

# A non-linear gray-box model of buildings connected to district heating systems

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## ABSTRACT

Traditional building automation controllers are having low performance in dealing with non-linear phenomena. In recent years, model predictive control (MPC) has become a notable control algorithm for building automation system capable of handling non-linear processes. Performance of model-based controllers, such as MPC, is depending on reasonably accurate process models. For a building using baseboard radiator heater, a non-linear model is a more reliable representation of heat distribution system. Therefore, this study aims to present a non-linear gray-box model for a residential building connected to the local district heating network that is equipped with radiator heat emitters. The model is supposed to forecast the indoor air temperature as well as the radiator secondary return temperature. The model is validated using measurements collected from a building in Västerås, Sweden. In addition to a better accuracy, another motivation behind using a non-linear heating circuit model is to enhance its generalization performance. With the added benefits of accuracy and generalization, this model is expected to extend practical MPC implementation for such buildings.

**Keywords:** District heating, Non-linear model, Gray-box modeling, Forecasting

## NONMENCLATURE

### Abbreviations

MPC	Model predictive control
BIPV	Building integrated photovoltaic panels
RC	Resistance-capacitance
RBS	Rule-based controller

### Symbols

$R_{ax}$	Heat transfer resistance between the indoor air and the building external environment
$R_{ae}$	Heat transfer resistance between the indoor air and the building envelope
$R_{ai}$	Heat transfer resistance between the indoor air and the building internal mass
$R_{ex}$	Heat transfer resistance between the building envelope and the building external environment

## 1. INTRODUCTION

Residential housing represents 15% of total final energy demand in Sweden of which the majority is attributed to space heating and domestic hot water demand [1]. Sitting at a market dominance of around 50%, district heating is currently the leader of heat supply to buildings [2]. The basic idea of district heating is to supply heat from centralized sources through a heat distribution network to satisfy customer demands. The climate goals and regulations encourage suppliers to strive towards using renewable sources in the heating sector. The integration of renewable sources into district heating system will require reductions in district heating supply and return temperatures [3]. In addition, the system becomes more sophisticated for control and management when various decentralized renewable energy sources are joined together. This integration emerges the need for advanced modeling and control strategies.

MPC is among advanced control techniques that has become an ideal strategy in research on intelligent building operation [4]. MPC controller has the potential of thermal comfort improvement with simultaneous increase in energy savings compared to other conventional rule-based controllers (RBCs). However,

practical applications are still in early stages [4]. In MPC, a model representation of the building is used to obtain a control signal that minimizes the errors between a desired setpoint and the simulated output. The quality of this control effort relies on the model accuracy. Due to non-linear processes occurring inside a building, an exclusively physical model known as white-box model ensures a high fidelity which is preferred. However, the amount of computational power required for the MPC implementation for these complex models makes them infeasible for control purposes. On the contrary, data-driven models, known as black-box models, only rely on measurement data which puts them at an advantage of a faster simulation time. The drawback lies in the fact that without inclusion of any physical laws, some patterns might remain undetected in case the number of features in observational data is insufficient. A third category for thermal modeling of buildings are known as gray-box modeling. It is a hybrid methodology in which the coefficients of the equations from physics-based models are evaluated using data. This method enables a trade-off between accuracy and simplicity which makes it suitable for control applications and hence is proposed in this paper for a building that is connected to a district heating system.

Determination of gray-box model structure depends heavily on prior knowledge about system dynamics, while the unknown parameters are evaluated by measurement data. Complex system dynamics are observed in ordinary households that are shifting toward prosumers; customers who both produce and consume energy such as electricity and district heating [5]. A linear model can approximate a non-linear process with reasonable accuracy if its non-linear characteristic is weak. However, linearization is not a suitable technique to analyze complexities of a prosumer. Prosumers entail non-linear phenomena such as radiative heat transfer when equipped with building integrated photovoltaic panels (BIPV) or free convection in case of using natural ventilation. A non-linear gray box model is supposed to adequately predict dynamic behavior of prosumers.

The non-linear behavior is not only limited to prosumers. A non-linear model is a more precise dynamical description of the heat exchange process for a baseboard radiator heater. However, only a few studies considered that in the literature [6-8]. A non-linear gray-box model is introduced for a building with a radiator-based heating system in this paper. This non-linear modeling has a desirable effect on generalization performance of the model. The aim of the present study is to create a model that can further be integrated with an MPC controller and facilitate the practical implementation of MPC for buildings.

## 2. DESCRIPTION OF BUILDING AND DATA

The case study is a nine-story building with 31 residential units located in Västerås, Sweden. Apartments are ventilated by means of an FTX system. The building uses district heating for domestic hot water and space heating. Two months data (from December 2019 to January 2020) were collected with an interval of 15 minutes from the sensors installed inside the apartments and the substation. The collected data consists of indoor temperature data for all apartments, supply and return temperature of the water circuit on both primary and secondary side, the valve openings, and the outdoor air temperature. The data is divided into two parts. One for training the model and another set for testing model forecasting performance. The training dataset is composed of 2998 elements collected from the December 1,2019 to December 31,2019. The testing dataset includes 3004 observations recorded from January 1,2020 to January 31,2020. This data is used to identify unknown coefficients of the gray-box model.

## 3. MODEL STRUCTURE

Resistance-capacitance (RC) technique which is widely used for thermal modeling of buildings is opted. The concept is modeling different building elements based on an analogy to their equivalent electric circuit component. Heat load calculation and control are two major topics in which RC models are mostly utilized in the building energy domain. A great amount of research is done on the subject to find the optimal model order capturing the building thermal dynamics [9-11]. However, a unified standard framework for choosing the appropriate structure is not available yet [12].

We proposed a third-order RC model which represents the linear thermal inertia structure for building coupled with a non-linear heating system model. The energy balance for the indoor air, the exterior walls, the internal mass, and the baseboard heating radiator are given as follows:

$$C_{air} \frac{dT_{air}}{dt} = \frac{1}{R_{ax}} (T_{ext} - T_{air}) + \frac{1}{R_{ae}} (T_{env} - T_{air}) + \frac{1}{R_{ai}} (T_{int} - T_{air}) + \dot{Q}_{rad} + \dot{Q}_{ven} + S_A \dot{Q}_{int} \quad (1)$$

$$C_{env} \frac{dT_{env}}{dt} = \frac{1}{R_{ex}} (T_{ext} - T_{env}) + \frac{1}{R_{ae}} (T_{air} - T_{env}) \quad (2)$$

$$C_{int} \frac{dT_{int}}{dt} = \frac{1}{R_{ai}} (T_{air} - T_{int}) \quad (3)$$

$$C_{rad} \frac{dT_{ret}}{dt} = \dot{Q}_{sub} - \dot{Q}_{rad} \quad (4)$$

where  $T_{air}$  is the mean indoor temperature and  $C_{air}$  is its corresponding capacitance,  $T_{env}$  is the envelope

temperature and  $C_{env}$  is its corresponding capacitance, and  $T_{int}$  is the internal thermal mass temperature with  $C_{int}$  as its capacitance.  $R_{ax}$ ,  $R_{ae}$ ,  $R_{ai}$ ,  $R_{ex}$  are thermal resistances between heat mediums.  $S_A$  defines the floor area according to which the building's primary energy was calculated. Building's primary energy was available in its energy report.

$\dot{Q}_{int}$  is the internal heat gain that is determined by the identification process. Internal heat gain approximate range was obtained based on historical measurements entailing equipment, lightning, and number of occupants. Solar heat gain was discarded as it was almost zero during the whole simulation period.

Ventilation heat exchange occurs by purposeful hot inside air exchange with the cold outside air during heating season which can be estimated as:

$$\dot{Q}_{ven} = V_{inf}(A + B(T_{air} - T_{ext})) \quad (5)$$

according to [13] by a linear relationship between the inside and outside environment.  $A$  and  $B$  are coefficients that are found by the identification process and their range were selected based on historical measurements.  $T_{ext}$  is the building external environment temperature.  $V_{inf}$  carries out the information of design flow rates and schedules associated with the ventilation system. Since there were no certainty and no historical measurements regarding the effect of wind speed and infiltration, they were excluded from the model.

Heat from the secondary side water loop is transferred to the zone via radiators. The term that represents the heat flow from the heating circuit at the substation into the baseboard heater is:

$$\dot{Q}_{sub} = \varphi_v \dot{m}_r c_p (T_{sup} - T_{ret}) \quad (6)$$

where  $\varphi_v$  is the valve opening at the substation,  $T_{sup}$  is the secondary supply temperature of the radiator,  $c_p$  denotes water specific heat capacity, and  $\dot{m}_r$  is the rated water mass flow rate that is obtained from the manufacturer's data sheet.

The discrete-element radiator used has the following characteristic equation [7]:

$$\dot{Q}_{rad} = F_1 (T_{ret} - T_{air})^{a+1} \quad (7)$$

where  $T_{ret}$  denotes circulating secondary return temperature of the radiator,  $F_1$  is the surface area of the radiator, and  $a$  is the characteristic coefficient of the radiator. Overall, the model has:

- Four variable states ( $dT_{air}$ ,  $dT_{env}$ ,  $dT_{int}$ ,  $dT_{ret}$ )
- Three input variables ( $T_{ext}$ ,  $\varphi_v$ ,  $T_{sup}$ )

- Two output variables ( $yT_{air}$ ,  $yT_{ret}$ )
- Fourteen parameters to be identified.

The identification process was carried out in MATLAB by following its format to create an *idnlgrey* model object due to its suitability for identifying non-linear systems. The unknown parameters were found by fitting measured data to the predicted model response. Optimal values should minimize the cost function, which is a mean square error, given by a sum of systematic error (bias) and random error (variance). This penalty function is thus a tradeoff in creating the model [14]. Therefore, optimal set of parameters ( $\theta^{opt}$ ) in vectorized format is obtained from:

$$\theta^{opt} = \operatorname{argmin} \left( \frac{1}{N} \sum_{t=1}^N \varepsilon^2(t, \theta) + \frac{1}{N} \lambda \theta^T R \theta \right) \quad (8)$$

where  $t$  is the time variable,  $N$  is the number of data samples, and  $\varepsilon(t, \theta)$  is the predicted error computed as the difference between the observed output and the predicted output of the model. The second term is regularization which modifies the cost function by adding a term proportional to the square of the norm of the parameter vector  $\theta$ .  $\lambda$  and  $R$  are tools to find a good model that balances complexity and provides the best tradeoff between bias and variance, and they were selected by trial and error in this work.

#### 4. RESULTS

Table 1 reports the parameter estimates of the model presented. All identified parameters had a low standard deviation compared to their value, except for  $V_{inf}$  and  $B$ . As mentioned in Section 3, there was no measurement regarding the wind speed and infiltration for the simulation period, which made an accurate identification hard as there was a high level of uncertainty.

Table 1. Identified parameters for the case-study building.

Parameter	Value	Standard deviation	Unit
$C_{air}$	3152	43.58	[J/°C]
$R_{ax}$	0.229	0.005	[W/°C]
$R_{ae}$	0.229	0.029	[W/°C]
$R_{ai}$	0.752	0.256	[W/°C]
$F_1$	0.182	0.011	[W/°C]
$a$	2.275	0.0399	
$V_{inf}$	0.002	0.101	[m <sup>3</sup> /sec]
$A$	0.103	0.002	[1/m <sup>3</sup> ]
$B$	0.005	0.02	[1/m <sup>3</sup> °C]
$\dot{Q}_{int}$	0.037	0.0007	[W]
$C_{env}$	920.9	29.71	[J/°C]

$R_{ex}$	0.983	0.119	$[W/^{\circ}C]$
$C_{int}$	543.9	30.60	$[J/^{\circ}C]$
$C_{rad}$	32.79	2.47	$[J/^{\circ}C]$

Fig. 1 illustrates how the system is simulated given only the initial states for the training dataset. It shows the time series plot of measured outputs and the simulation outputs. Except for the last day of the simulation, the difference between simulation and measured indoor temperature is less than  $0.5^{\circ}C$  which is normally not detected for human's thermal comfort. There is an abrupt change in supply temperature on December 29, which caused a large draft in simulating the indoor temperature. Since we were unable to confirm whether this behavior was caused by a measurement error, it was not omitted from the dataset.

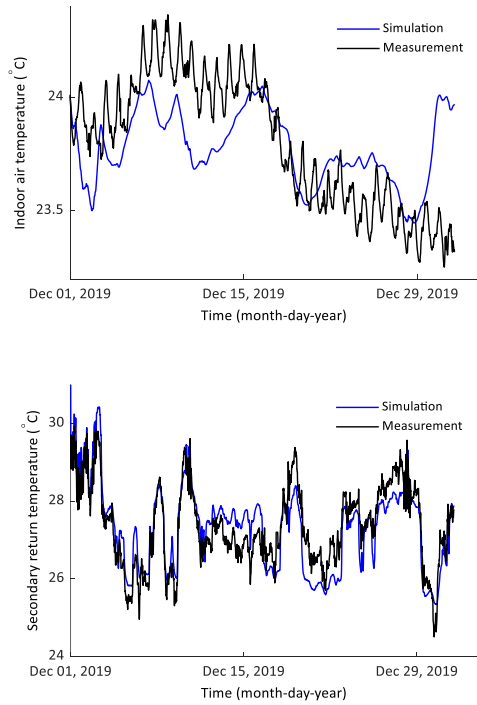


Fig. 1 Comparison between the simulated and the measured outputs for the training dataset.

The effectiveness of the model is also verified on the test dataset and the result is illustrated in Fig. 2. The model can forecast a long horizon in the future without losing accuracy. The zone air temperature difference between model and simulation is less than  $0.5^{\circ}C$  for all timesteps.

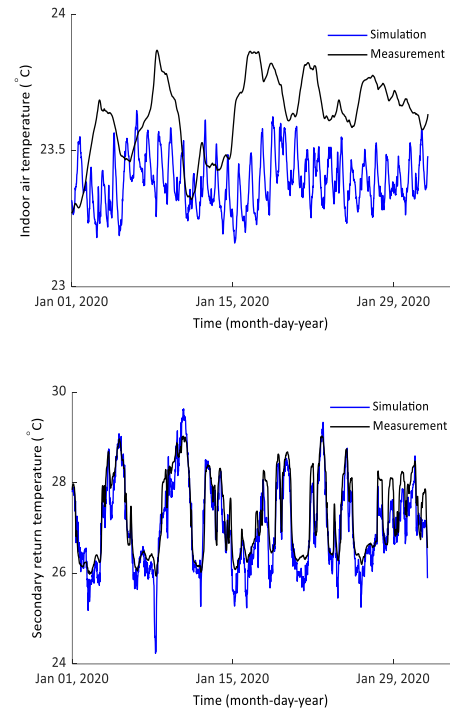


Fig. 2 Comparison between the simulated and the measured outputs for the test dataset.

The histogram of errors obtained from the identified model outputs are illustrated in Fig. 3. This error range indicates that the model has a suitable forecasting ability. The Gaussian distribution fit curve is also drawn to assess the mean and deviation of both outputs. The indoor temperature error has the mean value of 0.24 and a standard deviation of 0.16. For the secondary return temperature, the mean value and standard deviation of error are 0.24 and 0.33. The distribution confirms that forecasts are likely skewed, and the mean value might not be the best representative of the dataset.

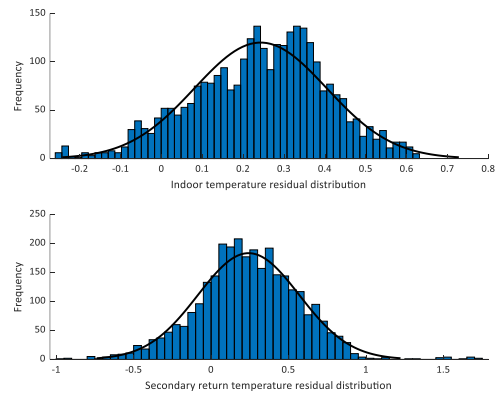


Fig. 3 Histogram of output distributions and their respective best-fit Gaussian distribution.

## 5. CONCLUSION

This paper presented a non-linear gray-box model to describe the heat dynamics of residential buildings connected to district heating systems. The model consists of two parts with particular attention to building thermal inertia and the heating system. The assumption of considering a linear behavior for baseboard radiator heaters, reduce the generalization ability of the model for other case studies. Therefore, a non-linear model was proposed in this study to represent a more realistic behavior of radiator-based heating system. The purpose of the model was to predict the indoor temperature and the radiator secondary return temperature. The model performance was evaluated for a training and a test dataset. The results showed an acceptable accuracy over a relatively long future horizon based on the histogram of errors.

The results of this case study are supposed to be integrated into an MPC controller as part of the project plan in our future publications.

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