired boiler of VUB. In this case, a shell and tube type counter-flow heat exchanger with baffles is selected [22]. The required heat transfer area for the device is calculated through the LMTD method [22], which results in 125 m^2 .

The performance of the condenser at different loads is studied with the ϵ -NTU method [23]. From this method the temperature of the outgoing fluid and the thermal power \dot{Q}_{fgc} is calculated. The condensation does not happen in the entire heat exchanger. Thus, the device is divided into a sensible and a condensing part, to yield an accurate temperature profile for the heat transferring fluids. The efficiency of the condensing boiler is given as

$$\eta_{\text{CB}} = \frac{\dot{Q}_{\text{boiler}} + \dot{Q}_{\text{fgc}}}{\dot{Q}_{\text{LHV}}} \quad (11)$$

The mass of avoided CO_2 emissions to the atmosphere is calculated as

$$m_{\text{CO}_2} = \frac{\dot{Q}_{\text{fgc}} M_{\text{CH}_4} \Delta t}{\eta_{\text{boiler}} \text{LHV} M_{\text{CO}_2}} \quad (12)$$

The economic analysis of the condenser is performed by considering steady state operation of the device. First, the required investment cost (CAPEX) is calculated following Smith's model [24]

$$\text{CAPEX} = C_E f_M (1 + f_{\text{PIP}}) + C_E (f_{\text{ER}} + f_{\text{INST}} + f_{\text{DEC}} + f_{\text{CONT}}) \quad (13)$$

The factors f in Eq. (13) are given in [24]. The initial investment cost is calculated as 248 000 € by using Eq. (13). The fuel cost is reduced by recovering sensible and latent heat with the condenser

$$B = \dot{Q}_{\text{fgc}} \Delta t C_{\text{NG}} \quad (14)$$

The operational costs correspond to the electricity consumed by the fan and pump and the chemical for condensate treatment:

$$\text{OC} = (\dot{W}_{\text{fan}} + \dot{W}_{\text{pump}}) \Delta t C_{\text{elec}} + \dot{V}_{\text{cond}} \Delta t C_{\text{ch tr}} \quad (15)$$

$\dot{W}_{\text{fan,pump}}$ is calculated via the pressure losses [24].

Finally, the Payback Period and the Net Present Value are calculated by using

$$\text{PBP} = \frac{\text{CAPEX}}{B - \text{OC}} \quad (16)$$

$$\text{NPV} = -\text{CAPEX} + \sum_{j=1}^n \frac{B_j - \text{OC}_j}{(1+z)^j} \quad (17)$$

3. RESULTS AND DISCUSSION

Since, the existing substation design of VUB with the PI controller does not provide the low return temperature to achieve the environmental and economic benefits in the DHS. In Fig.(3), the measured return temperature $T_{\text{R[measured]}}$ with PI design is compared with the return temperature T_{R} from the proposed MPC design.

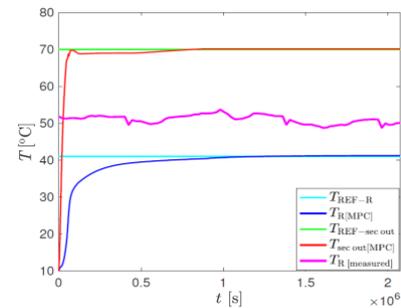


Fig.3. Temperature evolution in the substation; here: reference tracking of $T_{\text{sec out}}$ with MPC is shown and reduction in T_{R} with MPC design is compared with $T_{\text{R[measured]}}$ of PI.

It can be observed from Fig.(3) that T_{R} can be reduced up to 10°C with MPC design along with maintaining the desired set-point for the building supply ($T_{\text{sec out}}$ or T_{sri}) at 70°C .

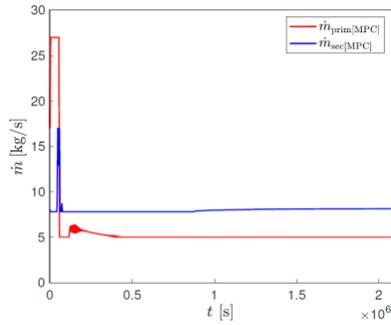


Fig.4. Regulation of flow rates in heating substation; here: \dot{m}_{prim} and \dot{m}_{sec} are controlled using MPC.

To achieve both the objectives, the designed MPC regulates the flow rates as shown in Fig.(4). The imposed constraints mentioned in optimal control form Eq. (6) are also satisfied and 1 MW of heat is transferred to the attach building as shown in Fig.(5).

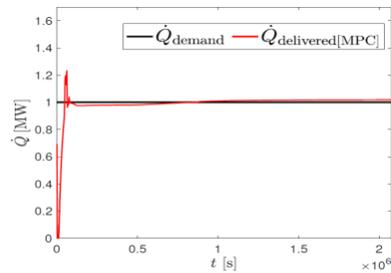


Fig.5. Thermal power demand of building; here: providing the heat demand of 1 MW with MPC based substation design.

The quantitative analysis of the VUB heating substation is also made in Table (1) to satisfy the heat demand of 1 MW.

Table 1: Performance comparison of the substation with existing PI and proposed MIMO MPC; by optimizing both flow rates \dot{m}_{prim} and \dot{m}_{sec} the substation return can be reduced by approximately 10°C.

controller	\dot{m}_{prim} kg/s	\dot{m}_{sec} kg/s	$T_{\text{prim in}}$ °C	$T_{\text{prim out}}$ °C	$T_{\text{sec in}}$ °C	$T_{\text{sec out}}$ °C
PI	5	26	90	52	70	51
MPC	5	8	90	42	70	42

The analysis of the condensing boiler is performed with low return of 40 °C to 42 °C from MPC-based substation control during the period of Dec-04-2020 to Dec-28-2020. The thermal power of the boiler during this period was between 1.7 MW to 2.3 MW and the condenser transferred 13% of the total thermal power. The efficiency of the heating system is calculated using Eq. (11) as 103%. The reduction of CO₂ emissions due to condensation is calculated using Eq. (12) as 5 ton. According to Eq. (16), the payback period of the installed condenser is 5 years, and the NPV is 76 000 € by using Eq. (17). Hence, the project is feasible in terms of investment returns. In Fig.(6), the comparison between the cost of a

traditional and a condensing boiler is shown which indicates cost saving of 4000 € during the month of December.

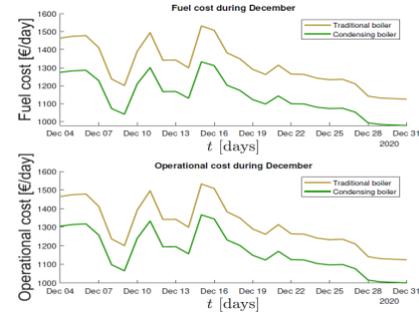


Fig.6. Evolution of the operational cost and the fuel cost. The condensing boiler reduces the operating cost by the 11%

4. CONCLUSIONS

In this work, a model-based constrained nonlinear MPC is developed for a heating substation. The nonlinear time domain model of the plate heat exchanger provides the accurate predictions of temperature evolutions in different control volumes to solve the formulated optimization problem of MPC efficiently. Consequently, MPC maintains the desired set-point for the building supply temperature while achieving a low substation return temperature of 42 °C. The simulation results and quantitative analysis demonstrate that the MPC-based design outperforms the existing PI-based design in achieving the low return from substation while satisfying the heat demand of 1 MW. Furthermore, a reduction of 10 °C in the substation return provokes the flue gas condensation in the condensing boiler. This increases the efficiency of the integrated DH system by 13%. The significant reduction in cost can also be obtained, along with 5 ton of CO₂ reduction when compared with the measured field data for December 2020.

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