

# An effective model for well-testing interpretation on carbonate reservoirs with complex cave connections

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## ABSTRACT

The fracture-caved gas carbonate reservoirs raise serious concerns in China. However, there are numerous large-scale caves in the fractured-caved gas reservoirs due to the structural fracture and dissolution. To solve the problems of fracture-caved gas reservoirs, like strong heterogeneity, obvious differences in fracture-caved connection characteristics, and great difficulty in well testing. In this paper, a novel pressure transient analysis model under a complex fracture-caved connection mode is established to estimate fracture region and cave parameters. First, governing equations considering the coupling of the fracture linear flow and the cave storage flow are derived. Furthermore, the dimensionless fracture pressure solution in Laplace space is solved after dimensionless treatment, and the dimensionless bottom-hole pressure solution in real space is produced using the Stehfest numerical inversion method. Finally, the flow phases are divided, and a multi-parameter sensitivity analysis is performed after identifying the type curves of the complex fracture-caved connection mode. The results indicate that the conductivity and length of the fracture region have the greatest effect on the start time and duration of the linear flow, whereas the volume of the cave has the greatest effect on the storage flow. Matching with the recorded pressure and flowrate data, the established pressure transient analysis model can effectively invert the fracture length, conductivity, and cave volume of fracture-caved reservoirs, which provides a basis for evaluating the geological characteristics and underground reserves of fracture-caved gas reservoirs and has significant guiding significance for the development of carbonate reservoirs.

**Keywords:** carbonate reservoirs, large caves, pressure transient analysis, fracture-caved gas reservoirs

## NONMENCLATURE

<i>Symbols</i>	
$p_f$	Pressure in the fracture regions, MPa
$\phi_f$	Fracture porosity
$t$	Well test time
$\mu_g$	Viscosity of gas, mPa·s
$c_{if}$	Total compressibility of fracture regions, 1/MPa
$k_f$	Permeability in the fracture regions, $\mu\text{m}^2$
$q$	Gas well production, $\text{m}^3/\text{d}$
$B$	Volume factor, $\text{m}^3/\text{m}^3$
$w_f$	Width of fracture region, m
$h$	Reservoir thickness, m
$p_v$	Pressure in the cave, MPa
$R_v$	Radius of cave, m
$\phi_v$	Cave porosity
$c_{iv}$	Total compressibility of the cave region, 1/MPa
$p_{fD}$	Dimensionless pressure of fracture region
$t_D$	Dimensionless time
$F_{CD}$	Dimensionless fracture conductivity
$C_{vD}$	Dimensionless storage coefficient of the cave
$x_D$	Dimensionless length
$r_w$	Wellbore radius
$k_r$	Reference permeability, $\mu\text{m}^2$
$\mu_r$	Reference viscosity, mPa·s
$\phi_r$	Reference porosity
$c_{ir}$	Reference total compressibility, 1/MPa
$\bar{p}_{v\phi D}$	Dimensionless phase redistribution pressure of cave in Laplace space

## 1. INTRODUCTION

In order to reduce energy waste and achieve the goal of carbon neutrality, oil and gas resources should be developed reasonably. Therefore, the reservoir parameters should be predicted to avoid environmental pollution and resource waste in the development process. Effective pressure transient analysis is an effective means to predict formation parameters.

After structural deformation, dissolution and collapse, fracture-caved gas reservoirs exhibit considerable heterogeneity [1,2]. Large caves and fracture regions make up the majority of the flow space, and it is evident that caves and fractures have different connection characteristics [3]. By using traditional pressure transient analysis methods, it is challenging to adequately describe the factors associated with fracture regions and caves [4,5]. In order to analyze the main parameters of fractures and caves using the pressure transient analysis method, a multi-media coupled pressure transient interpretation model under various fracture-caved connection modes is constructed in this paper. This method has significant practical implications for the development of fracture-caved gas reservoirs.

Because the matrix of fractured-caved gas reservoirs is mostly tight carbonate rocks, it is generally a non-reservoir space [6,7]. The major reservoir and flow space should be taken into account as the fracture regions and huge caves since the triple medium model taking into account matrix, fracture region, and cave fluid flow is insufficiently precise. It can be assumed that the pressure wave propagates quickly and extends to the entire cave unit since large-scale caves have an unusually high flow capacity [8,9]. As a result, a big cave can be thought of as an equipotential body, meaning that the pressure is the same across the entire cave.

At present, scholars have done a lot of research on pressure transient analysis methods for fracture-caved reservoirs. Wei Cao [10] established three kinds of well-fracture-cave connection modes according to the vertical beaded structure. Xin Du [11] proposed a more adaptable FCCRs model by combining PTA and RTA according to field geological information.

The triple interpretation continuum model is mostly used by commercial software and pressure transient analysis techniques to describe the parameters of fracture-caved gas reservoirs. The majority of pressure transient analysis models are simplistic and poorly applicable and should not consider the matrix system, which is non-main reservoir. As a result, pressure transient analysis models are built in this study under various fracture-caved connection modes, and governing

equations for linear flow in fracture regions and storage flow in caves are derived. Following dimensionless treatment, the dimensionless fracture pressure solution in Laplace space and the dimensionless bottom hole pressure analytical solution in real space are both solved. The typical curve is then obtained, and a sensitivity analysis is then performed. The pressure transient analysis confirms the logic and accuracy of the model when combined with the actual well production performance data. Therefore, the pressure transient analysis models developed under various fracture-caved connection modes have significant guiding significance for the development of fracture-caved gas reservoirs.

## 2. MATERIAL AND METHODS

### 2.1 physical model and assumptions

After long-term dissolution, the fracture region of carbonate reservoir has developed, making it easy to form huge caves. When exploiting this kind of reservoir, if the production well is drilled outside the cave, a fracture region exists in the formation to connect the wellbore and the cave, or an acid fracturing region is created by using acid fracturing technology to do so. In addition, if the well is drilled in a cave and connected to another cave through a fracture region, the physical model is displayed in Figure 1.

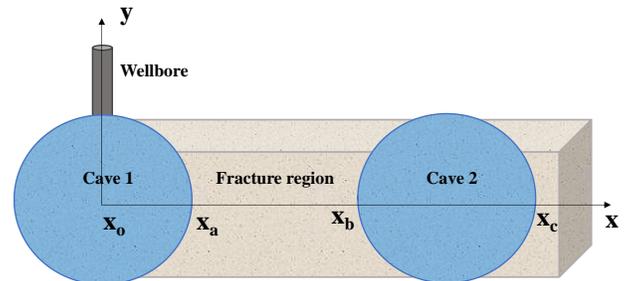


Fig. 1. Schematic diagram of Well-cave-fracture-cave-fracture physical model

The model's fundamental assumptions are as follows:

(1) Storage effect and skin effect are considered in wellbore.

(2) Ignoring the flow in the matrix, the fluid in the fracture regions and caves is the gas phase.

(3) Considering the pressure wave propagates rapidly and spreads to the whole cave unit. Therefore, a large-scale cave can be considered an equipotential body, that is, the pressure in each part of the cave is equal.

(4) Gas wells are taken into account for constant flowrate.

## 2.2 mathematical model and solution

The mathematical model of the well-testing interpretation appropriate for well-cave-fracture-cave-fracture mode is established and solved. And it is based on the linear flow in the fracture region following Darcy's law, and the large cave is an equipotential body with equal pressure and storage flow occurs inside.

### 2.2.1 Establishment of mathematical model

The partial differential equation describing the linear flow of gas in the fracture region ( $x_a \leq x \leq x_b$ ) is as follows:

$$\frac{\partial^2 p_f}{\partial x^2} = \frac{\phi_f \mu_g c_{if}}{3.6k_f} \frac{\partial p_f}{\partial t} \quad (x_a \leq x \leq x_b) \quad (1)$$

Where  $p_f$  is the pressure in the fracture regions, MPa;  $\phi_f$  is the fracture porosity,  $t$  is well test time, h;  $\mu_g$  is the viscosity of gas, mPa·s;  $c_{if}$  is the total compressibility of fracture regions, 1/MPa;  $k_f$  is permeability in the fracture regions,  $\mu\text{m}^2$ .

The fracture region is connected with the wellbore at  $x=x_o$ , and wellbore storage and skin effect are not considered for the time being:

$$\left. \frac{\partial p_f}{\partial x} \right|_{x=x_o} = \frac{q \mu_g B}{86.4k_f w_f h} \quad (2)$$

Where  $q$  is gas well production,  $\text{m}^3/\text{d}$ ;  $B$  is volume factor,  $\text{m}^3/\text{m}^3$ ;  $w_f$  is the width of the fracture region, m;  $h$  is the reservoir thickness, m.

Considering that the cave is an equipotential body filled with gas, the loss of pressure wave at the joint between the fracture region and the cave is ignored. Therefore, at  $x=x_b$ , the internal pressure  $p_v$  of the cave is equal to the pressure at the joint:

$$p_f \Big|_{x=x_b} = p_v \quad (3)$$

Considering the elastic expansion of gas in the cave, the flow at the joint ( $x=x_b$ ) can be considered as:

$$\left. \frac{\partial p_f}{\partial x} \right|_{x=x_b} = \frac{\mu_g}{86.4k_f w_f h} \left( -24 \times \frac{4}{3} \pi R_v^3 \phi_v c_{iv} \frac{dp_v}{dt} \right) \quad (4)$$

Where  $p_v$  is pressure in the cave, MPa;  $R_v$  is radius of cave, m;  $\phi_v$  is the cave porosity;  $c_{iv}$  is cave region total compressibility, 1/MPa.

At the initial moment, the pressure at the bottom of the well, the fracture region and the cave are equal:

$$p_f \Big|_{t=0} = p_w \Big|_{t=0} = p_v \Big|_{t=0} = 0 \quad (5)$$

### 2.2.2 Solution of the mathematical model

Introducing the following dimensionless variables:

$$p_{fD} = \frac{86.4 \times 2\pi k_r h (p_i - p_f)}{q \mu_r B} \quad (6)$$

$$t_D = \frac{3.6k_r t}{\phi_r \mu_r c_{ir} r_w^2} \quad (7)$$

$$\eta_{fD} = \frac{k_f / \phi_f \mu_g c_{if}}{k_r / \phi_r \mu_r c_{ir}} \quad (8)$$

$$F_{CD} = (k_f w_f)_D = \frac{k_f w_f}{k_r r_w} \quad (9)$$

$$C_{vD} = \frac{4/3 \pi R_v^3 \phi_v c_v}{2\pi h r_w^2 \phi_r c_{ir}} \quad (10)$$

$$x_D = \frac{x}{r_w} \quad (11)$$

Where  $p_{fD}$  is dimensionless pressure of fracture region;  $t_D$  is dimensionless time;  $F_{CD}$  is dimensionless conductivity of fracture region;  $C_{vD}$  is dimensionless storage coefficient of the cave;  $x_D$  is dimensionless length;  $r_w$  is the wellbore radius, m;  $k_r$  is reference permeability,  $\mu\text{m}^2$ ;  $\mu_r$  is reference viscosity, mPa·s;  $\phi_r$  is reference porosity;  $c_{ir}$  is reference total compressibility, 1/MPa.

Substituting the above dimensionless variables into the mathematical model, the dimensionless mathematical model is obtained as follows:

$$\left\{ \begin{array}{l} \frac{\partial^2 p_{fD}}{\partial x_D^2} = \frac{1}{\eta_{fD}} \frac{\partial p_{fD}}{\partial t_D} \quad (x_{aD} \leq x_D \leq x_{bD}) \\ \left. \frac{\partial p_{fD}}{\partial x_D} \right|_{x_D=x_{oD}} = -\frac{2\pi}{(k_f w_f)_D} \\ p_{fD} \Big|_{x_D=x_{bD}} = p_{vD} \\ \left. \frac{\partial p_{fD}}{\partial x_D} \right|_{x_D=x_{bD}} = -\frac{2\pi}{(k_f w_f)_D} C_{vD} \frac{dp_{vD}}{dt_D} \\ p_{fD} \Big|_{t=0} = p_{wD} \Big|_{t=0} = p_{vD} \Big|_{t=0} = 0 \end{array} \right. \quad (12)$$

The mathematical model in Laplace space can be obtained as follows after Laplace transformation:

$$\left\{ \begin{array}{l} \frac{\partial^2 \bar{p}_{fD}}{\partial x_D^2} = \frac{s}{\eta_{fD}} \bar{p}_{fD} \quad (x_{aD} \leq x_D \leq x_{bD}) \\ \left. \frac{\partial \bar{p}_{fD}}{\partial x_D} \right|_{x_D=x_{oD}} = -\frac{2\pi}{(k_f w_f)_D} \frac{1}{s} \\ \left. \frac{\partial \bar{p}_{fD}}{\partial x_D} \right|_{x_D=x_{bD}} = -\frac{2\pi}{(k_f w_f)_D} C_{vD} s \bar{p}_{vD} \\ \bar{p}_{fD} \Big|_{x_D=x_{bD}} = \bar{p}_{vD} \\ \bar{p}_{fD} \Big|_{t=0} = \bar{p}_{wD} \Big|_{t=0} = \bar{p}_{vD} \Big|_{t=0} = 0 \end{array} \right. \quad (13)$$

### 3. RESULTS

On the basis of the well-cave-fracture-cave-fracture model, the dimensionless fracture pressure solution in the Laplace space can be derived as follows using a similar solution method:

$$\bar{p}_{fD} = C_1 e^{ax_D} + C_2 e^{-ax_D} \quad (14)$$

Where

$$C_1 = \frac{c(a+b)}{a(a+b)e^{ax_{bD}} - a(a-b)e^{2ax_{bD} - ax_{aD}}}$$

$$C_2 = \frac{a-b}{a+b} e^{2ax_{bD}} C_3 \quad \left( a = \sqrt{\frac{s}{\eta_{fD}}}, \quad b = -\frac{2\pi}{(k_f w_f)_D} C_{vD} s \right)$$

$$c = -\frac{2\pi}{(k_f w_f)_D} \frac{1}{s}$$

Considering the storage flow of caves at the front end of fracture region, the dimensionless solution of cave pressure is obtained by substituting the pressure of fracture region into the following formula:

$$\bar{p}_{vD} = \frac{(s\bar{p}_{fD} + S)}{s + C_{vD} s^2 (s\bar{p}_{fD} + S)} \quad (15)$$

Where  $\bar{p}_{vD}$  is the dimensionless cave pressure in Laplace space.

Considering the influence of wellbore storage effect and skin factor, dimensionless bottom hole pressure in Laplace space is obtained:

$$\bar{p}_{wD} = \frac{s\bar{p}_{fD} + S}{s + C_D s^2 (s\bar{p}_{fD} + S)} \quad (16)$$

To derive the dimensionless solution of cave pressure under the well-cave-fracture-cave-fracture model, the storage flow of caves is taken into account at the front end of the fracture region, and the pressure of the fracture region is substituted into the equation(15). Subsequently, the dimensionless bottom hole pressure is solved using Equation(16), which takes into account the wellbore storage effect and skin factor.

### 4. DISCUSSION

#### 4.1 analysis of flow characteristics

Using the Stehfest numerical inversion method, the dimensionless bottom hole pressure solution in the Laplace space is transformed into the real space, and the type curve of the dimensionless bottom hole pressure and pressure derivative is depicted as shown in Figure 2.

Eight flow stages also can be identified:

(1) The first flow regime is the wellbore storage flow;

(2) The second flow regime is transition flow from the cave to the wellbore;

(3) The third flow regime is cave storage flow, which is a pseudo-steady flow with a slope of 1;

(4) The fourth flow regime is transition flow from the fracture region to the cave, which has a "concave" shape;

(5) The fifth flow regime is linear flow of fracture region, and the dimensionless pressure and pressure derivative curve has a slope of 1/2.

(6) The sixth flow regime is transition flow from cave 2 to the fracture region;

(7) The seventh flow regime is cave storage flow;

(8) The eighth flow regime is linear flow of fracture region, and the slope of the dimensionless pressure and pressure derivative curve is 1/2.

#### 4.2 sensitivity analysis of type curves

The impacts of fracture region length, fracture region conductivity, and cave volume on the type curves are investigated, which is utilized to examine the sensitivity of fracture region and cave parameters.

##### 4.2.1 Effect of length of fracture region

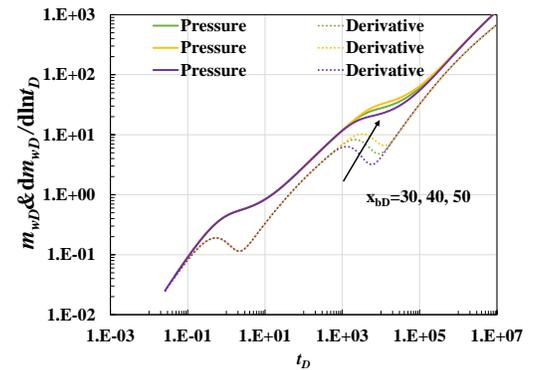


Fig. 3. Effect of fracture length on type curves

It can be seen from Figure 3 that the fracture length affects the duration of linear flow. The longer the fracture region is, the longer the linear flow lasts, and the later it enters the flow regime of cave storage flow.

#### 4.2.2 Effect of fracture conductivity

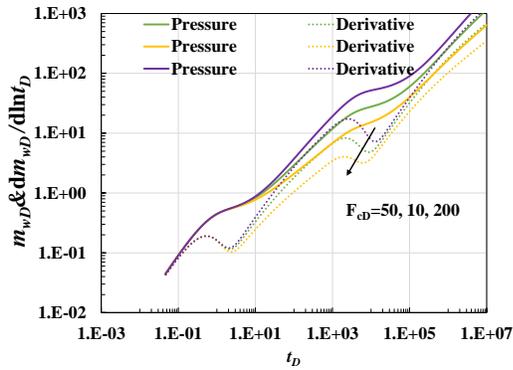
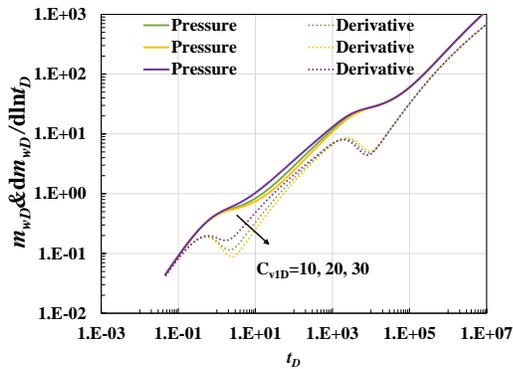


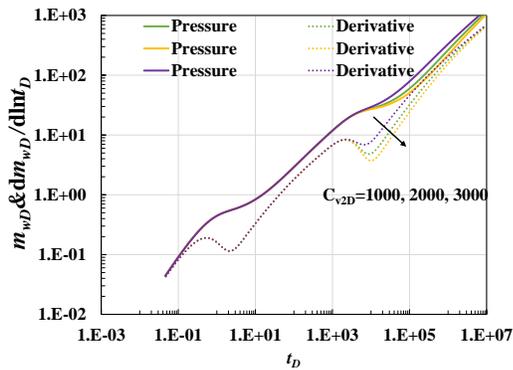
Fig. 4. Effect of fracture conductivity on type curves

It can be seen from Figure 4 that the fracture conductivity mainly affects the start of linear flow. If the fracture region has a higher conductivity, pressure will spread more quickly, cave 2 will supply more quickly. If the fracture region has a lower conductivity, pressure of cave 2 will supply more slowly, cave 1 will have longer storage flow, and linear flow may even stop altogether.

#### 4.2.3 Effect of volume of the cave



(a) Volume of cave 1



(a) Volume of cave 1

Fig. 5. Effect of volume of cave on type curves

From Figure 5, it can be seen that the volume of the cave affects the depth of the "concave". The volume of cave 1 affects the depth of the first "concave". The larger the volume, the stronger the supply capacity, the wider and deeper the "concave", and the closer the slope is to

1; The volume of cave 2 affects the depth of the second "concave". The larger the volume, the stronger the supply capacity, and the wider and deeper the "concave".

## 5. CONCLUSIONS

(1) This article is based on well-cave-fracture-cave-fracture connection modes, considering fracture regions and caves as the main storage spaces, and establishes pressure transient analysis models under complex fracture-caved connection modes, achieving effective inversion interpretation of wellbore, fracture regions, and caves related parameters.

(2) Sensitivity analysis is conducted on the length and conductivity of fracture regions and cave volume. It was found that the conductivity and length of fracture regions mainly affect the appearance and duration of linear flow, while the volume of caves mainly affects the storage flow.

(3) The longer the fracture region, the longer the duration of linear flow; The greater the conductivity of the fracture region, the earlier the appearance of linear flow; The larger the volume of the cave, the wider and deeper the "concave", and the stronger the supply capacity of the cave.

## ACKNOWLEDGEMENT

The authors acknowledge that this study was partially supported by the National Natural Science Foundation of China (No. 52174038 and No. 52004307), China Petroleum Science and Technology Project-major project-Research on tight oil-shale oil reservoir engineering methods and key technologies in Ordos Basin (ZLZX2020-02-04) and Science Foundation of China University of Petroleum, Beijing (No. 2462018YJRC015).

## DECLARATION OF INTEREST STATEMENT

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. All authors read and approved the final manuscript.

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