

Optimal Design of Complex Large-scale Heat Exchanger Networks

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ABSTRACT

Efficiently solving large-scale heat exchanger networks (HENs) remains a changing task due to the non-convex and nonlinearity characteristics. A novel methodology for optimal design of HEN combined structure design model based on pinch analysis and automatically converted the structure into associating topology in the superstructure is developed, subsequently as the initial point to optimize the superstructure-based model. Various InHEN structures obtained by changing ΔT_{\min} , and results of initial HEN (InHEN) model satisfy theoretical utility. The conversion model uses a temperature comparison-based method to locate the exchanger position, and distance-based method proposed in this study achieve updating the location. The reduced MINLP to NLP model through the strategy of replacing binary variables with absolute values in the superstructure is presented. A weakening strategy is proposed to simplify the computation of temperature differences. The TAC as the objective is then solved with various initial points, and the near globally optimal solution is obtained. The framework of optimization with initial points is named PinNLP model. The complex large-scale case is investigated for proving the method's applicability and advantages in solving quality and time, and the results show that the optimal is close to the global optimal solution. The proposed methodology can be applied to industrial HEN synthesis and thus has a wide space of applications.

Keywords: heat exchanger networks, pinch analysis, mathematical modeling, NLP, design optimization

NONMENCLATURE

Abbreviations

HENs	Heat exchanger networks
InHEN	Initial HEN
TAC	Total annual cost
NLP	Nonlinear programming
PinNLP	Pinch-based NLP
<i>Symbols</i>	
i	Set of hot streams
ia	Alias for set i
j	Set of cold streams
ja	Alias for set j
k	Set of stage in the superstructure
s	Set of splitting number for streams
sa	Alias for set s
ΔT_{\min}	Minimum approach temperature
$\Delta T_{i,j,k}^h$	Temperature difference at the hot side of exchanger ijk
$\Delta T_{i,j,k}^c$	Temperature difference at the cold side of exchanger ijk
$T_{i,k}$	Temperature of hot stream i between stage k and $k+1$
$T_{j,k}$	Temperature of cold stream j between stage k and $k+1$
$T_{i,j,k}^{hk}$	Temperature of hot stream branch after exchanger ijk
$T_{i,j,k}^{ck}$	Temperature of cold stream branch after exchanger ijk
OPS	Operation spending
OPSCU	Spending for cold utility
OPSHU	Spending for hot utility
CAPSP	Capital spending
ACAPSP	Additional capital spending
HEA	Fixed charge
HEB	Area expense coefficient
HEC	Exponent for area expense
Q	Heat load
$EAR_{i,j,k}$	Area of exchangers

$SHE_{i,s}$	Anticipated stages corresponding to hot stream sides of the exchanger
$SHE_{j,s}$	Anticipated stages corresponding to cold stream sides of the exchanger
NHE	Number of exchangers
AF	Annualized factor
CPINT	Cold pinch temperature
HPINT	Hot pinch temperature
T_{ci}	Inlet temperature of cold streams
T_{ho}	Outlet temperature of hot streams

1. INTRODUCTION

With the increasing demand of energy in human activities, energy consumes and environmental concerns such as greenhouse effect, highly efficient energy-saving methods have raised more and more attention in recent years. This behavior contributes to energy conservation and emission reduction under the background of the global hot topic with carbon neutrality and the requirement of sustainable development, especially in energy-intensive process enterprise [1], such as petrochemical sites [2], natural gas chemical complexes, carbon dioxide capture and based plants [3]. As one of the most extensively used heat recovery pathways in industries, heat exchanger networks (HENs) acting as a crucial role have been widely investigated, and yet HEN synthesis is an NP-hard problem in a strong sense [4]. Consequently, obtaining a global or near-globally optimal solution for HEN synthesis is an extremely complex task because of both integer and continuous variables are contained in the formulation of the model, which becomes even more complicated when encountering complex large-scale HEN synthesis problems.

The HEN synthesis technology aims to design a topology through optimizing mainly by maximizing heat recovery or minimizing TAC for chemical and petrochemical industries. Other indicator like environmental evaluation can also be used as the objective to optimize HEN synthesis for the purpose of sustainability [5], while the effects of unfriendly environmental are usually hard to evaluate. Gundersen and Naess [6], Jezowski [7,8], Morar and Agachi [9] both have reviewed a comprehensive development introduction and advancement for HEN synthesis technology. Although the investigation for small- and medium-scale HEN synthesis problems is accomplished in depth and the solution can approach the optimal HEN synthesis within a relevant shot time, solving complex large-scale problems generally difficult to acquire global

optimal solution, even local solutions without initial points, which is still a significance challenge.

This study is investigated for solving complex large-scale HEN synthesis problems efficiently. A pinch analysis-based and superstructure-based design method is presented. The stage-wise superstructure [10] is modified and MINLP-reduced NLP model applies new computation approach for temperature differences. The new method consists of the InHEN model, structure conversion model, modified NLP model, temperature difference weakening strategy and the solution of PinNLP model with modified objective. First, the InHEN model provides initial points by changing ΔT_{min} with theoretical utility. Then the structures acquired above automatically are converted to topology superstructure-based, which is innovative and technically. Furthermore, the PinNLP model with the weakening strategy under the proposed optimal method is solved a near global optimal solution in a short time. Finally, a complex large-scale case with $31H \times 5C$ is studied for testing the effectiveness of the proposed method.

2. REDUCED NLP MODEL

2.1 New method for nonlinearization

A Traditional HEN synthesis usually places utilities directly on the side of hot and cold streams to simplify model scale and facilitate the solving. In this study, utilities are introduced into HEN synthesis as usual streams to restore actual industrial operation. All streams are allowed to split and exchange heat, except that heat exchange is not allowed between utility streams. The modified HEN superstructure is shown in Figure 1.

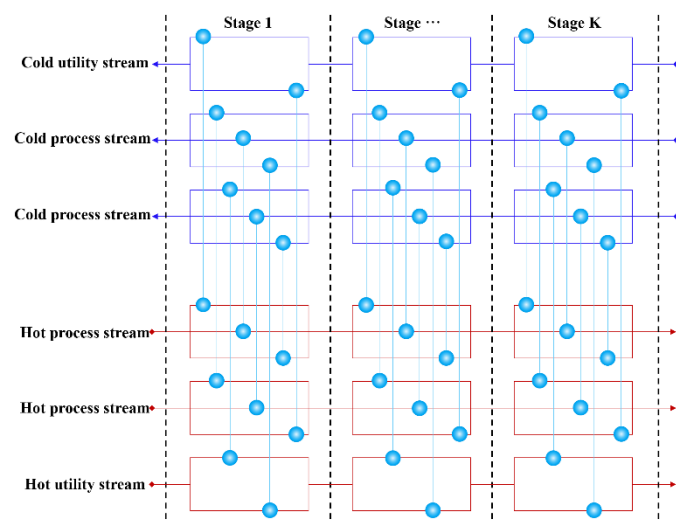


Fig. 1. Modified superstructure for HEN synthesis

The MINLP problem of HEN synthesis based on stage-wise superstructure is hardly difficult to solve due to highly non-convex and nonlinear, especially the temperature differences at both hot and cold sides. Duran and Grossmann [11] introduced a small positive number to overcome the non-differentiability bringing non-smooth term into objective function, but the premise is to ensure that the number is sufficiently small. Hence, a new simplified method of square first, then square root, and average is proposed in Equations. (1)-(2) to guarantee the nonnegativity of these terms.

$$\Delta T_{i,j,k}^h = \left[\sqrt{(T_{i,k} - T_{i,j,k}^{ck})^2} + (T_{i,k} - T_{i,j,k}^{ck}) \right] / 2 \quad (1)$$

$$\Delta T_{i,j,k}^c = \left[\sqrt{(T_{i,j,k}^{hk} - T_{j,k+1})^2} + (T_{i,j,k}^{hk} - T_{j,k+1}) \right] / 2 \quad (2)$$

2.2 Objective function

An economical trade-off between capital spending (CAPSP) and operation spending (OPS) is realized by optimizing the HEN synthesis, so to minimize TAC which includes hot and cold utilities and exchangers in whole HEN as follows.

$$TAC = OPS + CAPSP - ACAPSP \quad (3)$$

OPS includes the spending for cold utility (OPSCU) and hot utility (OPSHU), and both are annualized, indirectly determined by the product of utilities consumption and purchase spending (CCU and CHU), and the relevant computations are shown in Equations. (4)-(5). The expense for installment and equipment of hot and cold utility exchangers is straightly counted into purchase spending of utilities [12], which is consistent with practice.

$$OPSCU = \sum_{i \in PI} \sum_{j \in UJ} \sum_k (CCU \cdot Q_{i,j,k}) \cdot 3600 \cdot YHR \quad (4)$$

$$OPSHU = \sum_{i \in UI} \sum_{j \in PJ} \sum_k (CHU \cdot Q_{i,j,k}) \cdot 3600 \cdot YHR \quad (5)$$

CAPSP for exchangers except for utility exchangers contains HEA, HEB, HEC, as indicated in Equation. (6). In this reduced NLP model, CAPSP also counts the ACAPSP about non-existing exchangers in search domain, as represented in Equation. (7), Where NHE is the number of exchangers in optimal results. The increased HEA due to the absence of exchangers in the superstructure should be removed, as described in Equation. (3).

$$APSP = \sum_{i \in PI} \sum_{j \in PJ} \sum_k (HEA + HEB \cdot EAR_{i,j,k}^{HEC}) / AF \quad (6)$$

$$ACAPSP = \left\{ \frac{CARD(PI) \cdot CARD(PJ)}{(N_k - 1) - NHE} \right\} \cdot HEA / AF \quad (7)$$

3. CONVERSION METHODS

The InHEN model uses pinch analysis based on the principle of heat exchange matching from the pinch to both ends, and ultimately initial HEN flowsheet is obtained through the proposed method in this study. This flowsheet has the theoretical maximum heat exchange capacity. To make this flowsheet as the initial points to solve the reduced NLP model, it demands to be transformed into the corresponding topology in the superstructure.

Figure 2 illustrates the flowchart of determining the exchanger stage above the pinch. First, we distribute the exchanger stage away from the pinch, and then arrange it at the pinch. There are two situations in the relationship between $SHE_{i,s}$ and $SHE_{j,s}$, and it is also investigated whether the two streams heat exchange at the pinch are splitting or not. The alias of sets for hot streams, cold streams and exchanger location are indexed by ia, ja and sa, respectively. The conversion below the pinch also accomplished applies the similar methods.

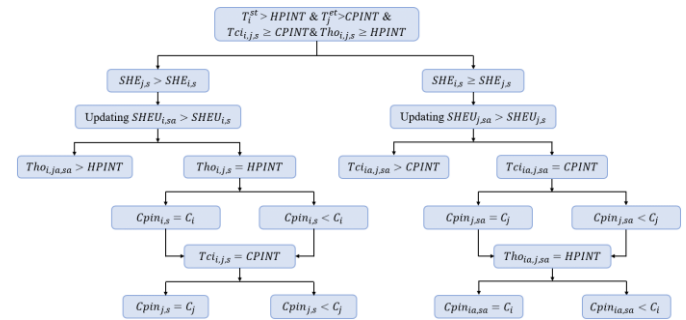


Fig. 2. Flowchart of conversion above the pinch

4. RESULTS AND DISCUSSION

4.1 Case description and data

This case is complex large-scale applying the heat medium water to recycle the low-grade heat from a petrochemical enterprise in China realized the energy saving, involving 31 hot streams (process streams i1-i30 and utility stream i31) and 5 cold streams (process streams j1-j4 and utility stream j5), which requires 72318 kW cold utility and 98840 kW hot utility to supply network energy, wherein the cold process streams are water. The streams data are given in Table 1.

Table 1 Streams data of case

Streams	T st (°C)	T ^{et} (°C)	Q (kW)	Cost (\$/MJ)
i1	128.4	48.6	2474	
i2	120.0	75.0	2295	
i3	117.0	51.0	2310	
i4	98.0	70.0	728	
i5	99.0	61.0	1026	
i6	95.0	75.0	380	
i7	245.0	170.0	8850	
i8	89.0	32.0	1083	
i9	110.0	58.0	2340	
i10	205.0	159.0	460	
i11	80.0	37.0	1720	
i12	95.0	68.0	1620	
i13	81.0	42.5	231	
i14	126.0	40.0	3784	
i15	130.0	40.0	3330	
i16	221.0	138.0	15936	
i17	110.0	80.0	330	
i18	110.0	35.0	1500	
i19	115.0	90.0	475	
i20	110.0	92.0	360	
i21	135.0	100.0	280	
i22	188.0	140.0	3600	
i23	125.0	44.0	1377	
i24	93.0	53.0	80	
i25	88.0	41.0	1175	
i26	121.0	35.0	2924	
i27	124.0	35.0	2937	
i28	170.0	45.0	7000	
i29	125.0	40.0	1020	
i30	159.0	60.0	693	
i31	1200.0	900.0	–	0.024
j1	60.0	90.0	20910	
j2	60.0	95.0	18305	
j3	60.0	105.0	26145	
j4	60.0	96.0	33480	
j5	20.0	30.0	–	0.003

4.2 A weakening strategy for temperature differences

Owing to the square root and bilinear characteristics in Equation (8), the difficulty of solving the model is greatly increased. We propose a weakening strategy to mitigate the calculation. The inequality Equation (9) is innovatively introduced to this study for simplifying the Equation (10). To ensure the $\Delta T_{i,j,k}$ in Equation (10) remains numerically stable, a very small positive tolerance ε , e.g. 10^{-10} , is added to the denominator to prevent division by zero. The $\Delta T_{i,j,k}$ is smaller than the value computed by Equation (8), and $EAR_{i,j,k}$ grows larger correspondingly. The objective is finally an upper bound of the original problem, and the solution of the primitive problem can be obtained by substituting the optimized

results into Equation (8). This strategy can solve complex large-scale HEN synthesis problems more conveniently.

$$\Delta T_{i,j,k} = \frac{2}{3} \cdot (\Delta T_{i,j,k}^h \cdot \Delta T_{i,j,k}^c)^{0.5} + \frac{1}{6} \cdot (\Delta T_{i,j,k}^h + \Delta T_{i,j,k}^c) \quad (8)$$

$$\sqrt{\Delta T_{i,j,k}^h \cdot \Delta T_{i,j,k}^c} \geq 2 / \left(\frac{1}{\Delta T_{i,j,k}^h} + \frac{1}{\Delta T_{i,j,k}^c} \right) \quad (9)$$

$$\Delta T_{i,j,k} = \frac{2}{3} \cdot 2 / \left(\frac{1}{\Delta T_{i,j,k}^h + \varepsilon} + \frac{1}{\Delta T_{i,j,k}^c + \varepsilon} \right) + \frac{1}{6} \cdot (\Delta T_{i,j,k}^h + \Delta T_{i,j,k}^c) \quad (10)$$

4.3 Optimization and solution

The solutions of the multiple scenarios in PinNLP model are investigated using topologies generated from InNLP model at different ΔT_{\min} as initial points, and several topologies are optimized within the reasonable time. Figure 3 illustrates the economic indicators obtained using the proposed method, while no feasible solution was found in several days, therefore it was not included in horizontal axis. Note that the ΔT_{\min} is continuous and only integers are taken, thus showing the discontinuous in horizontal axis. When the ΔT_{\min} is 12 °C, the objective TAC is minimum compared with other scenarios. The PinNLP model involves 19700 continuous variables, 23678 constraints including 17005 nonconvex nonlinear constraints, and 25484 nonlinear terms, of which 15673 are bilinear or quadratic.

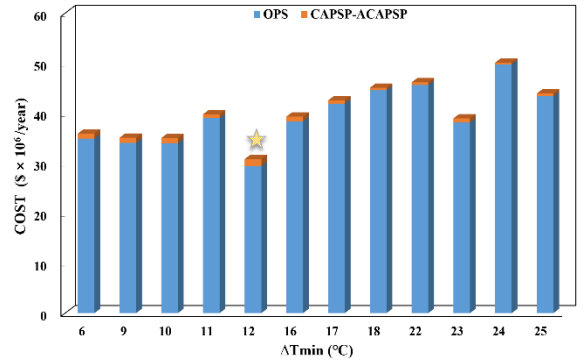


Fig. 3. Economic indicators with initial topology under different ΔT_{\min}

Figure 4 illustrates the topology of the optimal result in PinNLP model, 51 exchangers participating the heat exchange. It is reasonable that the hot process streams (i1 – i30) are without splitting as the result of excessive number, and multiple cold process streams are splitting into multiple branches.

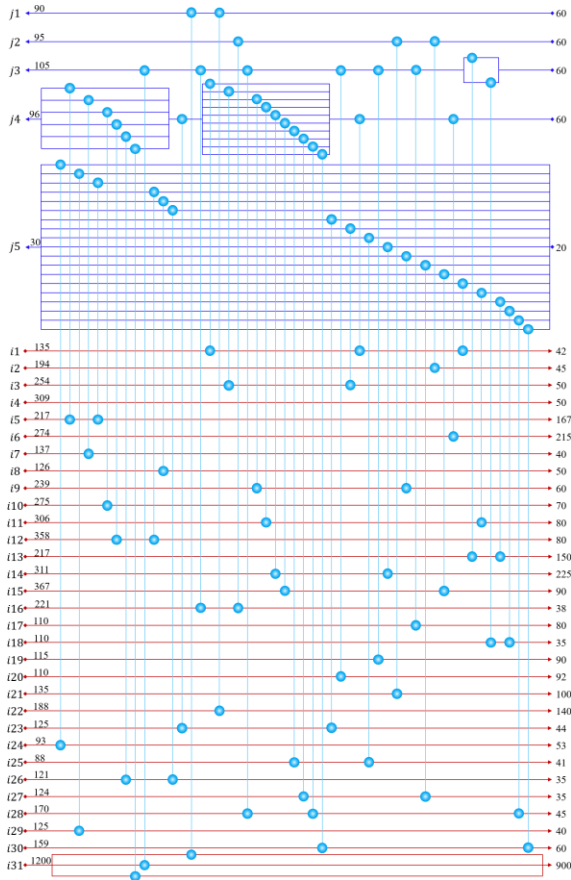


Fig. 4. The topology of solution in PinNLP model

4.4 Applicability and efficiency of proposed method

For proving that the proposed method is applicable for complex large-scale problems, the comparison of results in PinNLP and NLP models are listed in Table 2. When ΔT_{\min} is equal to 12 °C, the total stage number determined by InNLP model is 12, and the solution time of PinNLP model is 115.2 s, which is dramatically decreased in contrast to the NLP model. Various scenarios of NLP model are investigated, only four scenarios with small total stage number are obtained in reasonable time, while others are no feasible solution using MINOS and CONOPT within 100 hours. As we all know, as the total stage number in PinNLP model increases, the model scale will increase exponentially, especially for complex large-scale HEN synthesis problems, and the difficulty of solving also increase accordingly.

The NEX is 51 in the result of PinNLP, while it is reduced in the NLP model, resulting in more areas in PinNLP model. The more QR is achieved in PinNLP model, although the corresponding cost has the little increase of CAPSP–ACAPSP, and the OPS is greatly reduced, around 41.8%. The TAC is finally reduced to $\$30.82 \times 10^6$ /yr, up to 39.4% of the TAC in NLP model. When ΔT_{\min} is equal to

12 °C, the virtual temperature is 66 °C and the associating theoretical values of hot and cold utilities are 40618 kW and 10496 kW, respectively. Compared with the optimized results of PinNLP model, there are only a 0.3% and 0.8% difference. Consequently, the method proposed in this study can effectively provide various initial points to near the globally optimal solution, which can solve HEN synthesis problems especially complex large-scale situations.

Table 2 Comparison of results in PinNLP and NLP models

Items	PinNLP	NLP			
	$N_k = 12$	$N_k = 2$	$N_k = 3$	$N_k = 4$	$N_k = 5$
t (s)	116	209	644	5321	105482
NEX	51	35	35	36	39
EAR(m ²)	24380	17224	17141	17100	19134
QR (kW)	58109	33080	29087	30842	45545
OPS (\$ $\times 10^6$ /yr)	29.39	48.85	51.95	50.59	39.16
CAPSP–ACAPSP (\$ $\times 10^6$ /yr)	1.43	0.36	0.34	0.40	0.80
TAC (\$ $\times 10^6$ /yr)	30.82	49.21	52.29	50.99	39.96

5. CONCLUSIONS

The heat integration about HEN synthesis has become a widely used energy-saving technology applying for energy recovery and utilization, especially in energy-intensive process industry, but solving complex large-scale HEN synthesis has always been a challenge. This study proposes a novel method combing InHEN model based on pinch analysis and converting the generated flowsheet into topology in the superstructure as initial points to solve the complex large-scale HEN synthesis problems, and significantly improved the solution time and quality. Firstly, the MINLP-reduced NLP model based on superstructure for simplifying the problems is presented. Secondly, InHEN model based on pinch analysis is proposed for initial HEN design. Thirdly, the approach for topology obtained in InHEN model automatically converting into the associating topology in the superstructure is formulated. Ultimately, the PinNLP model is solved using various topologies through changing ΔT_{\min} in InHEN model as initial points. Through the optimal design and analysis for case study, several conclusions of this work are as follows:

(1) Multiple solution schemes for complex large-scale case with 31 hot streams and 5 cold streams are optimized within 100 hours under different ΔT_{\min} in InHEN model. When ΔT_{\min} is 12 °C, the solving time requires only 116 s, dramatically shorter than pure NLP model. More importantly, the optimal hot utility is 40732kW while theoretical value is 40618 kW, only a 0.3% deviation. The associating TAC is $\$30.82 \times 10^6$ /yr, resulting significantly reduction of 39.4% comparing with the NLP model.

(2) The optimal design for complex large-scale case acquires the feasible solution schemes verified practical significance and the proposed method can be well done. According to the results, both solution time and quality have been greatly improved, which would be valuable and credible for solving complex large-scale HEN synthesis in industrial practice.

To sum up, the new method of solving HEN synthesis especially for complex large-scale problems has been investigated in this study, efforts are still required for applying the method more realistically. In the future, this proposed methods for integration initial HEN design and automatically transforming into superstructure solving PinNLP model will concentrate on forming practical software involving the real-time optimization and control scheme optimal design for real industries.

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DECLARATION OF INTEREST STATEMENT

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. All authors read and approved the final manuscript.

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