

Robust online parameter identification for batteries using exponential resetting recursive least squares

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ABSTRACT

Parameter estimation methods based on recursive least squares (RLS) are extensively used in the online identification of battery models. Forgetting factor RLS (FFRLS), which can track the parameter changes online, has been regarded as an essential solution to real-time model adaptation. However, under the non-persistent excitation condition, the performance of FFRLS will degrade and the covariance windup phenomenon will be triggered, as a consequence, FFRLS will become numerically unstable and lose the capacity to provide reliable estimates. In this paper, a new scheme named exponential resetting RLS (ERRLS) is proposed to overcome the aforementioned shortcoming, the mechanism of information matrix updating is modified to guarantee exponential convergence towards a non-zero matrix under no excitation. This modification will result in a bounded covariance matrix whether the excitation is persistent or not, which implies an improved robustness in comparison with FFRLS. Experimental results indicate that ERRLS can achieve better performance than FFRLS when the persistent excitation condition is violated.

Keywords: battery modeling, equivalent circuit model, parameter identification, least squares, covariance resetting

NONMENCLATURE

Abbreviations

RLS	Recursive Least Squares
FFRLS	Forgetting Factor Recursive Least Squares
VFFRLS	Variable Forgetting Factor Recursive Least Squares
CNRLS	Condition Number based Recursive Least Squares
VDFRLS	Variable Direction Forgetting Recursive Least Squares

ERRLS	Exponential Forgetting Recursive Least Squares
BMS	Battery Management System
SoC	State of Charge
ECM	Electrical Circuit Model
PE	Persistent Excitation
<i>Symbols</i>	
λ	Forgetting Factor
e	Voltage Prediction Error
y	Output of estimator
ϕ	Input of estimator
x^*	Estimate of parameter vector
P	Covariance Matrix
R	Information Matrix
K	Gain Vector
I	Identity matrix
V_{oc}	Open Circuit Voltage
R_0	Ohmic Resistance
R_1	Polarization Resistance
C_1	Polarization Capacitance

1. INTRODUCTION

As one of the most prevalent energy storage devices, batteries are indispensable in our daily lives. The widespread deployment of batteries has promoted rapid development in relevant industry sectors, from consumer electronics, electrified transportation to grid energy storage [1]. Considering that batteries are vulnerable to harsh environments and abnormal usage, an advanced battery management system (BMS) is required to track battery states and optimize battery performance [2]. The model-based methods are appropriate choices for real-time monitoring and control purposes in BMS, particularly for the State of Charge (SoC) estimation [3]. The equivalent circuit models

(ECMs) consist of basic components like resistors, capacitors, and ideal voltage sources. They can be connected in different ways to form various topologies, which makes the modeling approach flexible and universal for various battery chemistries. Moreover, owing to the simple structure and the limited number of parameters, ECMs can be identified online with a low computational cost, and the timely update of model parameters will improve the accuracy and efficiency of state estimation [4].

The technique of least squares has long been applied in system identification problems. As the most practical version, recursive least squares (RLS) can achieve fast calculation with modest computational resources, and the obtained estimates is unbiased, consistent and efficient provided that the measurement noise is white. The characteristics of simple implementation, fast convergence and low complexity make RLS applicable to online time-invariant system identification. However, as has been pointed out by relevant literature, with the accumulation of successive measurements, the parameter correction ability of RLS will degenerate gradually, new observations can no longer contribute to updating model parameters, resulted in the phenomenon of “data saturation”. To overcome this inherent shortcoming and extend the utilization of RLS to real-time parameter tracking of time-variant systems, a forgetting factor λ is integrated into RLS. Through assigning heavier weights to more recent measurements, the estimates from RLS can converge to real parameter values exponentially under the persistent excitation condition, hence the slow-varying parameters can be estimated continuously to reveal their time-dependent variations [5, 6]. Since the change of battery states and environmental conditions will both influence the electrochemical processes inside batteries and lead to parameter variations in ECMs, the forgetting factor RLS (FFRLS) becomes an appropriate choice to achieve online parameter identification. It has been reported that SoC estimation frameworks based on FFRLS can fulfill sufficient accuracy for advanced battery management [7-9]. Nevertheless, the selection of a constant λ implies a trade-off between the ability of fast tracking and the robustness of estimator. FFRLS is able to provide smooth parameter trajectories when λ is close to 1, at the sacrifice of real-time capabilities. Otherwise, if a relatively small λ is used, parameter changes can be detected promptly due to accelerated forgetting, but the estimator also becomes sensitive to

measurement noises accordingly, conspicuous discontinuities in parameter trajectories will emerge, which reduces the reliability of parameter estimation and usually indicates a violation of prior knowledge on the corresponding system dynamics. Another serious problem lies in the numerical stability of FFRLS under non-ideal conditions, in case that the excitation is non-persistent, the old information is discarded exponentially but only a part of them can be supplemented by the newcoming measurements. At this time, the covariance matrix \mathbf{P}_k will grow continuously towards infinity, and the unbounded gain vector \mathbf{K}_k will amplify the disturbance of measurement noises greatly, inducing a numerically unstable parameter identification [10].

During the normal operation of batteries, it's impossible for the applied current profiles to always satisfy the persistent excitation (PE) condition. When batteries are under a quasi-steady state, e.g., constant current charging/discharging or idling, terminal voltage and load current are both stable in a relatively long time interval, which will trigger the aforementioned “covariance windup” phenomenon. In order to limit the expansion of the covariance matrix \mathbf{P}_k , a variety of variable forgetting factor RLS (VFFRLS) schemes have been proposed, with differences in the specific forgetting factor tuning mechanisms. For instance, at each time step, the forgetting factor can be determined to maintain an invariant information content within the covariance matrix \mathbf{P}_k [11], or adjusted according to the statistics of the error signal [12], etc. Particularly, when VFFRLS is adopted in the online parameter identification of battery ECMs, a prevalent strategy is shown by Eq. (1), in which the forgetting factor λ_k is explicitly correlated to the instantaneous voltage prediction error e_k [13].

$$\begin{cases} \alpha_k = 2^{\rho e_k^2} \\ \lambda_k = \lambda_{\min} + (1 - \lambda_{\min})^{\alpha_k} \end{cases} \quad (1)$$

Furthermore, the instantaneous error e_k is replaced by the average error within a preset sliding time window $\sum_{k=k_0}^{k_0+n} e_k^2 / n$, by this means, the variation of λ_k tends to be smooth and the dramatic change can be avoided when an impulsive error comes [14]. Eq. (2) shows another refined approach developed from Eq. (1), in which a sensitivity coefficient h is employed to represent the sensitivity of the forgetting factor λ_k w.r.t. the instantaneous error e_k , and the expected

voltage noise level e_{base} is integrated into the calculation of the exponent α_k . Due to an increased degree of freedom, the forgetting factor λ_k can be adjusted more meticulously [15].

$$\begin{cases} \alpha_k = \text{round} \left[\left(\frac{e_k}{e_{\text{base}}} \right)^2 \right] \\ \lambda_k = \lambda_{\text{min}} + (1 - \lambda_{\text{min}}) h^{\alpha_k} \end{cases} \quad (2)$$

However, such slight modifications doesn't alter the essential mechanism of VFRLS, so the achievable performance improvement is constrained, and the subjective intermediate parameter settings can greatly influence the behaviour of estimator, since no quantitative criteria have been proposed to guide the configuration of VFRLS algorithms.

Because λ_k is directly controlled by e_k rather than the mathematical properties of \mathbf{P}_k , the adaptive regulation of λ_k in VFRLS is aimed at reducing the instantaneous error, despite that it may contribute to alleviating the growth of \mathbf{P}_k to some extent concurrently, the actual effect is uncertain under practical applications. Several novel formulations of RLS have been reported to address ‘‘covariance windup’’ recently. For instance, a condition number based recursive least squares (CNRLS) have been proposed, in which the condition number of \mathbf{P}_k is utilized to evaluate the numerical stability of RLS estimator. By preventing the occurrence of an ill-conditioned covariance matrix, CNRLS can yield reliable estimates with less computational burden when the current excitation is non-persistent [16]. Variable-direction forgetting RLS (VDFRLS) is another example of improving the robustness of RLS from the perspective of \mathbf{P}_k . It have revealed that the correlation between the information matrix \mathbf{P}_k^{-1} and the system input ϕ_k can be analyzed in a vector space. Based on a geometric analysis, a variable-direction forgetting method has been presented to selectively forget old information according to the direction of newcoming data vector at each time step, thus the boundedness of the information matrix \mathbf{P}_k^{-1} can be ensured [17, 18]. In conclusion, covariance resetting and covariance modification are simple but effective measures to eliminate the risk of ‘‘covariance windup’’, since the covariance matrix \mathbf{P}_k will be corrected mandatorily once the upper bound is exceeded. However, theoretical guidance on the innovative algorithm design is still worth in-depth

exploration, and the potential impacts on the performance of RLS based algorithms need to be further clarified [19-22].

In this article, a new extension of FFRLS, namely exponential resetting recursive least squares (EFRLS), is adopted to cope with the reduced robustness of battery model parameter identification when the current excitation is non-persistent. The covariance matrix \mathbf{P}_k is endowed with well-designed resetting properties, it will spontaneously converge to a predefined positive definite matrix \mathbf{P}_∞ if the PE condition is not satisfied, rather than experience an explosive growth toward infinity. This significant change guarantees the boundedness of the covariance matrix \mathbf{P}_k under any circumstances. Actually, considering that the current profiles in practical battery applications are adaptive to real-time power demands, and can't be arbitrarily designed in advance to meet the PE condition, EFRLS shows superior practicality in comparison with conventional FFRLS for certain robustness without any external assistance.

2. EXPONENTIAL RESETTING RECURSIVE LEAST SQUARES

Eqs. (3)-(6) shows the main procedures of FFRLS, where e_k denotes the estimation error, \mathbf{y}_k and ϕ_k are the output and input of the RLS estimator, \mathbf{x}_k^* is the estimate of parameter vector, \mathbf{P}_k and \mathbf{K}_k represent the covariance matrix and the gain vector respectively.

$$\mathbf{e}_k = \mathbf{y}_k - \phi_k^T \mathbf{x}_{k-1}^* \quad (3)$$

$$\mathbf{P}_k = \frac{1}{\lambda} \left[\mathbf{P}_{k-1} - \frac{\mathbf{P}_{k-1} \phi_k \phi_k^T \mathbf{P}_{k-1}}{\lambda + \phi_k^T \mathbf{P}_{k-1} \phi_k} \right] \quad (4)$$

$$\mathbf{K}_k = \mathbf{P}_k \phi_k \quad (5)$$

$$\mathbf{x}_k^* = \mathbf{x}_{k-1}^* + \mathbf{K}_k \mathbf{e}_k \quad (6)$$

In order to get an intuitive understanding of the adaptive process, the information matrix $\mathbf{R}_k = \mathbf{P}_k^{-1}$ can be utilized, and an equivalent form of Eq. (4) can be expressed as

$$\mathbf{R}_k = \lambda \mathbf{R}_{k-1} + \phi_k \phi_k^T \quad (7)$$

the core mechanism of forgetting in RLS is exhibited explicitly in Eq. (7), with a forgetting factor $\lambda \in (0, 1)$, \mathbf{R}_k has a tendency to shrink exponentially to 0, while the addition of $\phi_k \phi_k^T$ at each time step contributes in the opposite direction. To put it simply, Eq. (7) describes the information update process, old information is forgotten gradually, and new information from recent measurement data is supplemented continuously to

ensure sufficient, fresh information content which plays an important role in online parameter estimation.

Motivated by Eq. (7), we would like to consider a new way of information matrix updating:

$$\begin{aligned} \mathbf{R}_k &= \mathbf{R}_{k-1} - (1 - \lambda)(\mathbf{R}_{k-1} - \mathbf{R}_\infty) + \boldsymbol{\phi}_k \boldsymbol{\phi}_k^T \\ &= \lambda \mathbf{R}_{k-1} + (1 - \lambda) \mathbf{R}_\infty + \boldsymbol{\phi}_k \boldsymbol{\phi}_k^T \end{aligned} \quad (8)$$

where \mathbf{R}_∞ is a specified positive infinite matrix, and it can be easily demonstrated that \mathbf{R}_k is positive definite for $k \geq 0$ provided that \mathbf{R}_0 is positive definite. A tight lower bound of \mathbf{R}_k can be given directly as

$$\mathbf{R}_k \geq \lambda^k \mathbf{R}_0 + (1 - \lambda^k) \mathbf{R}_\infty \quad (9)$$

Note that the persistent excitation condition is no longer needed in the derivation of Eq. (9). Similarly, a tight upper bound of \mathbf{R}_k can be obtained, suppose that there exists $\beta \geq 0$ such that $\boldsymbol{\phi}_k \boldsymbol{\phi}_k^T \leq \beta \mathbf{I}$ holds for $k \geq 0$, then

$$\mathbf{R}_k \leq \lambda^k \mathbf{R}_0 + (1 - \lambda^k) \mathbf{R}_\infty + \frac{1 - \lambda^k}{1 - \lambda} \beta \mathbf{I} \quad (10)$$

Rigorous proofs of Eqs. (9)-(10) can be found in a relevant paper [23].

According to the aforementioned conclusions, the boundedness of the information matrix \mathbf{R}_k can be guaranteed as long as the system input is bounded. Under a special case that $\boldsymbol{\phi}_k = 0$, \mathbf{R}_k will converge to \mathbf{R}_∞ rather than reduce to 0. As a result, the gain vector \mathbf{K}_k in Eq. (5) will be limited within an acceptable range. By this means, the sensitivity to noise can be well-controlled no matter the excitation is persistent or not. This valuable property will improve the performance of FFRLS based online parameter identification under non-ideal conditions, and enhance the applicability of ECM based state estimation methods designed for industrial use.

3. CASE STUDY

A 3 Ah Li-ion rechargeable cell INR18650/33V produced by EVE Energy CO. LTD. is tested on a high precision battery test platform, which consists of a programmable temperature chamber, a host computer and a battery test system. The experimental data are logged at a time interval of 1s. First, the cell is fully charged under constant current - constant voltage mode, then rested for 2h to achieve a steady state, finally, successive dynamic stress test (DST) cycles are conducted to discharge the cell until the cut-off voltage is reached, and a 60s rest period is inserted between two

adjacent discharging profiles. The load current and cell voltage are shown in Fig. 1.

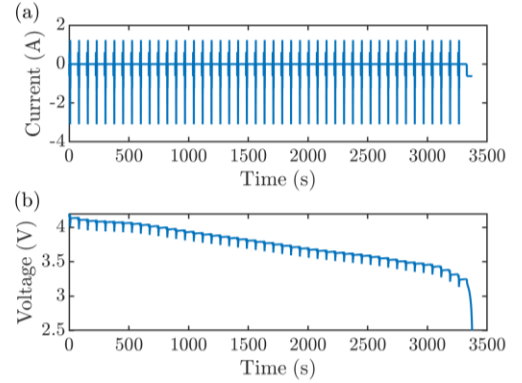


Fig. 1. Experimental results of (a) current and (b) voltage under DST cycles.

The first-order ECM that comprises an ideal voltage source (V_{oc}), an ohmic resistance (R_0) and a parallel RC network (R_1 and C_1) is selected to characterize battery voltage dynamics. After model discretization, a regression model expressed as

$$\begin{aligned} V_{t,k} &= a_1 V_{t,k-1} + (1 - a_1) V_{oc,k} + b_1 I_k + b_2 I_{k-1} \\ &= \boldsymbol{\phi}_k^T \boldsymbol{\theta}_k \end{aligned} \quad (11)$$

can be obtained [24], where

$$\begin{aligned} \boldsymbol{\phi}_k &= [V_{t,k-1} \quad I_k \quad I_{k-1} \quad 1]^T \\ \boldsymbol{\theta}_k &= [a_1 \quad b_1 \quad b_2 \quad (1 - a_1) V_{oc,k}]^T \end{aligned} \quad (12)$$

Generally, the model is similar to the autoregressive models with extra inputs in time series analysis, which can be identified by the proposed EFRLS. Let $\lambda = 0.99$, $\boldsymbol{\theta}_0 = [0 \quad 0 \quad 0 \quad 0]^T$, $\mathbf{P}_0 = \mathbf{P}_\infty = \mathbf{I}_{4 \times 4}$, the results of parameter identification is presented in Fig. 2.

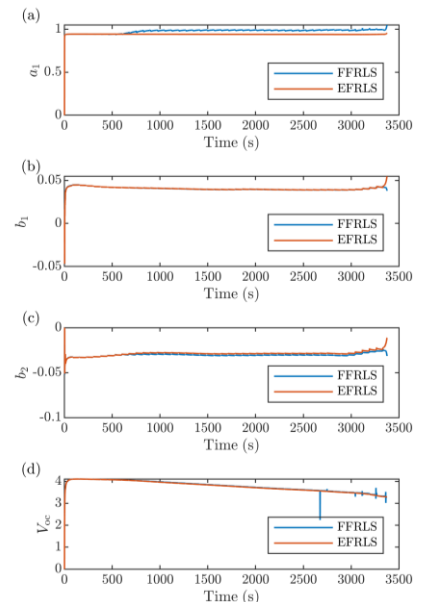


Fig. 2. Parameter estimates under DST cycles.

The Frobenius norm of P_k is used as a measure of numerical stability, along with cell voltage prediction error, are depicted in Fig. 3. By evaluating the volatility in parameter estimates and the norm of the covariance matrix, it can be concluded that ERRLS is more robust and accurate than FFRLS.

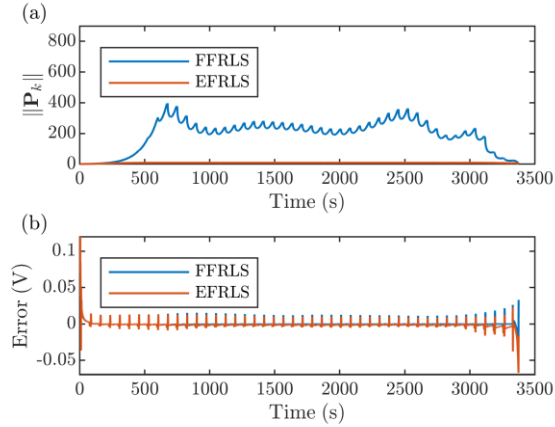


Fig. 3. (a) the Frobenius norm of covariance matrix and (b) voltage error under DST cycles.

4. CONCLUSIONS

In this paper, aimed at improving the robustness of FFRLS under non-persistent excitation, the conventional scheme of updating the covariance matrix is modified to prevent the covariance windup phenomenon. Instead of growing towards infinity, the covariance matrix will be limited within a preset range when the persistency of excitation is lost. A new extension of RLS based on the aforementioned covariance updating mechanism is proposed, thanks to its exponential resetting property, the covariance matrix is always bounded, regardless of the specific excitation condition. Hence, when compared with FFRLS, the proposed exponential resetting RLS (ERRLS) is more robust against noises, and has the ability to provide more reliable parameter estimates under complicated conditions, which is extremely important in online parameter identification of battery models. Theoretical analysis and experimental verification are conducted, and the superiority of ERRLS in robustness and accuracy is confirmed by the obtained results. It's believed that the proposed method will offer some new insight into the solution of real-time parameter and state estimation for batteries.

ACKNOWLEDGEMENT

This work is supported by the Science and Technology Project of State Grid Corporation of China (Grant No. 5419-202199552A-0-5-ZN) and the National

Natural Science Foundation of China (Grant No. 51977097, 52277217, 52337009).

DECLARATION OF INTEREST ATATEMENT

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. All authors read and approved the final manuscript.

REFERENCE

- [1] Olabi A. G., Abbas Q., Shinde P. A., & Abdelkareem M. A. Rechargeable batteries: Technological advancement, challenges, current and emerging applications. *Energy* 2023; 266:126408.
- [2] Hu X., Fei F., Liu K., Zhang L., Xie J., & Liu B. State estimation for advanced battery management: Key challenges and future trends. *Renewable and Sustainable Energy Reviews* 2019; 114:109334.
- [3] Shrivastava P., Soon T. K., Idris M. Y. I. B., & Mekhilef S. Overview of model-based online state-of-charge estimation using Kalman filter family for lithium-ion batteries. *Renewable and Sustainable Energy Reviews* 2019; 113:109233.
- [4] Wang Y., Tian J., Sun Z., Wang L., Xu R., Li M., & Chen Z. A comprehensive review of battery modeling and state estimation approaches for advanced battery management systems. *Renewable and Sustainable Energy Reviews* 2020; 131:110015.
- [5] Islam S. A. UL, & Bernstein D. S. Recursive least squares for real-time implementation. *IEEE Control Systems* 2019; 39(3):82-85.
- [6] Johnstone R. M., Johnson Jr C. R., Bitmead R. R., & Anderson B. D.O. Exponential convergence of recursive least squares with exponential forgetting factor. *System & Control Letters* 1982; 2(2):77-82.
- [7] Liu G., Xu C., Li H., Jiang K., & Wang K. State of charge and online model parameters co-estimation for liquid metal batteries. *Applied Energy* 2019; 250:677-684.
- [8] Xia B., Lao Z., Zhang R., Tian Y., & Chen G. Online parameter identification and state of charge estimation of lithium-ion batteries based on forgetting factor recursive least squares and nonlinear Kalman filter. *Energies* 2018; 11(1):3.
- [9] Xiao R., Hu Y., Jia X., & Chen G. A novel estimation of state of charge for the lithium-ion battery in electric vehicle without open circuit voltage experiment. *Energy* 2022; 243:123072.

- [10] Lai B., & Bernstein D. S. Generalized forgetting recursive least squares: stability and robustness guarantees. arXiv Preprint 2023.
- [11] Fortescue T. R., Kershenbaum L. S., & Ydstie B. E. Implementation of self-tuning regulators with variable forgetting factors. *Automatica* 2023; 17(6):831-835.
- [12] Paleologu C., Benesty J., & Ciocinã S. A robust variable forgetting factor recursive least-squares algorithm for system identification. *IEEE Signal Processing Letters* 2008; 15:597-600.
- [13] Lao Z., Xia B., Wang W., Sun W., Lai Y., & Wang M. A novel method for lithium-ion battery online parameter identification based on variable forgetting factor recursive least squares. *Energies* 2018; 11(6):1358.
- [14] Song Q., Mi Y., & Lai W. A novel variable forgetting factor recursive least square algorithm to improve the anti-interference ability of battery model parameters identification. *IEEE Access* 2019; 7:61548-61557.
- [15] Sun X., Ji J., Ren B., Xie C., & Yan D. Adaptive forgetting factor recursive least square algorithm for online identification of equivalent circuit model parameters of a lithium-ion battery. *Energies* 2019; 12(12):2242.
- [16] Kim M., Kim K., & Han S. Reliable online parameter identification of li-ion batteries in battery management systems using the condition number of the error covariance matrix. *IEEE Access* 2020; 8:189106-189114.
- [17] Goel A., Bruce A. L., & Bernstein D. S. Recursive least squares with variable-direction forgetting compensating for the loss of persistency. *IEEE Control Systems* 2020; 40(4):80-102.
- [18] Cao L., & Schwartz H. A directional forgetting algorithm based on the decomposition of the information matrix. *Automatica* 2000; 36(11):1725-1731.
- [19] Salgado M. E., Goodwin G. C., & Middleton R. H. Modified least squares algorithm incorporating exponential resetting and forgetting. *International Journal of Control* 1988; 47(2):477-491.
- [20] Goodwin G. C., Elliott H., & Teoh E. K. Deterministic convergence of a self-tuning regulator with covariance resetting. *IEE Proceedings D (Control Theory and Applications)* 1983; 130(1):6-8.
- [21] Kraus F. J. Stabilized least squares estimators for time-variant processes. *Proceedings of the 28th IEEE Conference on Decision and Control* 1989; 2:1803-1804.
- [22] Park D. J., & Jun B. E. Selfperturbing recursive least squares algorithm with fast tracking capability. *Electronics Letters* 1992; 28(6):558-559.
- [23] Lai B., & Bernstein D. S. Exponential resetting and cyclic resetting recursive least squares. *IEEE Control Systems Letters* 2023; 7:985-990.
- [24] Liu Z., Dang X., Jing B., & Ji J. A novel model-based state of charge estimation for lithium-ion battery using adaptive robust iterative cubature Kalman filter. *Electric Power Systems Research* 2019; 177:105951.