

# Frequency-Domain Transfer Function Modeling Method for Dynamic Natural Gas Flow and Application in Pipeline Leakage Location

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## ABSTRACT

Based on the derivation of frequency-domain transfer function, this work obtains natural gas pipelines lumped model directly reflecting the constraints of the inlet and outlet variables. This dynamic model transforms the original partial differential equations into linear lumped constraints of gas transport process and the nonlinear resistance characteristics of pipeline, which can be efficiently solved. Comparing with finite difference method, simulation on single tube and pipeline network indicates that the modeling and solution method proposed in this work has advantages in accuracy and calculation efficiency. On the basis of this method and the topology change caused by leakage, the location optimization calculation can accurately locate the leakage wherever the it occurs in single pipeline or pipeline network with illustrative cases.

**Keywords:** natural gas pipelines, Frequency domain, transfer function, lumped model, leakage location

$Q$	Leakage rate
$R$	Gas resistance
$x$	Leakage location
$\lambda$	Friction factor of pipeline
$\rho$	Density of natural gas

## 1. INTRODUCTION

As a kind of efficient and clean energy, natural gas becomes widely used in various industries of energy utilization in recent years. However, due to the poisonous and explosive characteristics, once the natural gas leaks, it may cause serious accidents. While in the practical engineering, the main factors, such as corrosion, cracks, manufacturing defects, geological changes and human operation errors, make leakage an inevitable problem in the transmission and distribution of natural gas [1]. Therefore, it is essential to predict and locate leakage rapidly and effectively to ensure the efficient and safe application of natural gas.

Among the leakage diagnosis and location methodologies of the gas pipelines, there are mainly two categories, the signal processing-based method and the mathematical model-based method. In the signal processing-based method, such as the acoustic wave method [2] and the negative pressure wave method [3], by processing the instant sound wave or negative pressure wave signal in pipeline detected by the sensors, it can judge whether leak happens and calculate the leakage position [4]. However, this method needs to improve its accuracy at the expense of complex installation and high hardware cost [5]. In addition, it still

## NONMENCLATURE

<i>Symbols</i>	
$A$	Cross-sectional area of pipeline
$b$	Branch in gas network model
$C$	Gas capacitance
$D$	Diameter of pipeline
$G$	Mass flow rate of natural gas
$L$	Gas inductance
$p$	Pressure of natural gas

presents difficulties in small leakages detection and location of natural gas systems [6].

While theoretically, the mathematical model-based leakage diagnosis and location method can locate the precise leakage point under slow and small leakages [7]. An intractable problem is that the pipeline dynamic transmission model is complicated to solve directly. Therefore, some assumptions are made to simplify the model. Assuming the input and the output flow rate of a pipeline are equal under normal operation condition, the volume or mass balance method can diagnose a leakage when the difference between the two flows goes beyond a threshold value [8]. This method usually provide little information about leakage location [2]. Another simplification, the pressure gradient method, assumes that the pressure gradient along the length of pipeline is linear, and locates leakage at the inflection point of the pressure gradient in the pipeline [9]. But in practice, pressure distribution along the pipeline is nonlinear as the density and velocity varies along the pipeline. Therefore, the linear pressure gradient assumption may reduce the location accuracy of this method.

Another mathematical model-based method, real-time model method, including inverse transient analysis approach [10] and state estimation method [11] and etc. In the real-time model simulation solving the dynamic gas transmission model in pipeline, the original nonlinear partial differential equations (PDE) are transformed into a series of linear algebraic equations in discrete time and space by finite difference method (FDM) after linearization [12]. The leak parameters are obtained by minimizing the error between the model numerical simulation and the available measured data using optimization method [5]. However, the system state variables and leakage parameters can only be obtained at discrete space points due to the introduction of FDM. It means that to get accurate leakage position and leakage rate, it is necessary to increase the number of pipeline segments [13], which simultaneously increases the number of variables and the dimension of equations in simulation model and further results high calculation amount and slow convergence speed of this method. Therefore, the real-time method needs to make a trade-off between high computational cost and low leakage location accuracy. Besides, there is less study on leakage location of natural gas networks.

Considering the compressibility of natural gas and the characteristics of pipeline dynamic flow, this paper aims to obtain natural gas pipeline lumped model directly reflecting the constraints of the inlet and outlet

variables by introducing frequency-domain transfer function, as elaborated in Section 2. Section 3 solves the lumped model and states its advantages in accuracy and calculation efficiency by comparison with a classical approach, FMD. Based on model simulation, the leakage location optimization algorithm can accurately locate when leak happens, wherever the leak occurs in single pipeline or pipeline network. As illustrative examples, case studies of leakage location in single tube and pipeline network are provided in Section 4.

## 2. FREQUENCY-DOMAIN TRANSFER FUNCTION MODEL OF PIPELINE GAS FLOW

This section establishes the gas dynamic flow model assuming that

- 1) The variation of natural gas temperature along the pipeline can be neglected.
- 2) Only considers one-dimensional gas flow along the pipeline.
- 3) The pipeline cross-sectional area is constant.

### 2.1 Pipeline dynamic gas flow model

The nonlinear PDEs, including mass conservation equation (1), the momentum conservation equation (2) and the gas state equation (3), describe the constraints among velocity, density and pressure of natural gas transported in pipeline [14].

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial x} = 0 \quad (1)$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial \rho v^2}{\partial x} = -\frac{\partial p}{\partial x} - \frac{\lambda \rho v^2}{2D} - \rho g \sin \theta \quad (2)$$

$$p = \rho c_s^2 / \gamma \quad (3)$$

In the three equations,  $\rho$ ,  $v$  and  $p$  respectively denote density, velocity and pressure of gas along pipeline.  $t$  is time, and  $x$  is the position in pipeline.  $\lambda$ ,  $D$  and  $\vartheta$  respectively represent friction factor, diameter and inclination angle of pipeline.  $g$ ,  $c_s$  and  $\gamma$  are gravitational acceleration, sound speed in natural gas and adiabatic compressibility factor of natural gas, respectively. The second term in Eq. (2) is the convective term. When the gas velocity is much smaller than the speed of sound, this convective term approaches zero and can be neglected in engineering practice. Inclination angle of pipeline is neglected here. As the mass flow rate  $G$  is the product of  $\rho$ ,  $v$  and the cross-sectional area of pipeline  $A$ , Eq. (1)(2) expressed with  $G$  are

$$\frac{\gamma A}{c_s^2} \frac{\partial p}{\partial t} + \frac{\partial G}{\partial x} = 0 \quad (4)$$

$$\frac{1}{A} \frac{\partial G}{\partial t} + \frac{\partial p}{\partial x} + \frac{\lambda c_s^2 G^2}{2\gamma A^2 D p} = 0 \quad (5)$$

Eq. (4)(5) can be expressed as

$$C \frac{\partial p}{\partial t} + \frac{\partial G}{\partial x} = 0 \quad (6)$$

$$L \frac{\partial G}{\partial t} + \frac{\partial p}{\partial x} + R G = 0$$

when making the following definition

$$C = \frac{\gamma A}{c_s^2}, \quad L = \frac{1}{A}, \quad R = \frac{\lambda c_s^2 G}{2\gamma A^2 D p} \quad (7)$$

where  $C$ ,  $L$  and  $R$  are respectively analogous to capacitance, inductance and resistance in the circuit. Capacitance reflects the compressibility of natural gas, inductance describes the inertia of natural gas flow in the pipeline, and resistance reflects the friction effect of pipeline on natural gas flow.

The boundary conditions are

$$p(x, t)|_{x=0} = p_{in}(t), \quad G(x, t)|_{x=0} = G_{in}(t) \quad (8)$$

The Fourier Transform of Eq. (6)(8) with respect to the time  $t$  are

$$\frac{d\tilde{G}}{dx} = -j\omega C \cdot \tilde{p} \quad (9)$$

$$\frac{d\tilde{p}}{dx} = -(j\omega L + R_f) \tilde{G}$$

$$\tilde{p}(x, \omega)|_{x=0} = \tilde{p}_{in}, \quad \tilde{G}(x, \omega)|_{x=0} = \tilde{G}_{in} \quad (10)$$

Variables with overline are their corresponding values in frequency domain.  $R_f$  is the tube frequency-domain flow resistance, and its expression is

$$R_f = F[RG]/\tilde{G} \quad (11)$$

$F[\ ]$  represents the Fourier Transform of variables. Eq. (9) shows that the original PDEs are transformed to ordinary differential equation (ODE) which can be solved theoretically. And the general solution of Eq. (9) is

$$p = \alpha \exp(\gamma_g x) + \beta \exp(-\gamma_g x) \quad (12)$$

$$G = -\frac{\alpha}{Z_g} \exp(\gamma_g x) + \frac{\beta}{Z_g} \exp(-\gamma_g x)$$

In this equation, the expression of  $Z_c$  and  $\gamma$  is

$$\gamma_g = \sqrt{j\omega C(j\omega L + R_f)}, \quad Z_g = \sqrt{\frac{j\omega L + R_f}{j\omega C}} \quad (13)$$

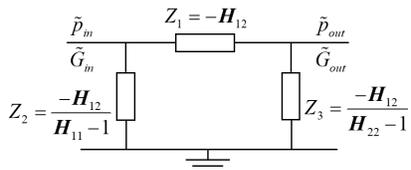


Fig. 1. Equivalent model of natural gas pipeline with frequency-domain lump parameters.

Substitute the boundary conditions Eq. (10) into Eq. (12), we obtained the expression of  $\alpha$  and  $\beta$  is

$$\alpha = (\tilde{p}_{in} - Z_g \tilde{G}_{in})/2, \quad \beta = (\tilde{p}_{in} + Z_g \tilde{G}_{in})/2 \quad (14)$$

Then the analytical solution is

$$\begin{aligned} \tilde{p}_{out} &= \tilde{p}_{in} \cosh(\gamma_g l) - Z_g \tilde{G}_{in} \sinh(\gamma_g l) \\ \tilde{G}_{out} &= -\frac{1}{Z_g} \tilde{p}_{in} \sinh(\gamma_g l) + \tilde{G}_{in} \cosh(\gamma_g l) \end{aligned} \quad (15)$$

$l$  is the length of pipeline. The transfer function matrix  $\mathbf{H}(\omega)$  between inlet and outlet parameters is

$$\begin{bmatrix} \tilde{p}_{out} \\ \tilde{G}_{out} \end{bmatrix} = \mathbf{H} \begin{bmatrix} \tilde{p}_{in} \\ \tilde{G}_{in} \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} \cosh(\gamma_g l) & -Z_g \sinh(\gamma_g l) \\ -\frac{1}{Z_g} \sinh(\gamma_g l) & \cosh(\gamma_g l) \end{bmatrix} \quad (16)$$

Eq. (16) gives the direct constraints between variables at the inlet and outlet of pipeline in frequency domain. By Fourier Transform and theoretically solving ODEs, the original PDEs are transformed into linear lumped gas transport constraints (16) and nonlinear flow resistance characteristics (11) of pipeline.

Meanwhile, Eq. (16) can also describe an equivalent  $\pi$ -type circuit shown in Fig.1. The branch constraints can be written in general form

$$\tilde{G}_b = y_b (\tilde{p}_b + \tilde{E}_b) \quad (17)$$

where  $G_b$  is the mass flow rate through the branch,  $p_b$  is the branch pressure drop,  $y_b$  is branch admittance and is the reciprocal of branch frequency-domain resistances  $Z_1$  to  $Z_3$  shown in Fig. 1.  $E_b$  is the pressure rise caused by compressor in the pipeline.

It is worth noting that different from the study in [14] and [15], this paper does not directly linearize the square term of velocity in the PDEs, that is, the velocity is not constant. And its variation with the natural gas flow along the pipeline at different locations and times can be considered in the model. Therefore, in pipelines with drastic velocity changes, the numerical solution of this model can still ensure high accuracy.

## 2.2 Pipeline network model

According to the topology of pipelines in network, connecting frequency-domain equivalent models of all pipelines at junctions, the network frequency-domain equivalent model is obtained.

For the pipeline network with  $n$  tubes, the branch constraints can be expressed as the matrix equations shown in Eq. (18).  $y_b$  is a  $3n \times 3n$  matrix

$$\tilde{G}_b = y_b (\tilde{p}_b + \tilde{E}_b) \quad (18)$$

Mass balance at the branch junction can be described as

$$\mathbf{A}_s \tilde{\mathbf{G}}_b = \tilde{\mathbf{G}}_n \quad (19)$$

$\mathbf{G}_n$  is the matrix of injection flow rate at all nodes.  $\mathbf{A}_s$  is node-branch incidence matrix in Equivalent model of pipeline network. Relation between the node pressure  $\mathbf{p}_n$  and the branch pressure drop  $\mathbf{p}_b$  is

$$\mathbf{A}_s^T \tilde{\mathbf{p}}_n = \tilde{\mathbf{p}}_b \quad (20)$$

Substituting (18) (20) into (19) can provide the relation among the pressure and mass flow rate of nodes in system equivalent model, as shown in Eq. (21).

$$\tilde{\mathbf{G}}_n - \mathbf{A}_s \mathbf{y}_b \tilde{\mathbf{E}}_b = \mathbf{A}_s \mathbf{y}_b \mathbf{A}_s^T \tilde{\mathbf{p}}_n \quad (21)$$

Set

$$\tilde{\mathbf{G}}_n^* = \tilde{\mathbf{G}}_n - \mathbf{A}_s \mathbf{y}_b \tilde{\mathbf{E}}_b, \quad \mathbf{Y}_s = \mathbf{A}_s \mathbf{y}_b \mathbf{A}_s^T \quad (22)$$

then Eq. (21) can be expressed as

$$\tilde{\mathbf{G}}_n^* = \mathbf{Y}_s \tilde{\mathbf{p}}_n \quad (23)$$

In Engineering practice, the nodes in pipeline network include pressure-given nodes (natural gas source) and injection-given nodes (natural gas load and middle transport nodes). We define the pressure of pressure-given nodes  $\mathbf{p}_p$  and the mass flow rate of injection-given nodes  $\mathbf{G}_g$  are the input of the system, and mass flow rate of pressure-given nodes  $\mathbf{p}_g$  and pressure of injection-given nodes  $\mathbf{G}_p$  are the output of the system. Then the network constraints (23) can be written as

$$\begin{bmatrix} \mathbf{Y}_{gg} & \mathbf{Y}_{gp} \\ \mathbf{Y}_{pg} & \mathbf{Y}_{pp} \end{bmatrix} \cdot \begin{bmatrix} \tilde{\mathbf{p}}_p \\ \tilde{\mathbf{p}}_g \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{G}}_g^* \\ \tilde{\mathbf{G}}_p^* \end{bmatrix} \quad (24)$$

Through matrix rearrangement and equation solving, the relation between the input and output of system is obtained.

$$\begin{bmatrix} \tilde{\mathbf{p}}_p \\ \tilde{\mathbf{G}}_p^* \end{bmatrix} = \mathbf{H}_s \begin{bmatrix} \tilde{\mathbf{p}}_g \\ \tilde{\mathbf{G}}_g^* \end{bmatrix}, \quad \mathbf{H}_s = \begin{bmatrix} -\mathbf{Y}_{gg}^{-1} \mathbf{Y}_{gp} & \mathbf{Y}_{gg}^{-1} \\ \mathbf{Y}_{pg} - \mathbf{Y}_{pg} \mathbf{Y}_{gg}^{-1} \mathbf{Y}_{gp} & \mathbf{Y}_{pg} \mathbf{Y}_{gg}^{-1} \end{bmatrix} \quad (25)$$

Here  $\mathbf{H}_s$  is system transfer function. It reflects the ratio between system output and input in frequency domain.

### 3. MODEL SOLUTION AND SIMULATION OF PIPELINE GAS FLOW

#### 3.1 Model solution

As is known to all, in linear system, if the input variable is a cosine function of a known frequency, the output variables are also cosine functions, its amplitude is the product of input function and transfer function, and its phase is the sum of input function and transfer function. That is, if the input is

$$X_{in} = a \cos(\omega_0 t) - b \sin(\omega_0 t) = \sqrt{a^2 + b^2} \cos(\omega_0 t + \varphi) \quad (26)$$

where  $\varphi = \arctan(b/a)$ . Considering the system transfer function  $H(\omega)$ , the output is

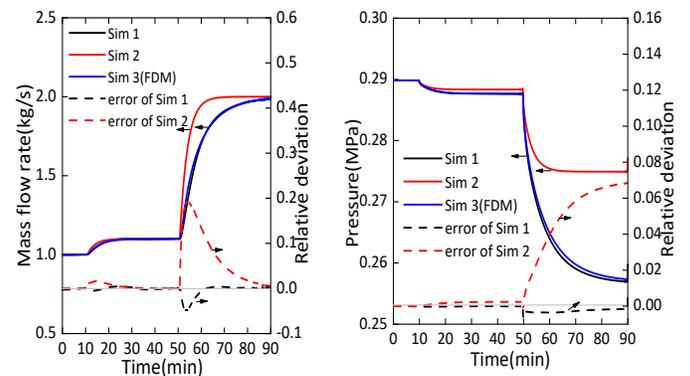
$$X_{out} = |H(\omega_0)| \sqrt{a^2 + b^2} \cos(\omega_0 t + \varphi + \angle H(\omega_0)) \quad (27)$$

Using fast Fourier transform (FFT) can transform the time-domain discrete input parameters of natural gas pipeline network into the sum of a series of cosine functions of known frequencies as shown in Eq. (26). With the help of transfer function given in Eq. (25), the output parameters can be obtained by simple operation of amplitude and phase at the corresponding frequency. Adding them together can offer the output results in time domain. The nonlinear pipeline resistance  $R_f$  in Eq. (11) needs to be updated according to the results of each solution.

It can be seen that frequency-domain transfer function modeling method provided in Section 2 greatly simplifies the solution process while considering the nonlinear components characteristics of pipeline.

#### 3.2 Dynamic simulation of single tube and comparison

Three simulations are performed in this part. Simulation 1 is based on the method proposed above, Simulation 2 is based on the proposed model with a constant pipeline resistance (defined by Eq.(7)), and Simulation 3 uses the Wendroff differential format [16] to obtain the finite difference solution as the reference. As the boundary condition, the tube inlet pressure is 0.3 MPa, and the outlet mass flow rate is 1 kg/s with an increase to 1.1 kg/s at  $t=10$  min and an increase to 2 kg/s at  $t=50$  min. Because the transfer function method has no consideration on initial conditions, a period of historical boundary conditions can be superimposed before time 0 to reflect the influence of the initial conditions. The calculated mass flow rate at inlet and pressure at outlet varying with time are shown in Fig 2.



(a) Inlet mass flow rate (b) Outlet pressure  
Fig. 2. Simulation results of single tube.

Comparison between the results of Simulation 1 and 3 verify the accuracy of the model and simulation method proposed in this work. Especially, the results at 50 min to 90 min indicates that the method can still maintain high accuracy when the gas velocity changes greatly. While at 50 min to 90 min, there is an obvious difference between result of Simulation 2 and 3. It indicates that when there is a relatively large variation of the boundary condition, the linearization method which assumes the coefficient of mass flow rate (i.e., resistance) in PDE is constant will bring great simulation errors.

### 3.3 Dynamic simulation of pipeline network

Simulation about pipeline network shown in Fig.3 is conducted in this part. The friction coefficient of each pipeline is 0.1, and the pressure rise in compressor is 0.2 MPa. The results are given in Fig.4.

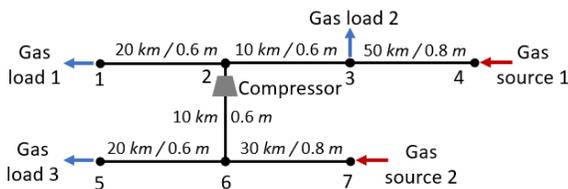


Fig. 3. Natural gas pipeline network.

Compared with the result using FDM [17] (the dash lines in Fig.4), the method proposed in this paper still keep high accuracy in simulation of pipeline network. However, the simulation using the method in this work takes only 2.8 s, while the simulation of FMD (time step 10 s and space step 1.25 km) needs 221.5 s in the same MATLAB simulation environment. It shows that the method proposed in this work has faster calculation speed and better calculation efficiency with the accuracy ensured.

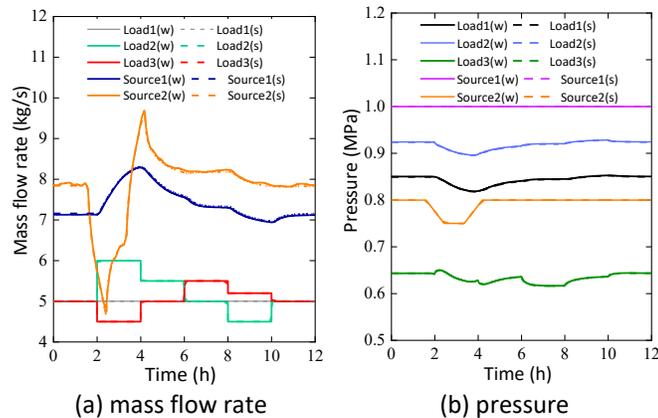


Fig. 4. Simulation results of pipeline network.

## 4. PIPELINE LEAKAGE LOCATION

### 4.1 Leakage location model

At the leakage point, following assumptions [1] are made.

- 1) Since the leakage occurs in the vertical direction, the momentum change caused by the leakage in the horizontal direction can be ignored.
- 2) The leakage rate at the leakage point is constant in time.

Then the leakage rate can be considered as a gas load with constant leakage rate at the location point. From Fig.5 it can be seen that the leakage changes the pipeline network topology to a new one.

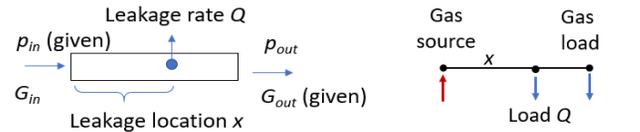
The leakage location problem can be described by an optimization problem

$$\min J(\mathbf{X}) = \sum_{i=1}^l [\mathbf{z}_i - \mathbf{X}_i]^T \mathbf{W}_i^{-1} [\mathbf{z}_i - \mathbf{X}_i] \quad (28)$$

s.t. equation(25)and(27)

$\mathbf{z}$  is measured value,  $\mathbf{X}$  is simulation value, and  $l$  is the sum of the number of time  $i$ . The element of  $\mathbf{W}_i^{-1}$  is the weight of each measurement. In this work, genetic algorithm is used to optimize this problem.

### 4.2 Single tube leakage location



(a) Diagram of leakage pipe (b) Equivalent pipe network

Fig. 5. Pipeline leakage model.

Fig.5(a) is the diagram of a leaking pipeline, its equivalent pipeline network model is shown in Fig.5(b). Assuming that the inlet pressure is 0.3 MPa, the outlet mass flow rate is 1.2 kg/s before leakage occurs, and the measured value of outlet pressure is the curve in Fig.6. Solving the leakage model using genetic algorithm offers the leakage location is 90.146% of the pipe length (60 km), and the error relative to the preset leakage location (90%) is 0.146%. The leakage rate is 0.4 kg/s which is equal to the supposed value. The case verifies accuracy of the leakage location method based on the transfer function model.

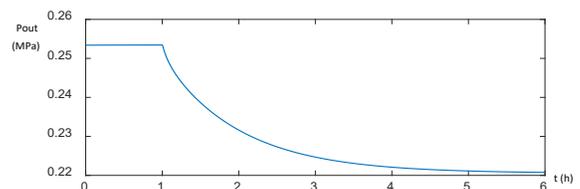


Fig. 6. Supposed measured value of tube outlet pressure

### 4.3 Pipeline network leakage location

Assuming that the pressure of two gas sources are 1 MPa and 0.8 MPa, the mass flow rate of three loads are all 5 kg/s before leakage occurs, and the measured value of outlet pressure is the curve in Fig.7. Solving the solution of network leakage location model gives the leakage pipeline number is 1 which is exactly the supposed one. The leakage location is 89.04% of the pipe length (20 km), and the error relative to the preset (90%) is 0.96%. The leakage rate is 1 kg/s, equal to the supposed value.

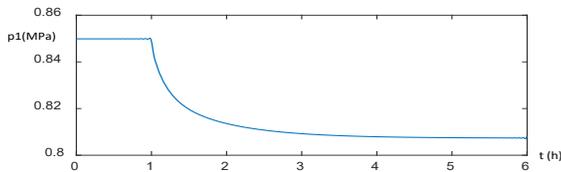


Fig. 7. Supposed measured value of load 1 pressure

## 5. CONCLUSION

With the benefit of Fourier Transform, the natural gas PDEs are transformed to ODEs with a frequency-domain nonlinear resistance. Theoretically solving the ODEs offers the transfer function of pipeline. Through matrix operation, the transfer function of pipeline network can also be obtained. Combining the FFT of boundary conditions with the transfer function gives the solution of the model. Comparing with FDM, simulations in Section 3.2 and 3.3 verify the accuracy and computation efficiency of this modeling and solution method by. Considering the topology change caused by leakage, this paper also proposes a leakage location optimization method. The results indicate that this method applies to location cases of both the single tube and the pipeline network with high accuracy.

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## REFERENCE

[1] Hou Q, Zhu W. An EKF-Based Method and Experimental Study for Small Leakage Detection and Location in Natural Gas Pipelines. *Applied Sciences*. 2019;9.  
 [2] Li S, Wen Y, Li P, Yang J, Dong X, Mu Y. Leak location in gas pipelines using cross-time–frequency spectrum of leakage-induced acoustic vibrations. *Journal of Sound and Vibration*. 2014;333:3889-903.

[3] Wang J, Ren L, Jiang T, Jia Z, Wang G. A novel gas pipeline burst detection and localization method based on the FBG caliber-based sensor array. *Measurement*. 2020;151.  
 [4] Xiao Q, Li J, Sun J, Feng H, Jin S. Natural-gas pipeline leak location using variational mode decomposition analysis and cross-time–frequency spectrum. *Measurement*. 2018;124:163-72.  
 [5] Torres L, Jiménez-Cabas J, González O, Molina L, López-Estrada F-R. Kalman Filters for Leak Diagnosis in Pipelines: Brief History and Future Research. *Journal of Marine Science and Engineering*. 2020;8.  
 [6] Wang L, Jian W, Gao X, Wang M. Summary of detection and location for oil and gas pipeline leak. *25th Chinese Control and Decision Conference*. 2013.  
 [7] Delgado-Aguiaga JA, Besanon G. EKF-based leak diagnosis schemes for pipeline networks. *IFAC-PapersOnLine*. 2018;51:723-9.  
 [8] Griebenow G, Mears M. Leak Detection Implementation: Modeling and Tuning Methods. *Journal of Energy Resources Technology*. 1989;111:66-71.  
 [9] S C, L D, B Z, X C. Leak detection and localization of gas pipeline system based on full dynamical model method. *Proceedings of the 30th Chinese Control Conference 2011*. p. 5894-8.  
 [10] Covas D, Ramos H. Case Studies of Leak Detection and Location in Water Pipe Systems by Inverse Transient Analysis. 2010;136:248-57.  
 [11] Delgado-Aguinaga JA, Besanon G, Begovich O, Carvajal JE. Multi-leak diagnosis in pipelines based on Extended Kalman Filter. *Control Engineering Practice*. 2016;49:139-48.  
 [12] Behrooz HA, Boozarjomehry RB. Modeling and state estimation for gas transmission networks. *Journal of Natural Gas Science Engineering*. 2015;22:551-70.  
 [13] Arifin BMS, Li Z, Shah SL. Pipeline Leak Detection Using Particle Filters. *IFAC-PapersOnLine*. 2015. p. 76-81.  
 [14] Chen Y, Guo Q, Sun H, Pan Z, Chen B. Generalized phasor modeling of dynamic gas flow for integrated electricity-gas dispatch. 2020;283:116153.  
 [15] Yang J, Zhang N, Botterud A, Kang C. Situation awareness of electricity-gas coupled systems with a multi-port equivalent gas network model. *Applied Energy*. 2020;258.  
 [16] Zheng Qiao, Jinhang Wang, Hongbin Sun, Yue Wu, Guo Q. Impact of natural gas infrastructure failure on electric power Systems considering the transmission dynamics of natural gas. *International Conference on Applied Energy*. 2019;574.