

Study on the Impact of Wind Turbine Relative Position on Power Output Performance

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Abstract— This paper systematically investigates how the wind turbine relative position influences the power output of the wind farm. Firstly, an engineering three-dimensional (3-D) wind turbine wake model is introduced. The novel 3-D wake model is relatively accurate, and can quickly predict wind speed at any downstream position and at any height. Then, based on the wake model, how the hub heights and relative position of wind turbines affect the wind speed and power are deeply studied. The influence depends largely on the specific situation. Especially when decreasing the hub height of downstream wind turbine, the impact of wake can be reduced, but the equivalent wind speed will be decreased as well. Therefore, the influence should be evaluated according to the specific hub height change and the relative downstream distance.

Keywords—wake effect, hub height, relative position, wind energy

I. INTRODUCTION

The demand of energy is an important factor for socio-economic development, which is annually increasing with the global population and economy increase [1]. Among all energies, renewable energy has become the most promising one duo to its inexhaustible and clean characteristics. It is the best energy source to solve the serious pollution, global warming, and energy shortage problems. The wind turbine position and hub height have significant influence on the power of wind farm, which attract an increasing attention.

Alam, et al. [2] reviewed a series of wind turbines and studied the relationship between energy production and hub height. They optimized the hub height for maximizing the energy yield in Saudi Arabian wind conditions. Chen, et al. [3] developed a method to investigate the optimal hub height based on iteration. It has been confirmed again that for the entire wind farm layout, the higher hub height is not always desirable for the optimality [4]. Compared with the identical hub height layout, the layout with several hub heights may increase the power and reduce the cost of unit power. This phenomenon is obvious in complex-terrain wind farms [5]. Chen, et al. [6] investigated the influence of the hub height on power output in a small wind farm. The study showed that for a wind farm, when the turbine number is fixed, the layout with various hub heights may increase the power, but the cost of unit power may also increase. Lee, et al. [7] presented an optimization method for wind turbine hub height. It has been found that when the wind shear exponent and the mean

wind speed increase, the optimized hub height will decrease. The rated and the cut-out wind speeds can seriously affect the optimized hub height. MirHassani and Yarahmadi [8] also studied the effect of hub height on total power. A mathematical algorithm was introduced to simulate the wake decay of Jensen model in terms of interaction matrix. Abdulrahman and Wood [9] optimized the layout of wind farm by considering the selection of wind turbine and the change of hub height. It has been found that applying various wind turbines or hub heights may achieve a better trade-off between power and cost. Vassel-Be-Hagh and Archer [10] assessed how the hub height optimization affects the power generation of the wind farm. Results from Large-Eddy Simulations showed that the hub height optimization was beneficial and the net gain was higher than that calculated by the linear model. Biswas, et al. [11] proposed a multi-objective evolutionary algorithm to obtain optimal windfarm layouts. The maximum power and the wind farm efficiency have been set as the optimization objectives. Song, et al. [12] optimized the wind farm on a flat terrain with multiple hub heights. It has been found that the 3-D optimization produces better solutions than the 2-D optimization.

So far, the research on the impact of wind turbines' relative position is still limited. Most of them only select two or three hub heights, which cannot obtain the actual best results. On the other hand, the wake effect becomes more important and more complicated in the wind turbine hub optimization. Therefore, this paper adopts a 3-D wake model to discuss the impact of relative position on the wind speed and power output of wind turbines.

II. THE WAKE MODEL

A. The three-dimensional wake model

A novel 3-D wake model has been presented by Sun and Yang [13]. The schematic diagram of the proposed model is shown in Fig. 1. In the wake model, the wind velocity $U(x, y, z)$ is relative to the downwind position of (x, y, z) .

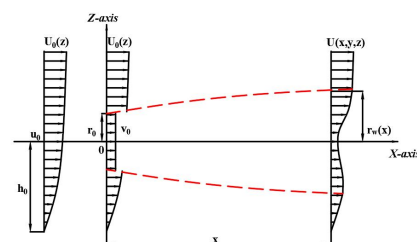


Fig. 1 Schematic diagram of the 3-D wake model [13]

There are three basic assumptions about the 3-D wake model. First, the wind speed deficit at the downstream distance x is Gaussian-distributed. Second, the model is based on the momentum conservation theory. Third, the wind velocity is continuous at the wake boundary. The function of the wake model is shown in Equation (1).

$$U(x, y, z) = A(x) \left(\frac{1}{2\pi\sigma(x)^2} e^{-\frac{y^2 + (z-h_0)^2}{2\sigma(x)^2}} \right) + B(x) + U_0(z) \quad (1)$$

h_0 is the hub height of WT. $A(x)$, $B(x)$, and $\sigma(x)$ are three important parameters that decide the Gaussian shape of the wind deficit. To simplify the calculation, $\sigma(x)$ is defined as $\sigma(x) = \frac{r_w(x)}{C}$. C is a constant and is to be determined with the real operating conditions. Finally, $A(x)$ and $B(x)$ can be expressed by constant C as Equation (2).

$$\begin{cases} A(x) = \frac{Q(x) - \int_{h_0-r_w(x)}^{h_0+r_w(x)} 2\sqrt{r_w(x)^2 - (z-h_0)^2} \cdot U_0(z) dz}{\left(1 - e^{-\frac{C^2}{2}} - \frac{C^2}{2} \cdot e^{-\frac{C^2}{2}}\right)} \\ B(x) = -\frac{A(x) \cdot C^2}{2\pi r_w(x)^2} \cdot e^{-\frac{C^2}{2}} \end{cases} \quad (2)$$

The log law [14] is adopted in this study, which is a common method to describe the inflow. The inflow wind speed varies in the vertical direction, and the downstream wake also varies in the vertical direction.

B. The wind turbine

The wake model has been validated by wind field experiments [15]. In this study the wake model is used for the WT of the UP77-1500 type. The rotor diameter of the WT is 77 m and the hub height is 65 m [16]. The power curve of the WT is shown in Fig. 2.

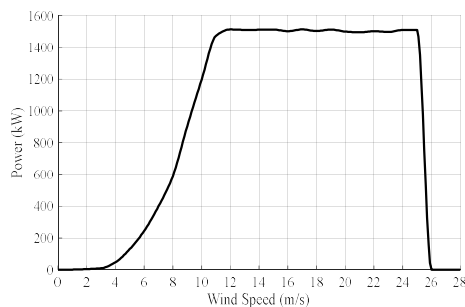


Fig. 2 Power curve

For this WT, the rated power is 1550 kW. The rated wind speed is 12 m/s. The cut-in wind speed is 3 m/s and the cut-off wind speed is 25 m/s. The WT is working in a wind farm. Therefore, the 3-D wake model is developed for this WT and then validated by the measurement data from the wind farm. For this case study, the selected working condition is when the incoming wind speed at the hub height of is 12.8 m/s.

III. IMPACT OF THE DOWNSTREAM DISTANCE

Due to the wake effect in wind farm, the change in hub height not only affects the wind turbine itself, but also other

nearby wind turbines. In the following part, the two adjacent wind turbines will be taken as an example to investigate the impact of hub height change of one wind turbine on another wind turbine. Changing the downstream distance between two wind turbines has no influence on the upstream wind turbine. However, increasing downstream distance between wind turbines can improve the power performance of the downwind turbine. Fig. 3 and Fig. 4 show the impact of downstream distance on the equivalent wind speed and power of the downstream turbine at different hub heights.

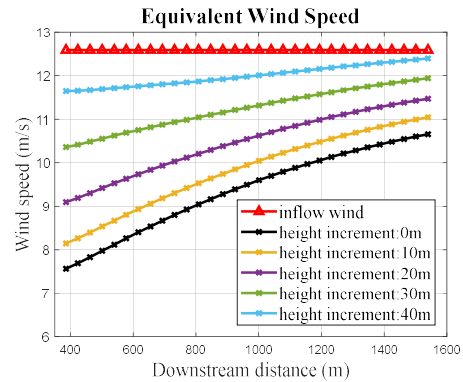


Fig. 3 Impact of downstream distance on the equivalent wind speed of the downstream wind turbine.

Due to the wake effect, the equivalent wind speed of the downstream turbine is much smaller than that of the upstream one. At the original hub height, the equivalent wind speed increases from 7.6 m/s at 385 m downstream position to 10.7 m/s at 1540 m downstream position. If increasing the hub height, the equivalent wind speed will also increase significantly. At 385 m downstream position, when hub height increases by 40 m, the equivalent wind speed increases to 11.7 m/s. At the 1540 m downstream position, even if with a 40 m hub height increment, the equivalent will not return to the incoming wind speed.

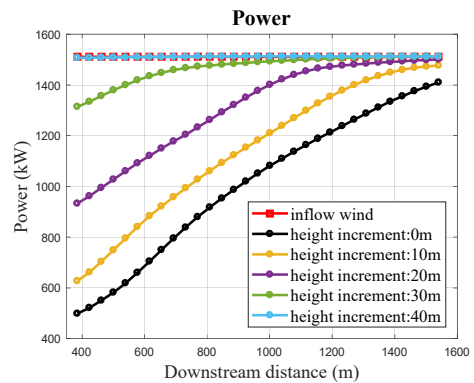


Fig. 4 Impact of downstream distance on the power of the downstream wind turbine.

The effect of downstream distance on power is similar to the effect on equivalent wind speed. At a certain hub height, the increase in the downstream distance can increase the power of downstream wind turbine to a certain extent. However, when the equivalent wind speed increases to the range of rated and cut-out wind speeds, the power performance will not continue to improve. With a hub height increment of 10 m, the power increases from 385 m to 1540 m downstream positions, because the power output has not yet reached the rated power. With the hub height increment of 20 m, the power increases from 940 kW at the downstream position of 385 m to the rated power at around

1400 m, which then maintains at this power. If the hub height further increases, less downstream distance is required to reach the rated power.

Increasing downstream distance is effective to improve the equivalent wind speed of the downstream wind turbine. However, it is not always effective to improve the power. If the equivalent wind speed is larger than the rated wind speed, the power will not continue to increase. With a higher hub height, the downwind turbine can reach the rated power at a relatively close downstream position.

IV. IMPACT OF THE HORIZONTAL DISTANCE

Changing the horizontal distance is another way to improve the performance of the downstream wind turbine. Fig. 5 and Fig. 6 show the impact of horizontal distance on the power output of the downstream wind turbine at various hub heights.

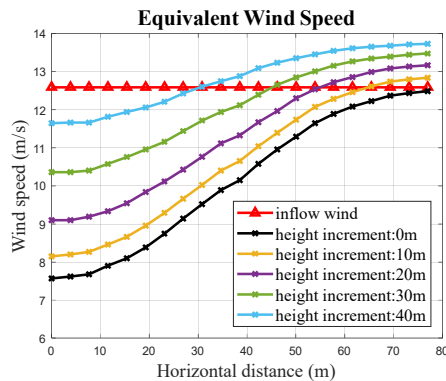


Fig. 5 Impact of horizontal distance on the equivalent wind speed of downstream wind turbine (5D downstream position).

It is clear that when a wind turbine is under the wake effect, increasing the horizontal distance will lead to an increase in the equivalent wind speed, as the wake effect from the upstream wind turbine can be avoided. For the original hub height, the equivalent wind speed increases from 7.6 m/s at the center position to near the incoming wind speed at the 77 m horizontal distance position. For a higher hub height, the turbine can have a larger equivalent wind speed at the same horizontal distance position. When the hub height increases to 40 m, it only needs to move 40 m horizontally to avoid the influence of the wake effect. If the horizontal distance further enlarges to 77 m, the equivalent wind speed will reach 13.7 m/s.

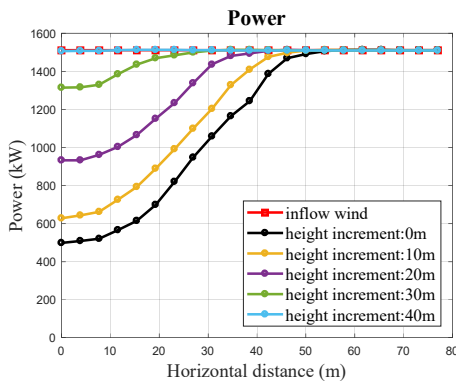


Fig. 6 Impact of horizontal distance on the power of the downstream wind turbine (5D downstream position).

The horizontal distance also has a significant influence on the power output of the downwind turbine. At lower hub heights, increasing the horizontal distance can significantly increase the power output. For example, at the original hub height, the power at the central position is only 500 kW, and as the horizontal distance increases to 55 m, it quickly increases to the rated power of 1513 kW. While when the hub height is larger than 40 m, which maybe different in other cases, the power output performance will not continue to improve.

V. IMPACT OF THE HUB HEIGHTS

Changing hub heights of either the upstream or downstream wind turbine may help improve the power generation performance of the two wind turbines.

A. Changing the upstream wind turbine

Changing the hub height of upwind turbine will have a significant impact on both itself and the downstream wind turbine. Fig. 7 and Fig. 8 show the impact of the upwind turbine hub height on the equivalent wind speed and the power of the downstream turbine. The downwind turbine has the smallest equivalent wind speed when it is fixed at the same height as the upstream one. The wind deficit is maximum at the hub height. Therefore, no matter increasing or decreasing the upstream hub height, the wake impact on the downstream will be reduced. This phenomenon is especially noticeable when the two wind turbines are close together.

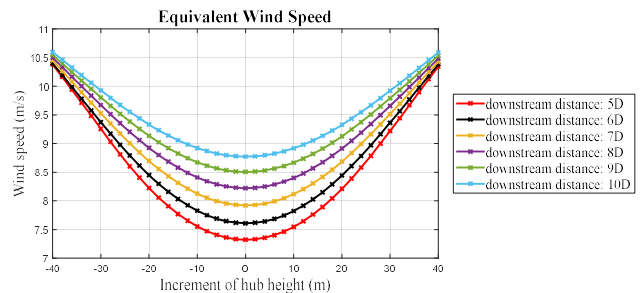


Fig. 7 Impact of the upstream wind turbine hub height on the equivalent wind speed of the downstream wind turbine.

The impact of the hub height of the upstream wind turbine on the power of the downstream one is even more pronounced than the equivalent wind speed. At the 5D downstream position, the power could be improved from 450 kW to 1330 kW. While at the 10D downstream position, the power output can be improved from 830 kW to 1400 kW. One thing to note is that moving the wind turbine a certain height may cause different impacts on the power output. At all downstream distances, the power increments caused by the hub height from 30 m to 40 m are much larger than those caused by the hub height from 0 m to 10 m.

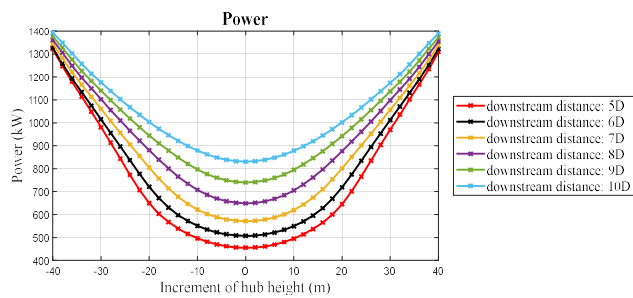


Fig. 8 Impact of the upstream wind turbine's hub height on the power of the downstream wind turbine.

B. Changing the upstream wind turbine

Changing the hub height of the downstream turbine has complicated influence on the downstream turbine itself. Fig. 9 and Fig. 10 demonstrate the impact of hub height of the downstream wind turbine on the equivalent wind speed and power of the downstream wind turbine. When increasing the hub height, the equivalent wind speeds and the power outputs at all downstream positions increase significantly. When decreasing the hub height, where the increment is smaller than zero, the equivalent wind speed does not change monotonically with the hub height. At the 10D downwind position, the equivalent wind speed decreases with hub height. While at the positions before 8D, there is a curvilinear relationship between the hub height increment and the equivalent wind speed. At the 5D downstream position, this phenomenon is most significant.

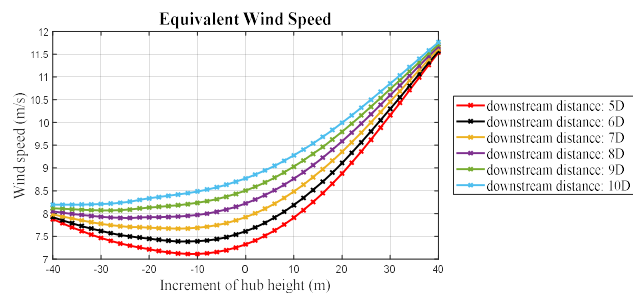


Fig. 9 Impact of the downstream wind turbine's hub height on the equivalent wind speed of the downstream wind turbine.

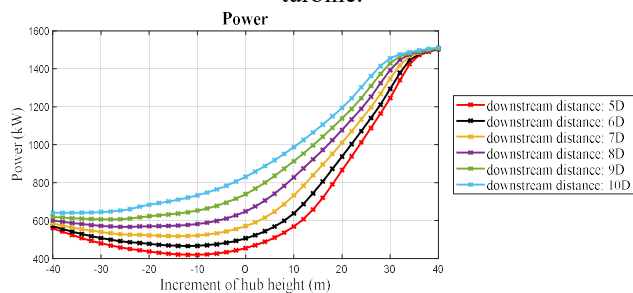


Fig. 10 Impact of the downstream wind turbine's hub height on the power of the downstream wind turbine.

Changing the wind turbine hub height has complicated impacts on the wind farm. Increasing the upstream hub height will decrease the wake impact on the downwind turbine, which will increase the equivalent wind speeds of both upwind and downwind turbines. Increasing the downstream hub height can increase the equivalent wind speed of the downwind turbine. Decreasing the upstream hub height will reduce the equivalent wind speed of the upwind turbine, but this may also reduce the wake's impact on the

downwind turbine and increase the equivalent wind speed. Whether the equivalent wind speed of the downstream wind turbine increases and decreases will depend on the relative distance between two turbines.

Generally, the performance of power output corresponds the equivalent wind speed of the wind turbine. However, due to the power curve, a higher equivalent wind speed will not always lead to a higher power output performance. When the equivalent wind speed is larger than the rated wind speed, the power will maintain the rated power. Therefore, the influence of hub height on the power performance of a wind turbine should be considered from aspects of both the equivalent wind speed and the power curve.

VI. CONCLUSION

This paper investigates the influence of relative positions of wind turbines on power output. The detailed conclusion is as follows. Firstly, an engineering three-dimensional (3-D) wake model is introduced. The 3-D wake model has high accuracy, and can quickly predict the wind speed at any downstream position and at any height. Secondly, the hub height and the layout have a significant impact on the wind speed and power performance of its own and its downstream wind turbines. For two wind turbines, increasing the relative distances in downwind direction and horizontal direction can both avoid the wake impact on the downstream wind turbine, thus increasing the wind speed and power output. Special attention should be paid when changing the hub heights of wind turbines. Increasing the hub height is definitely beneficial to power output. But decreasing the hub height may have different results. The influence from wakes can be reduced, but the equivalent wind speed will be decreased as well. Therefore, the influence should be evaluated according to the specific hub height increment and relative downstream distance.

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