

# Unified Energy-Space Modelling of Multi-Energy-Carrier Systems applied to Industrial HVAC

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**Abstract— Modern infrastructure reflects energy nexus of multiple sources: gas, electricity, water & air. Different energy systems are modelled by unique tools but suffer from complexity, high computational burden & low accuracy due to lack of measured data for modelling. Many involve closed tools, making them inapplicable for other energy systems. A unified modelling is needed which interfaces different energy carriers under common modelling variables. Energy & power are common parameters that are attributed with all energy systems: chemical, thermal, electrical & mechanical. This paper contributes towards the development of a unified modelling of the multi-energy systems followed by its direct application on an industrial HVAC system, which can further be used in efficient and optimal control of the system for cost, energy, and carbon savings.**

**Keywords— multi-energy carriers, unified modelling, industrial HVAC systems, energy modelling.**

## I. INTRODUCTION

Both developing and developed nations reflect an energy nexus of multiple carriers such as gas, electricity, water, and air. Different tools and methods have been used in the past to model these systems, however, they often suffer from complexity, high computational burden, and low accuracy due to lack of measured data for system modelling. In order to solve global energy crisis, reduce carbon emissions and save costs, a unified modelling approach, which can interface different energy carriers under a common umbrella of modelling variables, is the need of the hour. Energy and power are common parameters that all energy systems are attributed with, including chemical, thermal, electrical, or mechanical systems. A novel energy-power based modelling approach for multi-energy systems presented here builds on the long-term research and teaching based on multi-layered modelling which maps specific internal physical processes and their local automation into aggregate variables relevant for specifying interfaces between components [1]-[3]. The core of the proposed modelling recognizes well-known analogies of effort-flow variables across different energy

carriers: these are voltage and current for electrical energy; pressure and volume for fluid energy; temperature and entropy for thermal energy; force and momentum for mechanical energy [1]. Using these analogies power and energy are derived, and used as specifications at the component interfaces, much the same way as in well-known bond graphs founded on observing the First Law of Thermodynamics [4]. The proposed unified modelling, in addition, observes Second Law of Thermodynamics and, as such it is essential for establishing feasibility, stability and efficiency conditions when interconnecting novel technologies into an interconnected multi-energy system [5]. As such, it is essential for efficient control design. In the present work, we apply this novel energy-based approach to model a multi-energy industrial system which interfaces air-water-gas-electricity energy-nexus. This approach not only unifies modelling across different energy spectrum but also models the mutual effects of individual industrial sub-systems without extensive measurements which makes it a straight-forward retrofit for existing industrial systems to facilitate modelling with scarce measurement ports. With the help of the proposed energy model we show here, how different industrial systems such as: commercial heating, ventilation, and air conditioning (HVAC) units comprising of interconnected air, electricity, and water network; and energy stations comprising of interconnected gas, electricity, and water network with several intermediate heat exchanging coils; can all be modelled in a unified manner with energy and power interface variables. This is made possible by identification of different energy carriers interfacing each sub-system component. Different energy carriers may have one or more energy source interacting with each other, for example: electricity has electrical energy; gas has fluid energy; water has both thermal and fluid energy and air has both thermal and fluid energy. Each subsystem component like an electric water chiller interfaces electricity and water, with this basic understanding of the chiller operation, interface power and energy variables are derived using the energy model equations, thereby modelling the component without extensive knowledge of all internal component

modelling parameters and complete physical operation understanding. This unique feature of the unified energy model leads to an open modelling tool, applicable to multi-energy systems and accessible by people from different fields who merge towards common energy efficiency improvement goals. In this paper we provide a mathematical formulation of multi-energy systems using unified energy-space parameters, including the definition of power and energy variables for all electricity, gas, water, and air networks. We further provide a case study of industrial HVAC system and show its overall system energy flow diagram consisting of water flow network in the energy station consisting of: electric water chiller, gas absorption chiller, gas water boiler and storage tank; heat exchangers consisting of: coil for cooling or heating air using water coming from the energy station; air network consisting of: air handling unit (AHU), fan coil unit (FCU) and variable refrigerant volume (VRV) which take heating or cooling to the environmental zone. Overall, a novel approach is presented which maps specific internal physical processes into aggregate variables while specifying interfaces between components.

## II. UNIFIED ENERGY MODELLING

### A. Defining effort-flow variables in multi-energy systems

The proposed modelling recognizes analogies of effort-flow variables across different energy carriers [1]:

- Power hence derived is used as interface variable, like bond graphs based on First Law of Thermodynamics.
- Proposed model further observes Second Law of Thermodynamics for establishing feasibility, stability and efficiency conditions when interconnecting multi-energy system.
- It is essential for efficient control design and system optimization while achieving performance objective.

TABLE I. EFFORT AND FLOW VARIABLES WITH POWER AND RATE OF REACTIVE POWER VARIABLES IN MULTI-ENERGY

Energy Domain	Effort variable	Flow variable
Electric	Voltage	Current
Translational	Force	Velocity
Rotational	Torque	Angular velocity
Fluid	Pressure	Volume flow
Thermodynamic	Temperature	Entropy flow

- Real power and generalized reactive power rate computation for multi-energy systems:

$$\text{Electrical: } P = v \cdot i, Q = v \cdot di/dt - i \cdot dv/dt$$

$$\text{Mechanical: } P = F \cdot u, Q = F \cdot du/dt - u \cdot dF/dt$$

$$\text{Rotational: } P = \tau \cdot \omega, Q = \tau \cdot d\omega/dt - \omega \cdot d\tau/dt$$

$$\text{Fluid: } P = Pr \cdot vol, Q = Pr \cdot dvol/dt - vol \cdot dPr/dt$$

$$\text{Thermodynamic: } P = T \cdot S, Q = T \cdot dS/dt - S \cdot dT/dt$$

This novel energy-based approach is applied here to a multi-energy industrial HVAC system interfacing air-water-gas-electricity energy-nexus.

### B. Application to Industrial HVAC System

To apply the proposed unified energy modelling to the industrial heating ventilation and air conditioning (HVAC) system the decomposition of the full interconnected system into air flow, water flow and thermal subsystems is incorporated.

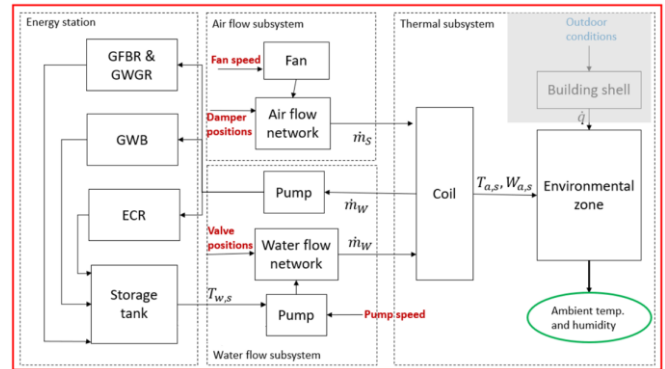


Fig. 1. Multi-energy industrial HVAC energy flow and interaction block diagram. Red denotes control inputs, blue exogenous disturbances and green output variables of interest.

The proposed approach models the commercial heating, ventilation, and air conditioning (HVAC) units with multi-energy stations in a unified manner with energy and power interface variables. This is made possible by the identification of different energy carriers interfacing each sub-system component. Different energy carriers may have one or more energy source interacting with each other as listed below:

- electricity has electrical energy.
- gas has fluid energy.
- water has both thermal and fluid energy.
- air has both thermal and fluid energy.

The airport HVAC system considered here has several subsystems, two major ones are namely: air-flow subsystem and water-flow subsystem. The energy model of the components of these two subsystems are discussed in the next two sections. The legend used in the diagrams to follow is shown in Fig. 2.

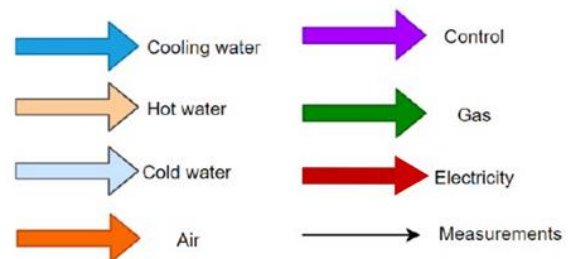


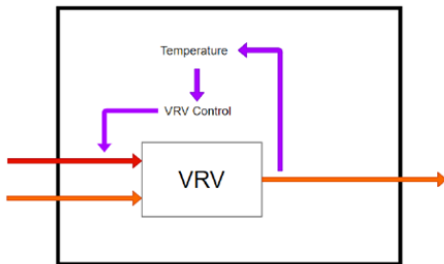
Fig. 2. System representation legend applied to energy flow diagrams in this paper for multi-energy systems.

III. AIR FLOW SUBSYSTEM COMPONENTS

In this section, we show the unified-energy modelling application to different subsystems of the air-flow subsystem components.

A. Variable refrigerant volume

A variable refrigerant volume (VRV) converts ambient air to cooled temperature-controlled air using electricity powered air-conditioning hardware. To categorize the multi-energy nature of this equipment, we specify the energy carrier, the energy input, and the controlled variable of interest in each equipment's case as shown below.



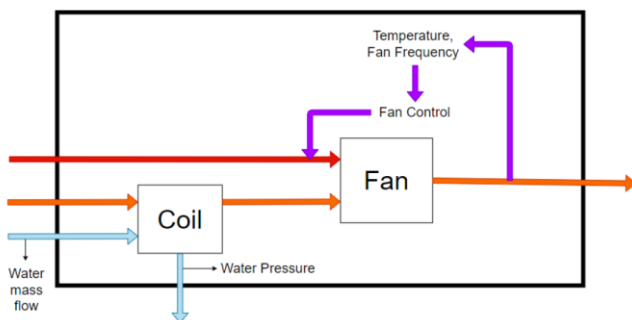
Energy carrier: Air (thermal energy)

Energy input: Electricity

Control variable: Output air temperature

B. Fan Coil Unit

Fan coil unit (FCU) is again a part of the air-flow network and is used to control the temperature of the air as per the fixed temperature set point. It takes in hot or cold water from the storage tank of the energy station and heats/cools down the air by passing it through heat exchangers within the coil. Fan then pushes this hot/cold air into the thermal zone by using electricity to power its motor. Therefore, for FCU electricity becomes the energy input, water and air become energy carriers and output air temperature becomes the controlled variable as stated below.



Equipment: Fan Coil Unit

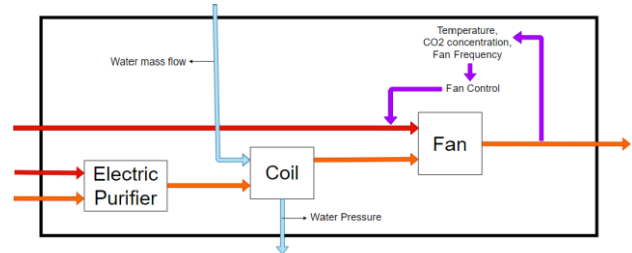
Energy carrier: Air and Water (thermal and fluid energy)

Energy input: Electricity

Control variable: output air temperature

C. Air Handling Unit

Air handling unit (AHU) is an air-conditioning equipment which takes in hot or cold water from the energy station and then exchanges heat with air which is either recycled within the industrial building or drawn from the ambience (outside the building) using valves. For this reason, unlike FCU, AHU has an electric air purifier. Other parts of an AHU are same as a FCU namely fan and coil. Thus, an AHU runs on electricity as its energy input, uses air and water as energy carriers and has output air temperature as its control variable.



Equipment: Air Handling Unit

Energy carrier: Air and Water (thermal and fluid energy)

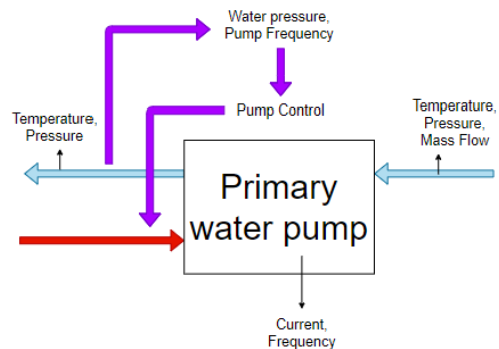
Energy input: Electricity

Control variable: Output air temperature

IV. COMPONENTS OF THE WATER FLOW SUBSYSTEM

A. Water Pump

Water pump is one of the most crucial part of the water flow network which takes in water from the storage tank and pumps it into the coil. In another application, the water pump takes in water from the coil and pumps it into the electric water chiller, gas water boiler, etc. The motors of the pump run on electricity and the output water pressure at a certain mass flow rate is the control variable which in turn is coupled to the pump frequency or the motor speed.



(a) Water Pump

Equipment: Water pump

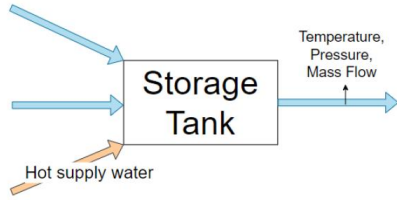
Energy carrier: Water (fluid energy)

Energy input: Gas

Control variable: Output water pressure

*B. Storage Tank*

Storage tank is the buffer of the water flow system in which the heated/cooled water is stored in the energy station till it is drawn by the coil for exchanging heat with the air to heat/cool a predefined thermal zone within the industrial building. Since water is the only energy carrier for the storage tank, it does not require a specific energy input and the temperature and pressure of the output water can be considered as its control variable.



(b) Water Storage Tank

Equipment: Storage tank

Energy carrier: Water (thermal and fluid energy)

Energy input: intrinsic

Control variable: output water temperature and pressure

V. CONVENTIONAL MODEL VS. UNIFIED ENERGY SPACE MODEL

To control a HVAC system, it is important to characterize its behavior which is the aim of modelling [6]. Conventional models typically look at the internal physical states of all components which for thermodynamic processes could be temperature, mass flow rate, pressure, and volume flow rate. These physical states are modelled through partial differential equations which can be linear or non-linear and implicit or explicit. A full-blown system model in terms of states and differential equations can become very complex posing problems of numerical solution and complex leading to time-consuming options which usually hit computational limits of present controllers.

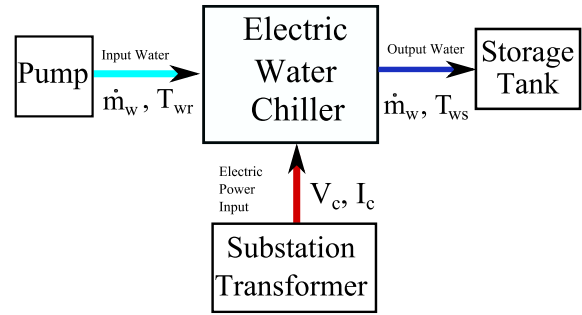
*A. Reduced decoupled model with constant mass flow rate:*

$$\frac{d}{dt} T_{ws} = \frac{1}{\rho_w V_{tc}} \times (-\dot{m}_w (T_{ws} - T_{wr}) - U_c E_c COP + \alpha_h (T_{\infty, t} - T_{ws}))$$

This physical chiller model is reduced and decoupled using constant mass flow rate assumption showing how conventional models get very complex with large number of modelling parameters to be estimated, with five parameters to be estimated from this single decoupled equation above. Another challenge in the conventional models is the lack of unified variables across multi-energy systems.

*B. First and Second Law of Thermodynamics in energy-model:*

An alternate energy-based approach was introduced in [10, 13] wherein based on the first and second law of thermodynamics all the multi-energy systems can be mapped using energy and power variables which result in linear differential equations as shown below:



$$\dot{E} = P - \frac{E}{\tau}$$

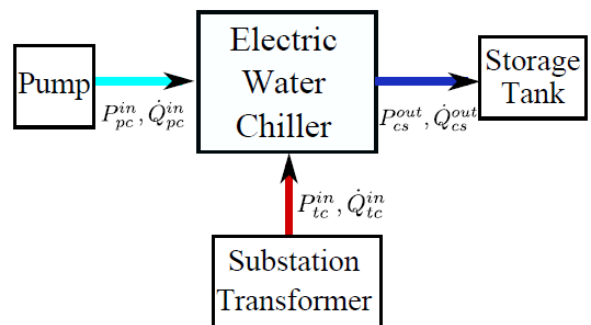
$$\dot{p} = 4Et - \dot{Q}$$

The first equation shows that the rate of change of stored energy in the system is a linear combination of the power input in the system and the energy losses. The second equation shows the dynamic behavior of the stored energy by conceptualizing tangent energy which is the ability of the system to do work and the energy lost in power exchange which is characterized as the rate of change of reactive power in the system. These equations are valid across multi-energy system and maintain their form across disciplines.

The key points of this unified modelling approach use the available terminal measurements to evaluate the interaction variables for the test system of electric water chiller:

- Real power and rate of change of reactive power.
- This avoids the modelling of complex internal states of the test system.

As a test case, the available measurements for the chiller were used for verifying the energy model for the chiller in [2] by validating the two fundamental energy space equations which in-effect reflect the first and second law of thermodynamics.



*C. Energy space model for Chiller with interface variables:*

In order to characterize all the physical components in the energy space, we convert the physical variables into interface variables which are real power and the rate of reactive power as shown below for the case of chiller:

$$\begin{aligned}
 P_{pc}^{in} &= c_w \dot{m}_w T_{wr} + (1/\rho_w) \dot{m}_w Pr_{ci} \\
 P_{cs}^{out} &= c_w \dot{m}_w T_{ws} + (1/\rho_w) \dot{m}_w Pr_{co} \\
 P_{tc}^{in} &= I_c V_c \\
 P &= P_{pc}^{in} + P_{tc}^{in} - P_{cs}^{out} \\
 \dot{Q}_{pc}^{in} &= c_w \left( \frac{d}{dt} \dot{m}_w T_{wr} - \dot{m}_w \frac{d}{dt} T_{wr} \right) \\
 &\quad + (1/\rho_w) \left( Pr_{ci} \frac{d}{dt} \dot{m}_w - \dot{m}_w \frac{d}{dt} Pr_{ci} \right) \\
 \dot{Q}_{cs}^{out} &= c_w \left( \frac{d}{dt} \dot{m}_w T_{ws} - \dot{m}_w \frac{d}{dt} T_{ws} \right) \\
 &\quad + (1/\rho_w) \left( Pr_{co} \frac{d}{dt} \dot{m}_w - \dot{m}_w \frac{d}{dt} Pr_{co} \right) \\
 \dot{Q}_{tc}^{in} &= V_c \frac{d}{dt} I_c - I_c \frac{d}{dt} V_c \\
 \dot{Q} &= \dot{Q}_{pc}^{in} + \dot{Q}_{tc}^{in} - \dot{Q}_{cs}^{out}
 \end{aligned}$$

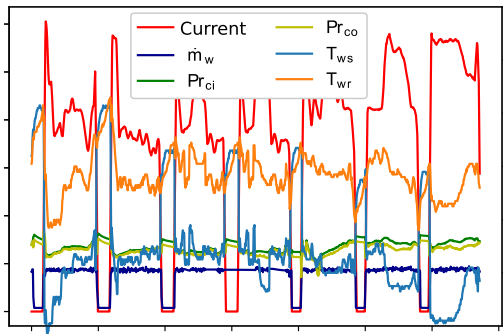
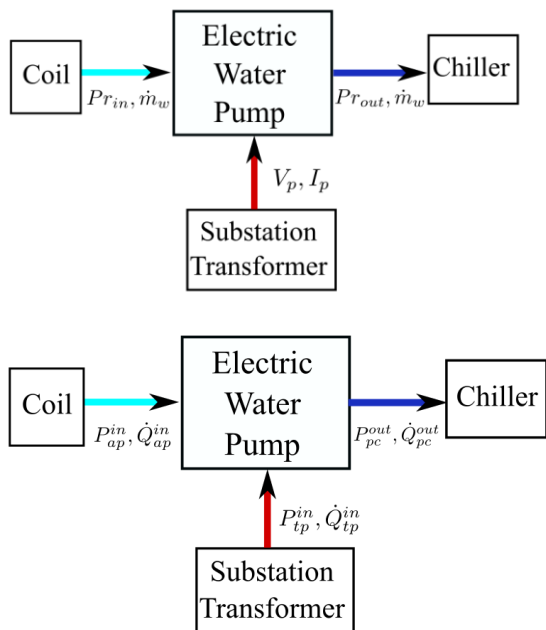


Fig. 3. Available Measurements versus time for electric water chiller [2].

*D. Derivation of unified interaction variables from measured parameters for electric water pump:*

Similar to electric water chiller, we evaluate the interface variables for the electric water pump as shown by the real power and rate of change of reactive power derivations:

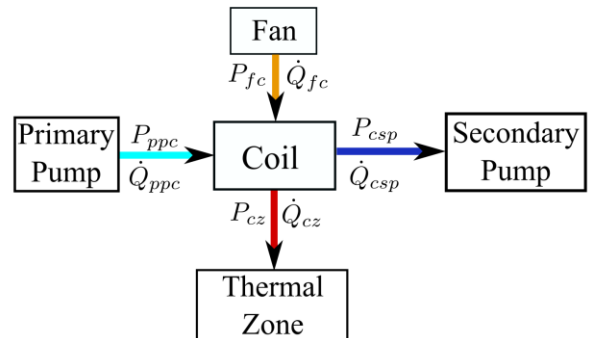


$$\begin{aligned}
 P_{ap}^{in} &= (1/\rho_w) \dot{m}_w Pr_{in} \\
 P_{pc}^{out} &= (1/\rho_w) \dot{m}_w Pr_{out} \\
 P_{tp}^{in} &= I_p V_p \\
 P &= P_{ap}^{in} + P_{tp}^{in} - P_{pc}^{out}
 \end{aligned}$$

$$\begin{aligned}
 \dot{Q}_{ap}^{in} &= (1/\rho_w) \left( Pr_{in} \frac{d}{dt} \dot{m}_w - \dot{m}_w \frac{d}{dt} Pr_{in} \right) \\
 \dot{Q}_{pc}^{out} &= (1/\rho_w) \left( Pr_{out} \frac{d}{dt} \dot{m}_w - \dot{m}_w \frac{d}{dt} Pr_{out} \right) \\
 \dot{Q}_{tp}^{in} &= V_p \frac{d}{dt} I_p - I_p \frac{d}{dt} V_p \\
 \dot{Q} &= \dot{Q}_{ap}^{in} + \dot{Q}_{tp}^{in} - \dot{Q}_{pc}^{out}
 \end{aligned}$$

It must be noted that all these interface variable equations are derived using analogies discussed in Table I above.

*E. Derivation of unified interaction variables from measured parameters for coil:*



Coil is a four-port variable as can be seen in its energy connection diagram below. Therefore, its energy model is more detailed with both air and water carrying both thermal and fluid energy. However, the linear energy space model helps us derive the coil model in energy model using analogies to derive real and rate of reactive powers below:

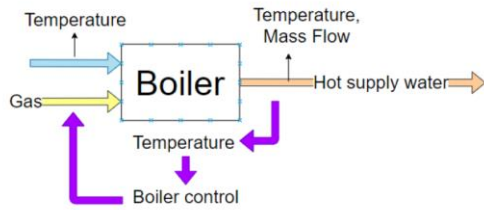
$$\begin{aligned}
 \dot{Q}_{ppc}^{in} &= c_w \left( \frac{d}{dt} \dot{m}_w T_{ws} - \dot{m}_w \frac{d}{dt} T_{ws} \right) \\
 &\quad + (1/\rho_w) \left( Pr_{wci} \frac{d}{dt} \dot{m}_w - \dot{m}_w \frac{d}{dt} Pr_{wci} \right) \\
 \dot{Q}_{cs}^{out} &= c_w \left( \frac{d}{dt} \dot{m}_w T_{ws} - \dot{m}_w \frac{d}{dt} T_{ws} \right) \\
 &\quad + (1/\rho_w) \left( Pr_{wco} \frac{d}{dt} \dot{m}_w - \dot{m}_w \frac{d}{dt} Pr_{wco} \right) \\
 \dot{Q}_{fc}^{in} &= c_a \left( \frac{d}{dt} \dot{m}_a T_{ao} - \dot{m}_a \frac{d}{dt} T_{ao} \right) \\
 &\quad + (1/\rho_a) \left( Pr_{aci} \frac{d}{dt} \dot{m}_a - \dot{m}_a \frac{d}{dt} Pr_{aci} \right) \\
 \dot{Q}_{cz}^{out} &= c_w \left( \frac{d}{dt} \dot{m}_w T_{ws} - \dot{m}_w \frac{d}{dt} T_{ws} \right) \\
 &\quad + (1/\rho_a) \left( Pr_{aco} \frac{d}{dt} \dot{m}_a - \dot{m}_a \frac{d}{dt} Pr_{aco} \right) \\
 \dot{Q} &= \dot{Q}_{ppc}^{in} + \dot{Q}_{fc}^{in} - \dot{Q}_{cs}^{out} - \dot{Q}_{cz}^{out}
 \end{aligned}$$

$$\begin{aligned}
 P_{ppc}^{in} &= c_w \dot{m}_w T_{ws} + (1/\rho_w) \dot{m}_w Pr_{wci} \\
 P_{cs}^{out} &= c_w \dot{m}_w T_{ws} + (1/\rho_w) \dot{m}_w Pr_{wco} \\
 P_{fc}^{in} &= c_a \dot{m}_a T_{ao} + (1/\rho_a) \dot{m}_a Pr_{aci} \\
 P_{cz}^{out} &= c_a \dot{m}_a T_{az} + (1/\rho_a) \dot{m}_a Pr_{aco} \\
 P &= P_{ppc}^{in} + P_{fc}^{in} - P_{csp}^{out} - P_{cz}^{out}
 \end{aligned}$$

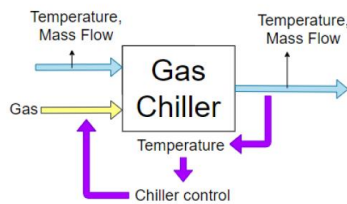
## VI. DISCUSSION

Important observations for simplifying complex multi-energy equipment using unified e-space:

- Fan and pump have analogous energy space model with air and water as respective energy carriers.
- FCU = coil + fan models combined.
- AHU is similar to FCU with additional air input from atmosphere, making input mass flow rate unequal to the output mass flow rate.
- Gas chiller is analogous to electric chiller in e-space, with gas input energy having pressure & volume flow rate as effort & flow variables.
- Gas boiler is modelled like chiller using e-space with negative difference between input and output water temperature.
- Cooling tower is analogous to chiller in e-space, but without electricity input.



(a) Gas Boiler



(c) Gas fired Bromide Refrigerator

## VII. CONCLUSIONS

This work contributes towards the development of a unified modelling theory of the multi-energy systems. The proposed model is applied to an industrial HVAC, which can further lead to designing a physically implementable optimal control of the system for cost, energy, and carbon savings. It finds direct application in characterizing component behavior using interaction variables to overcome modelling inaccuracy from limited measurements and lack of equipment specification from manufacturers. Unified

modelling optimizes physical efficiency in energy conversion processes through analogous derivations of rate of change of reactive power variable while achieving performance objective. Detailed modelling of complex multi-energy systems is avoided here using aggregate variables which are linearly relation in energy space. A common intuitive energy-based language across multi-disciplinary systems is invoked, optimizing chemical, thermal, electrical, and mechanical energy systems without in-depth understanding of complex physical processes.

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